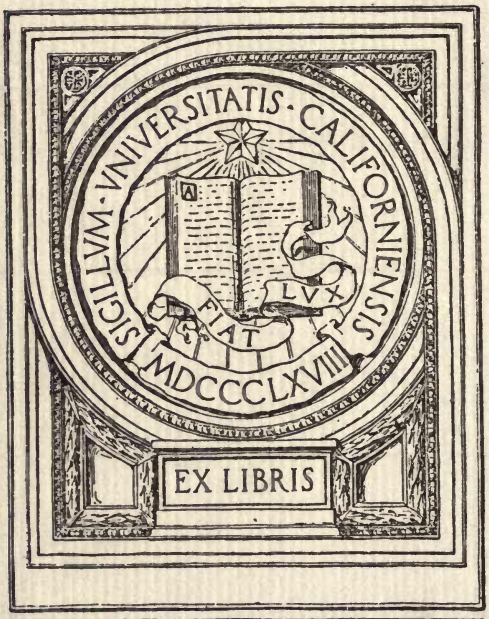

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THERMODYNAMICS

OF THE

STEAM-ENGINE

AND OTHER HEAT-ENGINES.

BY

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PREFACE.

THIS work is designed to give instruction to students in technical schools in the methods and results of the application of thermodynamics to engineering. While it has been considered desirable to follow commonly accepted methods, some parts differ from other text-books, either in substance or in manner of presentation, and may require a few words of explanation.

The general theory or formal presentation of thermodynamics is that employed by the majority of writers, and was prepared with the view of presenting clearly the difficulties inherent in the subject, and of giving familiarity with the processes employed.

In the discussion of the properties of gases and vapors the original experimental data on which the working equations, whether logical or empirical, must be based are given quite fully, to afford an idea of the degree of accuracy attainable in calculations made with their aid. Rowland's determination of the mechanical equivalent of heat has been adopted, and with it his determination of the specific heat of water at low temperatures. The author's "Tables of the Properties of Saturated Steam and Other Vapors" were calculated to accompany this work, and may be considered to be an integral part of it.

The chapters on the flow of gases and vapors and on the injector are believed to present some novel features, especially in the comparisons with experiments.

The feature in which this book differs most from similar works is in the treatment of the steam-engine. It has been deemed advisable to avoid all approximate theories based on

the assumption of adiabatic changes of steam in an engine cylinder, and instead to make a systematic study of steam-engine tests, with the view of finding what is actually known on the subject, and how future investigations and improvements may be made. For this purpose a large number of tests have been collected, arranged, and compared. Special attention is given to the investigations of the action of steam in the cylinder of an engine, considerable space being given to Hirn's researches and to experiments that provide the basis for them. Directions are given for testing engines, and for designing simple and compound engines.

Chapters have been added on compressed-air and refrigerating machines, to provide for the study of these important subjects in connection with the theory of thermodynamics.

Wherever direct quotations have been made, references have been given in foot-notes, to aid in more extended investigations. It does not appear necessary to add other acknowledgment of assistance from well-known authors, further than to say that their writings have been diligently searched in the preparation of this book, since any text-book must be largely an adaptation of their work to the needs of instruction.

C. H. P.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
May, 1889.

PREFACE TO FOURTH EDITION.

A THOROUGH revision of this work has been made to bring it into accord with more recent practice and to include later experimental work. Advantage is taken of this opportunity to make changes in matter or in arrangement which it is believed will make it more useful as a text-book.

C. H. P.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY.
July, 1898.

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THERMODYNAMICS OF THE STEAM-ENGINE.

CHAPTER I.

THERMAL CAPACITIES.

THE object of thermodynamics, or the mechanical theory of heat, is the solution of problems involving the action of heat, and, for the engineer, more especially those problems presented by the steam-engine and other thermal motors. In this work the discussion of the actual nature of heat and the rationale of its various actions will be purposely avoided, and attention will be given rather to the calculation of the results of such actions.

Effects of Heat.—In general the action of heat on a given substance changes all the characteristics of that substance, such as density, temperature, elasticity, conductivity, etc. A comprehensive theory of thermodynamics should make it possible to calculate such changes in any of the characteristics of any substance. In fact, the number of substances for which we have adequate theoretical and experimental knowledge is limited, and only a few characteristics of such substances are commonly included in our discussions.

The substances in which the engineer has the most interest are gases and vapors, more especially air and steam. Fortunately an adequate treatment can be given of these substances for engineering purposes.

First General Principle.—In the development of the theory of thermodynamics it is assumed that if any two characteristics or properties of a substance are known these two, treated as independent variables, will enable us to calculate any third property.

As an example we have from the combination of the laws of Boyle and Gay-Lussac the general equation for gases.

$$pv = RT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which p is the pressure, v is the volume, T is the absolute temperature by the air-thermometer, and R is a constant which for air has the value 53.22 when English units are used. It is probable that this equation led to the general assumption just quoted. That assumption is purely arbitrary, and is to be justified by its results. It may properly be considered to be the first general principle of the theory of thermodynamics; the other two general principles are the so-called first and second laws of thermodynamics, which will be stated and discussed later.

Characteristic Equation.—An equation which gives the relations of the properties of any substance is called the characteristic equation for that substance. The properties appearing in a characteristic equation are commonly pressure, volume, and temperature, but other properties may be used if convenient. The form of the equation must be determined from experiments, either directly or indirectly.

The characteristic equation for a gas is, as already quoted,

$$pv = RT,$$

which may be written also

$$p = \frac{RT}{v}, \qquad v = \frac{RT}{p}, \qquad pv - RT = 0.$$

A similar treatment may be applied to any characteristic

equation in terms of pressure, volume, and temperature, so that we may have

$$T = F(p, v), \qquad p = F_1(T, v), \qquad v = F_2(T, p),$$

or in general

$$f(p, v, T) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If x, y, and z represent any properties or characteristics of a substance, then the first general principle may be expressed algebraically by

$$f(x, y, z) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Specific Pressure.—The pressure is assumed to be a hydrostatic pressure, such as a fluid exerts on the sides of the containing vessel or on an immersed body. The pressure is consequently the pressure exerted *by* the substance under consideration rather than the pressure *on* that substance. For example, in the cylinder of a steam-engine the pressure of the steam is exerted on the piston during the forward stroke and does work on the piston; during the return stroke, when the steam is expelled from the cylinder, it still exerts pressure on the piston and abstracts work from it.

For the purposes of the general theory pressures are expressed in terms of pounds on the square foot for the English system of units. In the metric system the pressure is expressed in terms of kilograms on the square metre. A pressure thus expressed is called the *specific pressure*. In engineering practice other terms are used, such as pounds on the square inch, inches of mercury, millimetres of mercury, atmospheres, or kilograms on the square centimetre.

Specific Volume.—It is convenient to deal with one unit of weight of the substance under discussion, and to consider the volume occupied by one pound or one kilogram of the substance; this is called the *specific volume*, and is expressed in cubic feet or in cubic metres. The specific volume of air

at freezing-point and under the normal atmospheric pressure is 12.39 cubic feet; the specific volume of saturated steam at 212° F. is 26.6 cubic feet; and the specific volume of water is about $\frac{1}{62.4}$, or nearly 0.016 of a cubic foot.

Temperature is commonly measured by aid of a mercurial thermometer which has for its reference-points the freezing-point and boiling-point of water. A centigrade thermometer has the volume of the stem between the reference-points divided into one hundred equal parts called degrees. The Fahrenheit thermometer differs from the centigrade in having one hundred and eighty degrees between the freezing-point and the boiling-point, and in having its zero thirty-two degrees below freezing.

The scale of a mercurial thermometer is entirely arbitrary, and its indications depend on the relative expansion of glass and mercury. Indications of such thermometers, however carefully made, differ appreciably, mainly on account of the varying nature of the glass. For refined investigations thermometric readings are reduced to the air-thermometer, which has the advantage that the expansion of air is so large compared with the expansion of glass that the latter has little or no effect.

It is convenient in making calculations of the properties of air to refer temperatures to the absolute zero of the scale of the air-thermometer. To get a conception of what is meant by this expression we may imagine the air-thermometer to be made of a uniform glass tube with a proper index to show the volume of the air. The position of the index may be marked at boiling-point and at freezing-point as on the mercurial thermometer, and the space between may be divided into one hundred parts or degrees. If the graduations are continued to the closed end of the tube there will be found to be between 273 and 274 of them. It will be shown later that there is reason to suppose that the absolute zero of temperature is 273.7 degrees centi-

grade below the freezing-point of water. Speculations as to the meaning of absolute zero and discussions concerning the nature of substances at that temperature are not now profitable. It is sufficient to know that equations are simplified and calculations are facilitated by this device. For example, if temperature is reckoned from the arbitrary zero of the centigrade thermometer, then the characteristic equation for a perfect gas becomes

$$pv = \left(\frac{1}{\alpha} + t\right)R,$$

in which α is the coefficient of dilatation and $\frac{1}{\alpha} = 273.7$ nearly.

In order to distinguish the absolute temperature from the temperature by the thermometer we shall designate the former by T and the latter by t, bearing in mind that

$$T = t + 273.7^\circ \text{ centigrade},$$
$$T = t + 460.7^\circ \text{ Fahrenheit}.$$

It will appear in the course of the development of the theory of thermodynamics that a scale of temperature can be constructed depending on the fundamental units of length and weight, such as the foot and the pound. Such a scale is properly called the *absolute scale* of temperature, because it does not depend on the properties of any substance (glass, mercury, or air), and because degrees may be given the same value or significance in all parts of the scale. That a degree on the air-thermometer has not the same value in all parts of the scale is shown by the fact that the scale of the air-thermometer differs slightly from that of the absolute thermometer, as will be seen from the table on page 73. The irregularities of the scale of a mercurial thermometer are much greater, so that physical observations are reduced to the scale of the air-thermometer; in engineering tests it is usually suffi-

cient to take the readings of the mercurial thermometer without such a reduction.

In the development of the theory of thermodynamics it will be assumed that temperatures are referred to the absolute scale, though as yet we do not know the nature of that scale or how it is constructed. The apparent indefiniteness accompanying this suspension of judgment is much more than compensated for by the ease with which the absolute scale can be defined when we arrive at the proper place for doing so.

Graphical Representation of the Characteristic Equation.—Any equation with three variables may be represented by a geometrical surface referred to coördinate axes, of which surface the variables are the coördinates. In the case of a perfect gas which conforms to the equation

$$pv = RT$$

the surface is such that each section perpendicular to the axis of T is a rectangular hyperbola (Fig. 1).

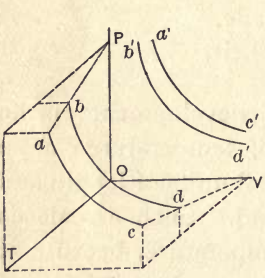

FIG. 1.

Returning now to the general case, it is apparent that the characteristic equation of any substance may be represented by a geometrical surface referred to coördinate axes, since the equation is assumed to contain only three variables; but the surface will in general be less simple in form than that representing the combined laws of Boyle and Gay-Lussac.

If one of the variables, as T, is given a special constant value it is equivalent to taking a section perpendicular to the axis of T; and a plane curve will be cut from the surface, which may be conveniently projected on the (p, v) plane. The reason for choosing the (p, v) plane is that the curves correspond with those drawn by the steam-engine indicator.

Considerable use is made of such thermal curves in explaining thermodynamic conceptions. As a rule, a graphical proc-

ess or representation is merely another way of presenting an idea that has been, or may be, presented analytically; there is, however, an advantage in representing a condition or a change to the eye by a diagram, especially in a discussion which appears to be abstract. A number of thermal curves are explained on page 18.

Standard Temperature.—For many purposes it is convenient to take the freezing-point of water for the standard temperature, since it is one of the reference-points on the thermometric scale; this is especially true for air. But the properties of water change rapidly at and near freezing-point and are very imperfectly known. It has consequently become customary to take 62° F. for the standard temperature for the English system of units; there is a convenience in this, inasmuch as the pound and yard are standards at that temperature. For the metric system 15° C. is used, though the kilogram and metre are standards at freezing-point.

Thermal Unit.—Heat is measured in calories or in British thermal units (B. T. U.). A British thermal unit is the heat required to raise one pound of water from 62° F. to 63° F.; in like manner a calorie is the heat required to raise one kilogram of water from 15° C. to 16° C.

The calorie is often defined as the heat required to raise a kilogram of water from freezing-point to one degree centigrade; and a B. T. U. is correspondingly defined as the heat required to raise one pound of water from freezing-point to 33° F. The objection to the use of the freezing-point as the standard temperature—namely, the uncertainty as to the properties of water at that temperature—applies even more forcibly here. The whole subject will be discussed later in connection with Rowland's determination of the mechanical equivalent of heat.

The thermal unit, or the calorie, should depend on the absolute scale of temperature, but for practical purposes the scale of the air-thermometer is sufficient.

Thermal Capacities.—The amount of heat required to change by unity any quality of a unit of weight of a substance

is called the thermal capacity corresponding to the given change.

Three thermal capacities have received names, i. e., specific heat at constant volume, specific heat at constant pressure, and latent heat of expansion.

Specific Heat is the number of thermal units required to raise a unit of weight of a given substance one degree of temperature. The specific heat of water at the standard temperature is, of course, unity.

If the specific heat of a given substance is constant, then the heat required to raise one pound through a given range of temperature is the product of the specific heat by the increase of temperature. Thus if c is the specific heat and $t - t_1$ is the range of temperature the heat required is

$$Q = c(t - t_1), \text{ and } c = \frac{Q}{t - t_1}.$$

If the specific heat varies the amount of heat must be obtained by integration—that is,

$$Q = \int c\,dt,$$

and conversely

$$c = \frac{dQ}{dt}.$$

It is customary to distinguish two specific heats for perfect gases; specific heat at constant pressure and specific heat at constant volume, which may be represented by

$$c_p = \left(\frac{dQ}{dt}\right)_p, \text{ and } c_v = \left(\frac{dQ}{dt}\right)_v.$$

The subscript attached to the parenthesis indicates the property which is constant during the change.

It is evident that the specific heats just expressed are partial differential coefficients. Partial differentials may often

be recognized by their positions in equations or by the context. Sometimes they are indicated by parentheses, as above, but without the subscripts; or they may be indicated by a special type, as

$$\frac{\partial Q}{\partial t}.$$

In thermodynamics several variables are often introduced, each of which may be a function of two independent variables; it is consequently convenient, if not essential, to indicate a partial differential by a parenthesis and a subscript, as above.

Latent Heat of Expansion is the amount of heat required to increase the volume of a unit of weight of the substance by one cubic foot, or one cubic metre, at constant temperature. It may be represented by

$$l = \left(\frac{dQ}{dv}\right)_t.$$

General Equations of the Effects Produced by Heat.— In conformity with the first general principle the heat required to produce a change in a unit of weight of a given substance may be expressed as a function of any two properties of the substance; thus we may have

$$Q = F_1(t, v), \quad Q = F_2(t, p), \text{ or } Q = F_3(p, v), \quad . \quad (4)$$

Differentiating the several equations (4), we have

$$dQ = \left(\frac{dQ}{dt}\right)_v dt + \left(\frac{dQ}{dv}\right)_t dv, \quad . \quad . \quad . \quad . \quad (5a)$$

$$dQ = \left(\frac{dQ}{dt}\right)_p dt + \left(\frac{dQ}{dp}\right)_t dp, \quad . \quad . \quad . \quad . \quad (6a)$$

$$dQ = \left(\frac{dQ}{dp}\right)_v dp + \left(\frac{dQ}{dv}\right)_p dv. \quad . \quad . \quad . \quad . \quad (7a)$$

In equation (5*a*) the partial differential coefficients $\left(\frac{dQ}{dt}\right)_v$ and $\left(\frac{dQ}{dv}\right)_t$ are the specific heat at constant volume and the latent heat of expansion; they may be replaced by c_v and l, giving

$$dQ = c_v dt + l dv. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In like manner the differential coefficient $\left(\frac{dQ}{dt}\right)_p$ in equation (6*a*) may be replaced by c_p. The differential coefficient $\left(\frac{dQ}{dp}\right)_t$ is also a thermal capacity, and represents the amount of heat that must be added to increase the pressure to the extent of one pound per square foot (or one kilogram per square metre). No name has been given to this thermal capacity; it may be represented by the letter m, and equation (6*a*) becomes

$$dQ = c_p dt + m dp. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Finally, $\left(\frac{dQ}{dp}\right)_v$ may be represented by n, and $\left(\frac{dQ}{dv}\right)_p$ by o, both being thermal capacities without names, and equation (7*a*) becomes

$$dQ = n dp + o dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Relations of the Thermal Capacities.—The three equations (5), (6), and (7) show the changes produced by the addition of an amount of heat dQ to a unit of weight of a substance, the difference coming from the methods of analyzing the changes. We may conveniently find the relations of the several thermal capacities by the method of undetermined coefficients. Thus equating the right-hand members of equations (5) and (6),

$$c_v dt + l dv = c_p dt + m dp. \quad . \quad . \quad . \quad . \quad (8)$$

From the first general principle we have

$$v = F(pT),$$

from which, by differentiating we have

$$dv = \left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp,$$

which substituted in (8) gives

$$c_p dt + m dp = c_v dt + l\left[\left(\frac{dv}{dt}\right)_p dt + \left(\frac{dv}{dp}\right)_t dp\right].$$

$$\therefore c_p dt + m dp = \left[c_v + l\left(\frac{dv}{dt}\right)_p\right] dt + l\left(\frac{dv}{dp}\right)_t dp. \quad . \quad . \quad (9)$$

It will be noted that, as T differs from t only by the addition of a constant, the differential dt may be used in all cases, whether we are dealing with absolute temperatures or temperatures on the ordinary thermometer.

In equation (9) p and T are independent variables, and each may have all possible values; consequently we may equate like coefficients.

$$\therefore c_p = c_v + l\left(\frac{dv}{dt}\right)_p, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$l\left(\frac{dv}{dt}\right)_p = c_p - c_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Again, equating the remaining coefficients,

$$l\left(\frac{dv}{dp}\right)_t = m. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Again, we have

$$p = F_1(T, v),$$

from which

$$dp = \left(\frac{dp}{dt}\right)_v dt + \left(\frac{dp}{dv}\right)_t dv, \quad . \quad . \quad . \quad . \quad (13)$$

which substituted in (8) gives

$$c_p dt + m\left[\left(\frac{dp}{dt}\right)_v dt + \left(\frac{dp}{dv}\right)_t dv\right] = c_v dt + l dv,$$

$$\left[c_p + m\left(\frac{dp}{dt}\right)_v\right] dt + m\left(\frac{dp}{dv}\right)_t dv = c_v dt + l dv.$$

Equating like coefficients,

$$c_p + m\left(\frac{dp}{dt}\right)_v = c_v, \quad \ldots\ldots \quad (14)$$

or

$$-m\left(\frac{dp}{dt}\right)_v = c_p - c_v. \quad \ldots\ldots \quad (15)$$

From equations (6) and (7)

$$c_p dt + m dp = n dp + o dv. \quad \ldots\ldots \quad (16)$$

And from the equation

$$T = F_3(v, p)$$

we have

$$dt = \left(\frac{dt}{dv}\right)_p dv + \left(\frac{dt}{dp}\right)_v dp,$$

which substituted in equation (16) gives

$$c_p\left(\frac{dt}{dv}\right)_p dv + c_p\left(\frac{dt}{dp}\right)_v dp + m dp = n dp + o dv.$$

Equating coefficients of dv,

$$o = c_p\left(\frac{dt}{dv}\right)_p. \quad \ldots\ldots \quad (17)$$

Finally, from equations (5) and (7) we have

$$c_v dt + l dv = n dp + o dv.$$

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Substituting for the value of dt, as above,

$$c_v\left(\frac{dt}{dv}\right)_p dv + c_v\left(\frac{dt}{dp}\right)_v dp + ldv = ndp + odv.$$

Equating coefficients of dp,

$$n = c_v\left(\frac{dt}{dp}\right)_v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

For convenience the several relations of the thermal capacities may now be assembled as follows:

$$l\left(\frac{dv}{dt}\right)_p = c_p - c_v, \qquad -m\left(\frac{dp}{dt}\right)_v = c_p - c_v,$$

$$n = c_v\left(\frac{dt}{dp}\right)_v, \qquad o = c_p\left(\frac{dt}{dv}\right)_p,$$

$$m = l\left(\frac{dv}{dp}\right)_t.$$

They are the necessary algebraic relations of the literal functions growing out of the first general principle, and are independent of the scale of temperature, or of any other theoretical or experimental principle of thermodynamics other than the one already stated—namely, that any two properties of a given substance, treated as independent variables, are sufficient to allow us to calculate any third property.

Of the six thermal capacities the specific heat at constant pressure is the only one that is commonly known by direct experiment. For perfect gases this thermal capacity is a constant, and, further, the ratio of the specific heats

$$\frac{c_p}{c_v} = k$$

is a constant, so that c_v is readily calculated. The relations of the thermal capacities allow us to calculate values for the other thermal capacities, l, m, n, and o, provided that we can

first determine the several partial differential coefficients which appear in the proper equations. But for a perfect gas the characteristic equation is

$$pv = RT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

from which we have

$$\left(\frac{dv}{dt}\right)_p = \frac{R}{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

consequently

$$l = \frac{p}{R}(c_p - c_v), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

so that l may be readily calculated for any pressure. Other partial differential coefficients can be deduced and substituted, if desired, to provide means of calculating the other thermal capacities, but that properly belongs in the discussion of perfect gases and will be considered in the proper place.

For a different substance—for example, superheated steam—it will appear that the ratio of the specific heats is not a constant, and, further, the form of the characteristic equation is different. The values of the partial differential coefficients must of course be found for each special case, and the use to be made of the relations of the thermal capacities will depend on circumstances.

CHAPTER II.

FIRST LAW OF THERMODYNAMICS.

THE formal statement of the first law of thermodynamics is:

Heat and mechanical energy are mutually convertible, and heat requires for its production and produces by its disappearance a definite number of units of work for each thermal unit.

This law, which may be considered to be the second general principle of thermodynamics, is the statement of a well-determined physical fact. It is a special statement of the general law of the conservation of energy, i. e., that energy may be transformed from one form to another, but can neither be created nor destroyed. It should be stated, however, that the general law of conservation of energy, though universally accepted, has not been proved by direct experiment in all cases; there may be cases that are not susceptible of so direct a proof as we have for the transformation of heat into work.

The best determinations of the mechanical equivalent of heat were made by Rowland, whose work will be considered in detail in connection with the properties of steam and water. From his work it appears that 778 foot-pounds of work are required to raise one pound of water from 62° to 63° Fahrenheit; this value of the mechanical equivalent of heat is now commonly accepted by engineers, and is verified by the latest determinations by Joule and other experimenters.

The values of the mechanical equivalent of heat for the English system and for the metric system are:

1 B. T. U. = 778 foot-pounds.
1 calorie = 426.9 metre-kilograms.

This physical constant is commonly represented by the letter J; the reciprocal is represented by A.

In older works on thermodynamics the values of J are commonly quoted as 772 for the English system and 424 for the metric system. The error of these values is about one per cent.

Effects of the Transfer of Heat.—Let a quantity of any substance of which the weight is one unit—i. e., one pound or one kilogram—receive a quantity of heat dQ. It will, in general, experience three changes, each requiring an expenditure of energy. They are: (1) The temperature will be raised, and, according to the theory that sensible heat is due to the vibrations of the particles of the body, the kinetic energy will be increased. Let dS represent this change of sensible heat or vibration work expressed in units of work. (2) The mean positions of the particles will be changed; in general the body will expand. Let dI represent the units of work required for this change of internal potential energy, or work of disgregation. (3) The expansion indicated in (2) is generally against an external pressure, and to overcome the same—that is, for the change in external potential energy—there will be required the work dW.

If during the transmission no heat is lost, and if no heat is transformed into other forms of energy, such as sound, electricity, etc., then the first law of thermodynamics gives

$$dQ = A(dS + dI + dW). \quad . \quad . \quad . \quad . \quad (22)$$

It is to be understood that any or all of the terms of the equation may become zero or may be negative. If all the terms become negative heat is withdrawn instead of added, and dQ is negative. It is not easy to distinguish between the vibration work and the disgregation work, and for many purposes it is unnecessary; consequently they are treated together under the name of intrinsic energy, and we have

$$dQ = A(dS + dI + dW) = A(dE + dW). \quad . \quad (23)$$

The inner work, or intrinsic energy, depends on the state of the body, and not at all on the manner by which it arrived at that state; just as the total energy of a falling body, with reference to a given plane consisting of kinetic energy and potential energy, depends on the velocity of the body and the height above the plane, and not on the previous history of the body.

The external work is assumed to be done by a fluid pressure; consequently

$$dW = pdv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$W = \int_{v_1}^{v_2} pdv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where v_2 and v_1 are the final and initial volumes.

In order to find the value of the integral v in equation (25) it is necessary to know the manner in which the pressure varies with the volume. Since the pressure may vary in different ways, the external work cannot be determined from the initial and final states of the body; consequently the heat required to effect a change from one state to another depends on the manner in which the change is effected.

Assuming the law of the variation of the pressure and volume to be known, we may integrate thus:

$$Q = A\left(E_2 - E_1 + \int_{v_1}^{v_2} pdv\right). \quad . \quad . \quad . \quad . \quad . \quad (26)$$

In order to determine E for any state of a body it would be necessary to deprive it entirely of vibration and disgregation energy, which would of course involve reducing it to a state of absolute cold; consequently the direct determination is impossible. However, in all our work the substances operated on are changed from one state to another, and in each state the intrinsic energy depends on the state only; consequently the change of intrinsic energy may be determined from the initial and final states only, without knowing the manner of change from one to the other.

All succeeding equations will be arranged to involve differences of energy only, and the hypothesis involved in a separation into vibration and disgregation work avoided.

Thermal Lines.—The external work can be determined only when the relations of p and v are known, or, in general, when the characteristic equation is known. It has already been shown that in such case the equation may be represented by a geometrical surface, on which so-called thermal lines can be drawn representing the properties of the substance under consideration. These lines are commonly projected on the (p, v) plane. It is convenient in many cases to find the relation of p and v under a given condition and represent it by a curve drawn directly on the (p, v) plane.

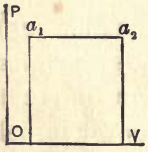

FIG. 2.

Lines of Equal Pressure.—The change of condition takes place at constant pressure, and consists of a change of volume, as represented in Fig. 2. The tracing-point moves from a_1 to a_2, and the volume changes from v_1 to v_2. The work done is represented by the rectangular area under a_1a_2, or by

$$W = p\int_{v_1}^{v_2} dv = p(v_2 - v_1).$$

During the change the temperature may or may not change; the diagram shows nothing concerning it.

Lines of Equal Volume.—The pressure increases at constant volume, and the tracing-point moves from a_1 to a_2. The temperature usually increases meanwhile. Since dv is zero,

$$W = \int_{v_1}^{v_2} p\,dv = 0.$$

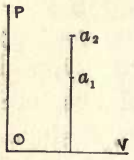

FIG. 3.

Isothermal Lines or Lines of Equal Temperature.—The temperature remains constant, and a line is drawn, usually convex, toward the axis OV. The pressure of a mixture of a

liquid and its vapors is constant for a given temperature; consequently the isothermal for such a mixture is a line of equal pressure, represented by Fig. 2. The isothermal of a perfect gas, on the other hand, is an equilateral hyperbola, as appears from the law of Boyle, which may be written

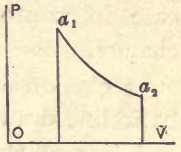

FIG. 4.

$$pv = C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Isodynamic or Isoenergic Lines are lines representing changes during which the intrinsic energy remains constant. Consequently all the heat received is transformed into external work. It will be seen later that the isodynamic and isothermal lines for a gas are the same.

Adiabatic Lines.—A very important problem in thermodynamics is to determine the behavior of a substance when a change of condition takes place in a non-conducting vessel. During the change—for example, an increase of volume or expansion—some of the heat in the substance may be changed into work; but no heat is transferred to or from the substance through the walls of the containing vessel. Such changes are called *adiabatic* or *isoentropic* changes.

Very rapid changes of dry air in the cylinder of an air-compressor or a compressed-air engine are very nearly adiabatic. Adiabatic changes never occur in the cylinder of a steam-engine on account of the rapidity with which steam is condensed on or vaporized from the cast-iron walls of the cylinder.

Since there is no transmission of heat to (or from) the working substance, equation (26) becomes

$$Q = A(E_2 - E_1 + \int_v^{v_2} p dv),$$

$$E_1 - E_2 = \int_{v_1}^{v_2} p dv;$$

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that is, the external work is done wholly at the expense of the intrinsic energy of the working substance, as must be the case in conformity with the assumption of an adiabatic change.

Relation of Adiabatic and Isothermal Lines.—An adiabatic line drawn on the (p, v) plane is steeper than an isothermal line at the point of intersection. This is easily shown for a substance that expands with a rise of temperature. Thus let ab and cd (Fig. 5) represent an adiabatic and an isothermal line crossing at p. The substance when in the condition represented by the point p has a certain volume, pressure, and temperature. The isothermal change represented by pd, is at constant temperature. On the other hand, the adiabatic change represented by pb, is accompanied by a loss of intrinsic energy; but the intrinsic energy is the sum of the vibration energy and the disgregation energy, and, in general, a loss of intrinsic energy means a diminution in both vibration and disgregation energy. Now the vibration energy is represented by the temperature, and the temperature will fall when the vibration energy decreases. Consequently the temperature at b is less than the temperature at p, and therefore is less than the temperature at d. But b and d are at the same pressure, and consequently the volume at b is less than the volume at d; that is, the adiabatic line is the steeper.

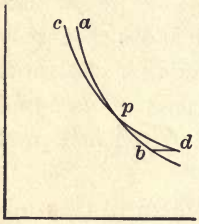

FIG. 5.

Graphical Representations of Change of Intrinsic Energy.—Professor Rankine first used a graphical method of representing a change of intrinsic energy, employing adiabatic lines only, as follows:

Suppose that a substance is originally in the state A (Fig. 6), and that it expands adiabatically; then the external work is done at the expense of the intrinsic energy, hence if the expansion has proceeded to A_1 the area AA_1a_1a, which repre-

sents the external work, also represents the change of intrinsic energy. Suppose that the expansion were to continue indefinitely: then the adiabatic will approach the axis OV indefinitely, and the area representing the work will be included between the curve $A\alpha$ produced indefinitely, the ordinate Aa, and the axis OV; this area will represent all the work that can be obtained by the expansion of the substance; and if it be admitted that during the expansion all the intrinsic energy is transformed into work, so that at the end the intrinsic energy is zero, it represents also the intrinsic energy. In cases for which the equation of the adiabatic can be found it is easy to show that

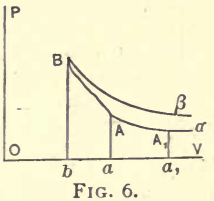

FIG. 6.

$$E_1 = \int_{v_1}^{\infty} p dv$$

is a finite quantity; and in any case, if we admit an absolute zero of temperature, it is evident that the intrinsic energy cannot be infinite. On the other hand, if an isothermal curve were treated in the same way the area would be infinite, since heat would be continually added during the expansion.

Now suppose the body to pass from the condition represented by A to that represented by B, by any path whatever—that is, by any succession of changes whatever—for example, that represented by the irregular curve AB. The intrinsic energy in the state B is represented by the area $VbB\beta$. The change of intrinsic energy is represented by the area $\beta BbaA\alpha$, and this area does not depend on the form of the curve AB. This graphical process is only another way of saying that the intrinsic energy depends on the state of the substance only, and that change of intrinsic energy depends on the final and initial states only.

Another way of representing change of intrinsic energy by

aid of isodynamic lines avoids an infinite diagram. Suppose the change of state to be represented by the curve AB (Fig. 7). Draw an isodynamic line AC through the point A, and an adiabatic line BC through B, intersecting at C. Then the area $ABba$ represents the external work, and the area $bBCc$ represents the change of intrinsic energy; for if the body be allowed to expand adiabatically till the intrinsic energy is reduced to its original amount at the condition represented by A the external work $bBCc$ will be done at the expense of the intrinsic energy. And further, since the intrinsic energy is constant for all points on the isodynamic line through A, and in like manner is constant for points on the line through B, there will be the same change of intrinsic energy in passing from a condition represented by any point of the line through A to any point of the line through B; consequently if through any point, as D of the upper line, an adiabatic DE be drawn the area $dDEe$ will be equal to $bBCc$, and will equally represent the change of intrinsic energy from the point A to the point B.

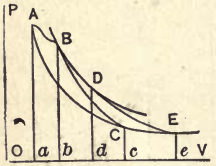

FIG. 7.

Entropy.—If a body have its condition represented by the point e of the isothermal aa_1 (Fig. 8) it will have a definite temperature, which will be the same so long as its condition is represented by some point on aa_1, as, for example, e_1, though the volume and pressure will meanwhile have varied. Should the temperature change, the condition will be represented by some point, as f, on another isothermal bb_1. There will evidently be the same change of temperature in passing from e to f as from e_1 to f_1; that the changes of volume and pressure, external work, and intrinsic energy are different does not affect the statement concerning the temperature. In like manner it is indifferent how or at what part of the diagram the transfer from bb_1 to cc_1 is accomplished; the same change of temperature must occur.

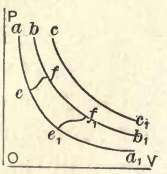

FIG. 8.

In the same way isoenergic changes will be represented by

the motion of a point along a curve of constant energy; and there will be a definite change of energy in passing from a curve of constant energy to the next curve of a system of isoenergic curves.

Conversely, if we have any system of curves it is reasonable to suppose that there must be a constant change of some sort in passing from one such curve to the next of the same system. A series of adiabatic curves as represented by Fig. 9, is such a system of curves, and we may consider that there is the same change in passing from e to f as in passing from e_1 to f_1, e being any point on the curve aa_1 and f being any point on the next curve bb_1. It will appear in our future work that a definite form may be assigned to the function representing the change that occurs in passing from one adiabatic line of a given substance to another adiabatic line of the same substance, and that a numerical calculation may be made representing that change. It will further appear that the form of the function and the corresponding numerical calculation will depend on the nature of the substance, and will be different for different substances. For example, the form of the function for steam is radically different from the form for air. The form of the function is consequently a property of the substance, just as are specific volume and intrinsic energy. The fact that the form of the function cannot be intelligently explained now, and that the nature is different from other properties thus far discussed, is no reason why we should not provide for the function, give it a name, and include it in our equations. In the proper place the form and use of the function will be explained.

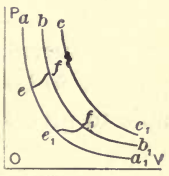

FIG. 9.

The name given to this function or property which remains constant during an adiabatic change is *entropy*. It is commonly represented by ϕ.

In the process of establishing an absolute scale of temperature we shall show how a system of isothermal lines can be

drawn at intervals of one degree of temperature. At the same time, and by a similar method, a system of adiabatics will be drawn, each one unit of entropy from the next. Both systems of lines will be made to depend on the foot-pound.

It may be suggested further that thus far no adequate idea of the nature of temperature has been obtained, the assertion that temperature is in some way connected with the vibration energy of a body being too indefinite for this purpose. Though an accepted dictum of science, it remains to a certain extent speculative. Again, for the engineer it is more important to be able to calculate changes and effects than to be able to give a philosophical account of their real nature.

CHAPTER III.

SECOND LAW OF THERMODYNAMICS.

Heat-engines are engines by which heat is transformed into work. All actual engines used as motors go through continuous cycles of operations, which periodically return things to the original conditions. All heat-engines are similar in that they receive heat from some *source*, transform part of it into work, and deliver the remainder (minus certain losses) to a *refrigerator*.

The source and refrigerator of a condensing steam-engine are the furnace and the condenser. The boiler is properly considered as a part of the engine, and receives heat from the source.

Carnot's Engine.—It is convenient to discuss a simple ideal engine, first described by Carnot.

Let P of Fig. 10 represent a cylinder with non-conducting walls, in which is fitted a piston, also of non-conducting material, and moving without friction; on the other hand, the bottom of the cylinder is supposed to be of a material that is a perfect conductor. There is a non-conducting stand C on which the cylinder can be placed while adiabatic changes take place. The source of heat A at a temperature t is supposed to be so maintained that in operations during which the cylinder is placed on it, and draws heat from it, the temperature is unchanged. The refrigerator B at the temperature t_1 in like manner can with-

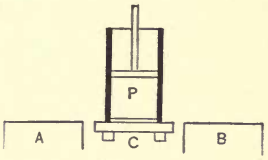

FIG. 10.

draw heat from the cylinder, when it is placed on it, at a constant temperature.

Let there be a unit of weight (for example, one pound) of a certain substance in the cylinder at the temperature t of the source of heat. Place the cylinder on the source of heat A (Fig. 10), and let the substance expand at the constant temperature T, receiving heat from the source A.

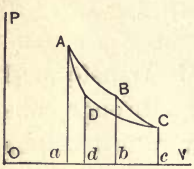

FIG. 11.

If the first condition of the substance be represented by A (Fig. 11), then the second will be represented by B, and AB will be an isothermal. If E_a and E_b are the intrinsic energies at A and B, and if W_{ab}, represented by the area $aABb$, be the external work, the heat received from A will be

$$Q = A(E_b - E_a + W_{ab}).$$

Now place the cylinder on the stand C (Fig. 10), and let the substance expand adiabatically until the temperature is reduced to T_1, that of the refrigerator, the change being represented by the adiabatic BC (Fig. 11). If E_c is the intrinsic energy at C, then, since no heat passes into or out of the cylinder,

$$0 = A(E_c - E_b + W_{bc}),$$

where W_{bc} is the external work represented by the area $bBCc$. Place the cylinder on the refrigerator B, and compress the substance till it passes through the change represented by CD, yielding heat to the refrigerator so that the temperature remains constant. If E_d is the intrinsic energy at D, then

$$-Q_1 = A(E_d - E_c - W_{cd})$$

is the heat yielded to the refrigerator, and W_{cd}, represented by the area $cCDd$, is the external work, which has a minus sign, since it is done on the substances.

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The point D is determined by drawing an adiabatic from A to intersect an isothermal through C. The process is completed by compressing the substance while the cylinder is on the stand C (Fig. 10) till the temperature rises to T, the change being represented by the adiabatic DA. Since there is no transfer of heat,

$$0 = A(E_a - E_d - W_{da}).$$

Adding together the several equations, member to member,

$$Q - Q_1 = A(W_{ab} + W_{bc} - W_{cd} - W_{da});$$

or, if W be the resulting work represented by the area $ABCD$, then

$$Q - Q_1 = AW;$$

that is, the difference between the heat received and the heat delivered to the refrigerator is the heat transformed into work.

A Reversible Engine is one that may run either in the usual manner, transforming heat into work, or reversed, describing the same cycle in the opposite direction, and transforming work into heat.

A Reversible Cycle is the cycle of a reversible engine.

Carnot's engine is reversible, the reversed cycle being $ADCBA$ (Fig. 11), during which work is done by the engine on the working substance. The engine then draws from the refrigerator a certain quantity of heat, it transforms a certain quantity of work into heat, and delivers the sum of both to the source of heat.

No actual heat-engine is reversible in the sense just stated, for when the order of operations can be reversed, changing the engine from a motor into a pump or compressor, the reversed cycle differs from the direct cycle. For example, the valve-gear of a locomotive may be reversed while the train is running, and then the cylinders will draw gases from the smoke-box, compress them, and force them into the boiler.

The locomotive as ordinarily built is seldom reversed in this way, as the hot gases from the smoke-box injure the surfaces of the valves and cylinders. Some locomotives have been arranged so that the exhaust-nozzles can be shut off and steam and water supplied to the exhaust-pipe, thus avoiding the damage from hot gases when the engine is reversed in this way. Such an engine may then have a reversed cycle, drawing steam into the cylinders, compressing and forcing it into the boiler; but in any case the reversed cycle differs from the direct cycle, and the engine is not properly a reversible engine.

A Closed Cycle is any cycle in which the final state is the same as the initial state. Fig. 12 represents such a cycle made up of four curves of any nature whatever. If the four curves are of two species only, as in the diagram representing the cycle of Carnot's engine, the cycle is said to be simple. In general, we shall have for a cycle like that of Fig. 12

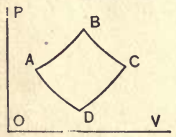

FIG. 12.

$$Q_{ab} + Q_{bc} - Q_{cd} - Q_{da} = \Sigma Q = A\Sigma W$$
$$= A(W_{ab} + W_{bc} - W_{cd} - W_{da}).$$

A closed curve of any form may be considered to be the general form of a closed cycle, as that in Fig. 13. For such a cycle we have $\int dQ = A \int dW$, which is one more way of stating the first law of thermodynamics.

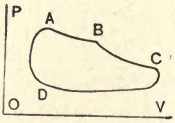

FIG. 13.

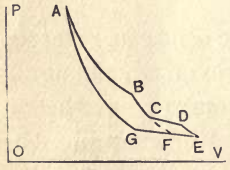

FIG. 14.

It may make this last clearer to consider the cycle of Fig. 14, composed of the isothermals *AB*, *CD*, and *EG*, and the adiabatics *BC*, *DE*, and *GA*. The cycle may be divided by drawing the curve through from *C* to *F*. It is indifferent whether the path followed be *ABCDEGA* or *ABCFCDEGA* or, again, *ABCFGA* + *CDEFC*.

Again, an irregular figure may be imagined to be cut into elementary areas by isothermals and adiabatic lines, as in Fig. 15. The summation of the areas will give the entire area, and the summation of the works represented by these will give the entire work represented by the entire area.

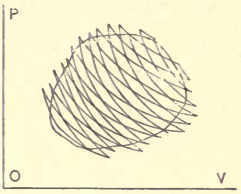

FIG. 15.

The Efficiency of an engine is the ratio of the heat changed into work to the entire heat applied; so that if it be represented by η,

$$\eta = \frac{AW}{Q} = \frac{Q - Q'}{Q} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

for the heat Q' rejected to the refrigerator is what is left after AW thermal units have been changed into work.

Carnot's Principle.—It was first pointed out by Carnot that the efficiency of a reversible engine does not depend on the nature of the working substance, but that it depends on the temperatures of the source of heat and the refrigerator.

Let us see what would be the consequence if this principle were not true. Suppose there are two reversible engines R and A, each taking Q thermal units per second from the source of heat, of which A is the more efficient, so that

$$\frac{AW_a}{Q} = \frac{Q - Q_a'}{Q} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

is larger than

$$\frac{AW_r}{Q} = \frac{Q - Q_r'}{Q}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

this can happen only because Q_a' is less than Q_r', for Q is assumed to be the same for each engine. Let the engine R be reversed and coupled to A, which can run it and still have left the useful work $W_a - W_r$. This useful work cannot come from the source of heat, for the engine R when reversed gives

to the source Q thermal units per second, and A takes the same amount in the same time. It must be assumed to come from the refrigerator, which received Q_a' thermal units per second, and gives up Q_r' thermal units per second, so that it loses

$$Q_r' - Q_a' = A(W_a - W_r)$$

thermal units per second. This equation may be derived from equations (28) and (29) by subtraction.

Now it cannot be proved by direct experiment that such an action as that just described is impossible. Again, the first law of thermodynamics is scrupulously regarded, and there is no contradiction or formal absurdity of statement. And yet when the consequences of the negation of Carnot's principles are clearly set forth they are naturally rejected as improbable, if not impossible. The justification of the principle is found in the fact that theoretical deductions from it are confirmed by experiments.

Second Law of Thermodynamics.—The formal statement of Carnot's principle is known as the second law of thermodynamics. Various forms are given by different investigators, none of which are entirely satisfactory, for the conception is not simple, as is that of the first law.

The following are some of the statements of the second law:

(1) *All reversible engines working between the same source of heat and refrigerator have the same efficiency.*

(2) *The efficiency of a reversible engine is independent of the working substance.*

(3) *A self-acting machine cannot convey heat from one body to another at a higher temperature.*

The second law is the third general principle of thermodynamics; it differs from each of the others and is independent of them. Summing up briefly, the first general principle is a pure assumption that thermodynamic equations may con-

tain only two independent variables; the second is the statement of an experimental fact; the third is a choice of one of the two propositions of a dilemma. The first and third are justified by the results of the application of the theory of thermodynamics.

Carnot's Function.—Carnot's principle asserts that the efficiency of a reversible engine is independent of the nature of the working substance; consequently the expression for the efficiency will not include such properties of the working substance as specific volume and specific pressure. But the principle asserts also that the efficiency depends on the temperatures of the source of heat and the refrigerator, which indeed are the only properties of the source and refrigerator that can affect the working of the engine.

We may then represent the efficiency as a function of the temperatures of the source of heat and the refrigerator, or, what amounts to the same thing, as a function of the superior temperature and the difference of the temperatures, and may write

$$\eta = \frac{AW}{Q} = \frac{Q - Q'}{Q} = F(t, t - t'), \quad . \quad . \quad (30)$$

where Q is the heat received, Q' the heat rejected, and t and t' are the temperatures of the source of heat and of the refrigerator on any scale whatsoever, absolute or relative.

If the temperature of the refrigerator approaches near that of the source of heat $Q - Q'$ and $t - t'$ become ΔQ and Δt, and at the limit dQ and dt, so that

$$\frac{dQ}{Q} = F(t, dt). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

But dt is itself a function of t, so that at the limit the efficiency depends on t only. It is convenient to express the equation in the form

$$\frac{dQ}{Q} = f(t)dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

An algebraic reduction from equation (31) to (32) may be made as follows: divide, and multiply by dt, so that

$$\frac{dQ}{Q} = \frac{F(t, dt)}{dt} dt = f(t)dt.$$

$f(t)$ is called Carnot's function and is represented by μ, so that the efficiency may be represented by

$$\frac{dQ}{Q} = \mu dt.$$

The form of the function will depend on the scale of temperature selected and will vary from one scale to another scale. The logical method appears to be to choose some scale of temperature and deduce the form of Carnot's function corresponding. The easier way is to define or establish an arbitrary scale, independent of any special thermometer or material, and find the difference between that scale and the scale of a thermometer such as the air thermometer or a mercurial thermometer.

Absolute Scale of Temperature.—The simplest form that can be assigned to Carnot's function is $\frac{1}{T}$, where T is the absolute temperature on the arbitrary scale corresponding to that form of Carnot's function. The scale of temperature so determined is the absolute scale referred to on page 5, and depends only on the fundamental units the foot and the pound, or, what amounts to the same thing, on the foot-pound. The most ready way of showing this is by Thomson's graphical method.

Thomson's Graphical Method.—The method just given of arriving at an absolute scale independent of any substance was first given by Lord Kelvin, who further explained it by the following graphical construction:

In Fig. 16 let *ak* and *bi* be two adiabatic lines, and let

the substance have its condition represented by the point *a*. Through *a* and *d* draw isothermal lines; then the diagram *abcd* represents the cycle of a simple reversible engine. Draw the isothermal line *fe*, so that the area *dcef* shall be equal to *abcd*; then the diagram *dcef* represents the cycle of a reversible engine, doing the same amount of work per stroke as that engine whose cycle is represented by *abcd*; and the difference between the heat drawn from the source and delivered to the refrigerater—i. e., the heat transformed into work—is the same. The refrigerator of the first engine might serve for the source of heat for the second.

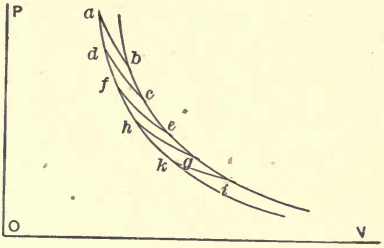

FIG. 16.

Suppose that a series of equal areas are cut off by isothermal lines, as *fegh*, *hgik*, etc., and suppose there are a series of reversible engines corresponding: then there will be a series of sources of heat of determinate temperatures, which may be chosen to establish a thermometric scale. In order to have the scale correspond with those of ordinary thermometers one of the sources of heat must be at the temperature of boiling water, and one at that of melting ice; and for the centigrade scale there will be one hundred, and for the Fahrenheit scale one hundred and eighty, such cycles, with the appropriate sources of heat, between boiling-point and freezing-point. To establish the absolute zero of the scale the series must be imagined to be continued till the area included between an isothermal and the two adiabatics, continued indefinitely, shall not be greater than one of the equal areas.

This conception of the absolute zero may be made clearer by taking wide intervals of temperature, as on Fig. 17, where the cycle *abcd* is assumed to extend between the isothermals of 0° and 100° C.; that is, from freezing-point to boiling-point. The next cycle, *cdef*, extends to — 100° C., and the third cycle, *efgh*,

extends to −200° C. The remaining area, which is of infinite length and extremely attenuated, is bounded by the isothermal *gh* and the two adiabatics *hα* and *gβ*. The diagram of course cannot be completed, and consequently the area cannot be measured; but when the equations to the isothermal and the adiabatics are known it can be computed. So computed, the area is found to be $\frac{73 \cdot 7}{100}$ of one of the three equal areas *abcd*, *cdfe*, and *efhg*. The absolute zero is consequently 273°.7 C. below freezing-point. Further discussion of the absolute scale will be deferred till a comparison is made with the air-thermometer.

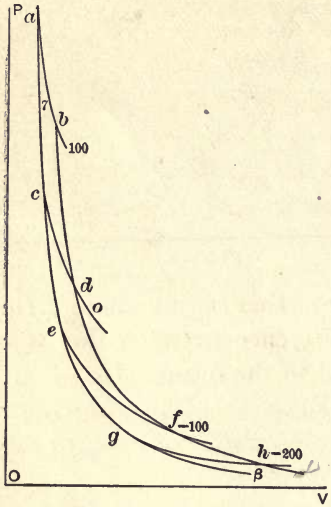

FIG. 17.

Scale of Entropy.—A similar treatment may be given to the scale of entropy. Thus in Fig. 18 let the isothermals *ab* and *cd* be extended indefinitely, and let a series of adiabatics be drawn, cutting off equal areas *abcd*, *blmc*, and *lnom*, etc.; we shall then have a series of intervals of entropy depending on the foot-pound. To make the scale of entropy definite we will assume that the isothermals *an* and *do* are one degree apart, and that the initial cycle *abcd* is so drawn that it represents the change of one thermal unit into work; for the English system the area *abcd* represents 778 foot-pounds.

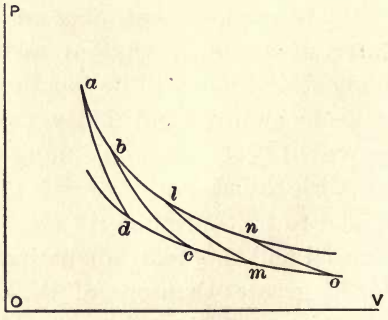

FIG. 18.

Now the area beneath an isothermal line extending indefinitely is infinite, for heat is continually added to the working substance to keep up the temperature, and there is no limit to the amount of work that can be done by expansion. In this regard the isothermal is radically different from the adiabatic, for an infinite adiabatic expansion is supposed to change all the intrinsic energy of the working substance into external work; and as the intrinsic energy is finite, so also must be the area representing external work. We may conclude that the area bounded by the adiabatic *ad*, and the two isothermals *an* and *do*, is infinite, and that there is no limit to the number of equal areas that can be cut off by equally spaced adiabatics.

It will be noted that on the diagram the adiabatic *no* is higher than the adiabatic *ad*, so that the entropy increases from *a* towards *n*. The area of the strip between the adiabatic *bc* and the isothermals *ba*, and *cd*, both extended indefinitely towards the left, is infinite; consequently there is no absolute zero of entropy, and we shall therefore be able to calculate differences of entropy only. The area of the typical cycle chosen for measuring intervals of entropy is large (equal to 778 foot-pounds); consequently the numbers expressing changes of entropy will be found to be small.

Efficiency of Reversible Engines.—The general differential equation for the efficiency of a reversible engine is

$$\frac{dQ}{Q} = f(t)dt = \mu dt,$$

or, making Carnot's function equal to $\frac{1}{T}$,

$$\frac{dQ}{Q} = \frac{dt}{T}.$$

Integrating between limits,

$$\log_e \frac{Q'}{Q} = \log_e \frac{T'}{T}.$$

$$\therefore \quad \frac{Q'}{Q} = \frac{T'}{T}.$$

$$\therefore \quad \eta = \frac{Q - Q'}{Q} = \frac{T - T'}{T} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The absolute scale of temperature consequently depends only on the efficiency of a reversible engine, and since the efficiency of such an engine is independent of the properties of any substance, so also is the absolute scale. The efficiency is expressed in thermal units, which are equivalent to the proper number of mechanical units (foot-pounds); consequently the absolute scale of temperature may be made to depend directly on the foot-pound—that is, on the fundamental units the foot and the pound. This discussion is only another way of viewing the ideas discussed by Thomson's graphical method.

Graphical Representation of Efficiency.—Let Fig. 19 represent the cycle of a reversible heat-engine. For convenience it is supposed there are four degrees of temperature from the isothermal AB to the isothermal DC, and that there are three intervals or units of entropy between the adiabatics AD and BC. First it will be shown that all the small areas into which the cycle is divided by drawing the intervening adiabatics and isothermals are equal. Thus we have to begin with $a = b$ and $a = c$ by construction. But engines working on the cycles a and b have the same efficiency and reject the same amounts of heat. These heats rejected are equal to the heats supplied to engines working on the cycles c and d, which consequently take in the same amounts of heat. But these engines work between the same limits of temperature and have the same efficiency, and consequently change the same amount of heat into work. Therefore the areas c and d are equal. In like manner all the small areas are equal,

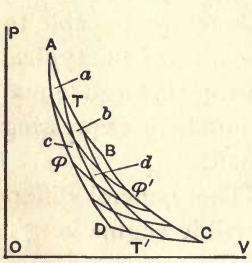

FIG. 19.

and each represents one thermal unit, or 778 foot-pounds of work.

It is evident that the heat changed into work is represented by

$$(T-T')(\phi'-\phi), \quad . \quad . \quad . \quad . \quad . \quad (34)$$

and, further, that the same expression would be obtained for a similar diagram, whatever number of degrees there might be between the isothermals, or intervals of entropy between the adiabatics, and that it is not invalidated by using fractions of degrees and fractions of units of entropy. It is consequently the general expression for the heat changed into work by an engine having a reversible cycle.

It is clear that the work done on such a cycle increases as the lower temperature T' decreases, and that it is a maximum when T' becomes zero, for which condition all of the heat applied is changed into work. Therefore the heat applied is represented by

$$Q = T(\phi' - \phi), \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and the efficiency of the engine working on the cycle represented by Fig. 19 is

$$\frac{AW}{Q} = \frac{Q - Q'}{Q} = \frac{(T - T')(\phi' - \phi)}{T(\phi' - \phi)} = \frac{T - T'}{T},$$

as found by equation (33). The deduction of this equation by integration is more simple and direct, but the graphical method is interesting and may give the student additional light on this subject.

Temperature—Entropy Diagram.—Thermal diagrams are commonly drawn with pressure and volume for the coördinates, but for some purposes it is convenient to use temperature and entropy. Thus Carnot's cycle, when temperature and

entropy are used for coördinates, becomes a rectangle, as shown by Fig. 20. As in Fig. 19, four degrees of temperature and three intervals of entropy have been chosen, and the diagram is subdivided into twelve equal areas, which in Fig. 20 are evidently equal, as they are all rectangles.

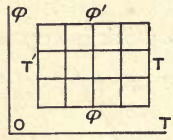

FIG. 20.

Expression for Entropy. — Resuming the expression (35) for the heat supplied along the isothermal line AB (Fig. 19), it appears that the amount of heat supplied depends on the temperature T and on the difference of entropy $\phi' - \phi$. The amount of heat will decrease as the interval of entropy decreases, and will approach dQ when $\phi' - \phi$ approaches $d\phi$, as shown by Fig. 21, so that

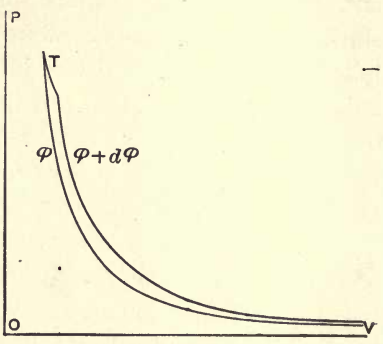

FIG. 21.

$$dQ = T d\phi,$$

or

$$d\phi = \frac{dQ}{T}, \quad \ldots\ldots \quad (36)$$

or, integrating between limits,

$$\phi' - \phi = \int \frac{dQ}{T}. \quad \ldots\ldots \quad (37)$$

Equation (37) may be used for calculating the change of entropy during any reversible operation, provided that the heat added may be expressed as a function of the temperature.

For example, we may calculate the change of entropy from A to B (Fig. 19) as follows:

The temperature during the isothermal expansion from A to B is constant, therefore the heat added is due entirely to the latent heat of expansion, and we have from equation (5)

$$dQ = ldv.$$

For a perfect gas the latent heat of expansion is given by equation (66), page 63, so that

$$dQ = (c_p - c_v)\frac{p}{R}dv.$$

But $R = \frac{pv}{T}$; consequently

$$d\phi = \frac{dQ}{T} = (c_p - c_v)\frac{dv}{v}.$$

$$\therefore \quad \phi' - \phi = (c_p - c_v)\log_e \frac{v_b}{v_a}.$$

Suppose that v_b is 8 cubic feet, and v_a is 4 cubic feet, then

$$\phi' - \phi = (0.2375 - 0.1690)\log_e \frac{8}{4},$$

when the values of c_p and c_v for air are used, giving finally

$$\phi - \phi' = 0.0685 \times 0.6931 = 0.0475$$

for the increase of entropy corresponding to the isothermal expansion from A to B.

Application to a Reversible Cycle.—A very important result is obtained by the application of equation (37) to the calculation of entropy during a reversible cycle. In the first place, it is clear that the entropy of a substance having its condition represented by the point a (Fig. 22), depends on the

adiabatic line drawn through it; in other words, the entropy depends only on the condition of the substance. In this regard entropy is like intrinsic energy and differs from external work. Suppose now that the substance is made to pass through a cycle of operations, represented by the point a tracing the diagram on Fig. 22: it is clear that the entropy will be the same at the end of the cycle as at the beginning, for the tracing-point will then be on the original adiabatic line. As the tracing-point moves toward the right from adiabatic to adiabatic the entropy increases, and as it moves to the left the entropy decreases, the algebraic sum of changes of entropy being zero for the entire cycle. This conclusion holds whether the cycle is reversible or non-reversible.

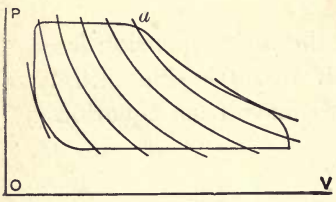

FIG. 22.

If the cycle is reversible, then equation (37) may be used for calculating the several changes of entropy, and for calculating the change for the entire cycle, giving for the cycle

$$\int \frac{dQ}{T} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

This is a very important conclusion from the second law of thermodynamics, and is considered to represent that law. The second law is frequently applied by using this equation in connection with a general equation or a characteristic equation, in a manner to be explained later.

Though the discussion just given is simple and complete, there is some advantage in showing that equation (38) holds for certain simple and complex reversible cycles.

Thus for Carnot's cycle, represented by Fig. 19, the increase of entropy during isothermal expansion is

$$\phi' - \phi = \int \frac{dQ}{T} = \frac{1}{T} \int dQ = \frac{Q}{T},$$

because the temperature is constant. In like manner the decrease during isothermal expansion is

$$\phi - \phi' = \frac{Q'}{T'},$$

so that the change of entropy for the cycle is

$$\frac{Q}{T} - \frac{Q'}{T'}$$

But from the efficiency of the cycle we have

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T}. \qquad \therefore \frac{Q'}{Q} = \frac{T'}{T}; \qquad \therefore \frac{Q}{T} - \frac{Q'}{T'} = 0.$$

A complex cycle like that represented by Fig. 23 may be broken up into two simple cycles *ABFG* and *CDFE*, for each of which individually the same result will be obtained—that is, the increase of entropy from *A* to *B* is equal to the decrease from *F* to *G*, and the increase from *C* to *D* is equal to the decrease from *E* to *F*, so that the summation of changes for the entire cycle gives zero.

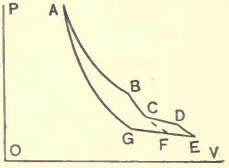

FIG. 23.

Fig. 24 represents the simplified ideal diagram of a hot-air engine, in which by the aid of a regenerator the adiabatic lines of Carnot's cycle are replaced by vertical lines without affecting the reversibility or the efficiency of the cycle. We may replace the actual diagram by a series of simple cycles made up of isothermals and adiabatics, so drawn that the perimeter of the complex cycle includes the same area and corresponds approximately with that of the actual diagram. The summation of

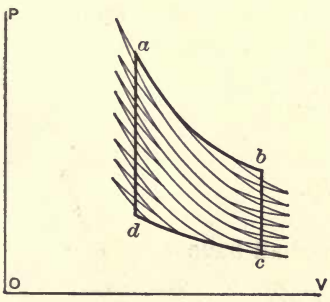

FIG. 24.

the change of entropy for the complex cycle is clearly zero, as before. But by drawing the adiabatic lines near enough together we may make the perimeter approach that of the actual diagram as nearly as we please, and we may therefore conclude that the integration for the changes of entropy for that cycle is also zero.

Non-reversible Cycles.—If a process or a cycle is non-reversible, then the change of entropy cannot be calculated by equation (37), and equation (38) will not hold. The entropy will, indeed, be the same at the end as at the beginning of the cycle, but the integration of $\frac{dQ}{T}$ for the cycle will not give zero. On the contrary, it can be shown that the integration of $\frac{dQ}{T}$ for the entire cycle will give a negative quantity. Thus let the non-reversible engine A take the same amount of heat per stroke as the reversible engine R which works on Carnot's cycle, but let it have a less efficiency, so that

$$\frac{Q - Q_1'}{Q} < \frac{Q - Q'}{Q}, \quad . \quad . \quad . \quad . \quad (39)$$

where Q_1' represents the heat rejected by the engine A. Then

$$Q - Q_1' < Q - Q' = (T - T')(\phi' - \phi). \quad . \quad . \quad (40)$$

Suppose now that T' approaches zero and that ϕ' approaches ϕ, then at the limit we shall have

$$dQ_1 < dQ = Td\phi,$$

or

$$\frac{dQ_1}{T} < d\phi.$$

Integrating for the entire cycle, we shall have

$$\int \frac{dQ_1}{T} < 0. \quad \therefore \int \frac{dQ_1}{T} = -N, \quad . \quad . \quad (41)$$

where N represents a negative quantity. The absolute value of N will, of course, depend on the efficiency of the non-reversible engine. If the efficiency is small compared with that of a reversible engine, then the value of N will be large. If the efficiency approaches that of a reversible engine, then N approaches zero. It is scarcely necessary to point out that N cannot be positive, for that would infer that the non-reversible engine had a greater efficiency than a reversible engine working between the same temperatures.

Some non-reversible operations, like the flow of gas through an orifice, result in the development of kinetic energy of motion. In such case the equation representing the distribution of energy contains a fourth term K to represent the kinetic energy, and equation (22) becomes

$$dQ = A(dS + dI + dW + dK). \quad . \quad . \quad . \quad (42)$$

As before S represents vibration work, I represents disgregation work, and W represents external work. If the vibration and disgregation work cannot be separated, then we may write

$$dQ = A(dE + dW + dK). \quad . \quad . \quad . \quad . \quad (43)$$

CHAPTER IV.

FUDNAMENTAL EQUATIONS.

Application of the First Law.—The application of the first law of thermodynamics consists in uniting an equation resulting from that law to some general or some characteristic equation. For example, equations (5) and (23) give

$$dQ = A(dE + dW) = c_v dt + l dv,$$

or, replacing dW by pdv,

$$A(dE + pdv) = c_v dt + l dv.$$

$$\therefore\ dE = \frac{c_v}{A} dt + \left(\frac{l}{A} - p\right) dv. \quad . \quad . \quad . \quad (44)$$

Now E depends on the state of the body only, and not on the method of changing from one condition to another; that is, dE is an exact differential, and consequently

$$\frac{d^2E}{dt\,dv} = \frac{d^2E}{dv\,dt},$$

which may be written

$$\left\{ \frac{d\left(\frac{dE}{dt}\right)_v}{dv} \right\}_t = \left\{ \frac{d\left(\frac{dE}{dv}\right)_t}{dt} \right\}_v,$$

in which the partial differential coefficients are those of the equation

$$dE = \left(\frac{dE}{dt}\right)_v dt + \left(\frac{dE}{dv}\right)_t dv.$$

Comparing with equation (44), it appears that

$$\left(\frac{dE}{dt}\right)_v = \frac{c_v}{A}, \quad \text{and} \quad \left(\frac{dE}{dv}\right)_t = \left(\frac{l}{A} - p\right),$$

and that consequently

$$\frac{d}{dv}\left(\frac{c_v}{A}\right)_t = \frac{d}{dt}\left(\frac{l}{A} - p\right)_v.$$

$$\therefore \frac{1}{A}\left[\left(\frac{dl}{dt}\right)_v - \left(\frac{dc_v}{dv}\right)_t\right] = \left(\frac{dp}{dt}\right)_v. \quad . \quad . \quad . \quad (45)$$

Equation (45) represents the relation which must exist between the latent heat of expansion and the specific heat at constant volume in consequence of the first law of thermodynamics. In order that the relation may be developed for some particular substance, such as air, the partial differential coefficient $\left(\frac{dp}{dt}\right)_v$ must be deduced from the characteristic equation. The use of this and of similar succeeding equations can be determined only by the application of the theory of thermodynamics to various substances.

In a similar manner the first law may be applied to equation (6), as follows:

$$dQ = A(dE + pdv) = c_p dt + mdp.$$

Substituting the value of dv from the equation

$$dv = \left(\frac{dv}{dp}\right)_t dp + \left(\frac{dv}{dt}\right)_p dt,$$

$$A\left[dE + p\left(\frac{dv}{dp}\right)_t dp + p\left(\frac{dv}{dt}\right)_p dt\right] = c_p dt + m dp;$$

$$\therefore\ dE = \left[\frac{c_p}{A} - p\left(\frac{dv}{dt}\right)_p\right] dt + \left[\frac{m}{A} - p\left(\frac{dv}{dp}\right)_t\right] dp.$$

But

$$\frac{d^2E}{dt\,dp} = \frac{d^2E}{dp\,dt};$$

$$\therefore\ \frac{d}{dp}\left[\frac{c_p}{A} - p\left(\frac{dv}{dt}\right)_p\right]_t = \frac{d}{dt}\left[\frac{m}{A} - p\left(\frac{dv}{dp}\right)_t\right]_p;$$

$$\frac{1}{A}\left(\frac{dc_p}{dp}\right)_t - \left(\frac{dv}{dt}\right)_p - p\left(\frac{d\left(\frac{dv}{dt}\right)_p}{dp}\right)_t = \frac{1}{A}\left(\frac{dm}{dt}\right)_p - p\left(\frac{d\left(\frac{dv}{dp}\right)_t}{dt}\right)_p.$$

But

$$\frac{d^2v}{dp\,dt} = \frac{d^2v}{dt\,dp};$$

$$\therefore\ \frac{1}{A}\left[\left(\frac{dc_p}{dp}\right)_t - \left(\frac{dm}{dt}\right)_p\right] = \left(\frac{dv}{dt}\right)_p, \quad . \quad . \quad . \quad (46)$$

which is the relation between the thermal capacity m and the specific heat at constant pressure developed by the application of the first law to equation (6).

Again, the same law may be applied to the equation (7).

$$dQ = A(dE + p\,dv) = n\,dp + o\,dv;$$

$$\therefore\ dE = \frac{n}{A}dp + \left(\frac{o}{A} - p\right)dv. \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Since

$$\frac{d^2E}{dp\,dv} = \frac{d^2E}{dv\,dp},$$

$$\therefore\ \frac{1}{A}\left(\frac{dn}{dv}\right)_p = \frac{1}{A}\left(\frac{do}{dp}\right)_v - 1;$$

$$\therefore\ \frac{1}{A}\left[\left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p\right] = 1. \quad . \quad . \quad . \quad . \quad (48)$$

Application of the Second Law.—The second law of thermodynamics is expressed by making $\frac{dQ}{T}$ an exact differential as indicated by equations (36) and (38). Applying this to equation (5) in the same way as was done with the first law,

$$\frac{T}{dQ} = \frac{c_v}{T}dt + \frac{l}{T}dv.$$

But

$$\frac{d^2\phi}{dt\,dv} = \frac{d^2\phi}{dv\,dt} \quad \text{or} \quad d^2\frac{\int\frac{dQ}{T}}{dt\,dv} = \frac{d^2\int\frac{dQ}{T}}{dv\,dt};$$

$$\therefore \quad \frac{d}{dv}\left(\frac{c_v}{T}\right)_t = \frac{d}{dt}\left(\frac{l}{T}\right)_v.$$

$$\frac{1}{T}\left(\frac{dc_v}{dv}\right)_t = \frac{T\left(\frac{dl}{dt}\right)_v - l}{T^2};$$

$$\therefore \quad \left(\frac{dl}{dt}\right)_v - \left(\frac{dc_v}{dv}\right)_t = \frac{l}{T}, \quad \ldots \quad (49)$$

the relation between l and c_v developed by the application of the second law to equation (5).

Applying to equation (6), we have

$$\frac{dQ}{T} = \frac{c_p}{T}dt + \frac{m}{T}dp.$$

$$\frac{d}{dp}\left(\frac{c_p}{T}\right)_t = \frac{d}{dt}\left(\frac{m}{T}\right)_p;$$

$$\therefore \quad \frac{1}{T}\left(\frac{dc_p}{dp}\right)_t = \frac{T\left(\frac{dm}{dt}\right)_p - m}{T^2}$$

$$\therefore \quad \left(\frac{dc_p}{dp}\right)_t - \left(\frac{dm}{dt}\right)_p = -\frac{m}{T} \quad \ldots \quad (50)$$

Again, applying to equation (7),

$$\frac{dQ}{T} = \frac{n}{T}dp + \frac{o}{T}dv,$$

$$\therefore \quad \frac{d}{dv}\left(\frac{n}{T}\right)_p = \frac{d}{dp}\left(\frac{o}{T}\right)_v;$$

$$\therefore \quad \frac{T\left(\frac{dn}{dv}\right)_p - n\left(\frac{dt}{dv}\right)_p}{T^2} = \frac{T\left(\frac{do}{dp}\right)_v - o\left(\frac{dt}{dp}\right)_v}{T^2};$$

$$\therefore \quad \frac{1}{T}\left[o\left(\frac{dt}{dp}\right)_v - n\left(\frac{dt}{dv}\right)_p\right] = \left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p. \quad . \quad . \quad . \quad (51)$$

First and Second Laws Combined.—The result of applying both the first and second laws of thermodynamics simultaneously to the fundamental equations is deduced by uniting the equations obtained by applying each separately.

For the equation (5), in terms of c_v and l, the comparison of equations (45) and (49) gives

$$\left(\frac{dp}{dt}\right)_v = \frac{1}{A}\frac{l}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

For equation (6) the comparison of equations (46) and (50) gives

$$\left(\frac{dv}{dt}\right)_p = -\frac{1}{A}\frac{m}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

For equation (7) the comparison of equations (48) and (51) gives

$$A = \frac{1}{T}\left[o\left(\frac{dt}{dp}\right)_v - n\left(\frac{dt}{dv}\right)_p\right]; \quad . \quad . \quad . \quad (54)$$

or, substituting the values of n and o from equations (17) and (18),

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p\left(\frac{dp}{dt}\right)_v. \quad . \quad . \quad . \quad . \quad (55)$$

Alternative Method.—Some writers on thermodynamics reserve the discussion of temperature until they are ready to define or assume an absolute scale independent of any substance, and depending only on the fundamental units of length and weight. Of the three general equations (5), (6), and (7) they use at first only the latter:

$$dQ = ndp + odv.$$

Now from the equation (23), representing the first law of thermodynamics,

$$dQ = A(dE + pdv),$$

it is evident that dQ is not an exact differential, since the equation cannot be integrated directly. The fact that in certain cases p may be expressed as a function of v, and the integral for external work can be deduced, does not affect this general statement. Suppose that there is some integrating factor, which may be represented by $\frac{\text{I}}{S}$, so that

$$\frac{dQ}{S} = \frac{n}{S}dps + \frac{o}{S}dv$$

may be integrated directly: we may then consider that we have a series of thermal lines represented by making

$$\frac{\text{I}}{S} = \text{const.}, \quad \frac{\text{I}}{S'} = \text{const.}, \quad \frac{\text{I}}{S''} = \text{const., etc.}$$

These lines with a series of adiabatic lines represented by

$$\phi = \text{const.}, \quad \phi' = \text{const.}, \quad \phi'' = \text{const., etc.,}$$

allow us to draw a simple cycle of operations represented by

Fig. 25, in which AB and CD are represented by the equations

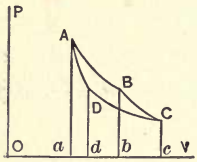

FIG. 25.

$$\frac{I}{S} = C, \quad \text{and} \quad \frac{I}{S'} = C',$$

while AD and BC are adiabatics. The efficiency of a reversible engine receiving the heat Q during the operation AB, and rejecting the heat Q' during the operation CD, will be

$$\eta = \frac{Q - Q'}{Q} = \frac{AW}{Q}.$$

But $\frac{dQ}{S}$ is an exact differential, and depends on the state of the substance only, and consequently is the same at the end as at the beginning of the cycle, so that for the entire cycle

$$\int \frac{dQ}{S} = 0.$$

Now during the operations represented by the adiabatics AD and BC no heat is transmitted, and during the operations represented by the lines AB and CD $\frac{I}{S}$ is constant; consequently the integration of the above equation for the cycle gives

$$\frac{Q}{S} - \frac{Q'}{S} = 0.$$

$$\therefore \quad \frac{Q - Q'}{Q} = \frac{S - S'}{S};$$

that is, the efficiency of an engine working on such a cycle depends on S and S', and on nothing else.

Let us now define absolute temperature by making

$$T = S,$$

so that we have

$$\frac{Q - Q'}{Q} = \frac{T - T'}{T},$$

and we will have a scale of temperature depending only on the efficiency of a reversible engine, and consequently independent of the properties of any substance.

The discussion just given comes properly after the statement of the first and second laws of thermodynamics, and is followed by the application of those laws.

Zeuner's Equations.—In his *Mechanische Wärmetheorie* Zeuner employs the alternative method so far as to deducing equation (41). Then, instead of assuming that S is the absolute temperature, or giving such a definition of temperature, he assumes that the similarity of the thermodynamic equations to certain gravitation equations indicates an essential similarity, and thereby avoids the second law of thermodynamics. Without discussing his method, there appears no reason why it might not be applied to deduce equations of the same form as those given on pages 44 to 48. He, however, gives equations of a different form which may readily be deduced from our own, and which it may be convenient to write down here. Comparing equation (47) with

$$dE = \left(\frac{dE}{dp}\right)_v dp + \left(\frac{dE}{dv}\right)_p dv,$$

it is evident that

$$\frac{n}{A} = \left(\frac{dE}{dp}\right)_v;$$

$$\frac{o}{A} = \left(\frac{dE}{dv}\right)_p + p;$$

These Zeuner writes:

$$X = \left(\frac{dE}{dp}\right)_v;$$

$$Y = p + \left(\frac{dE}{dv}\right)_p.$$

Solving equation (54) for o and for n,

$$o = \frac{AT + n\left(\frac{dt}{dv}\right)_p}{\left(\frac{dt}{dp}\right)_v};$$

$$-n = \frac{AT - o\left(\frac{dt}{dp}\right)_v}{\left(\frac{dt}{dv}\right)_p}.$$

Substituting the value for o in equation (7), we have

$$dQ = \frac{1}{\left(\frac{dt}{dp}\right)_v}\left[n\left(\frac{dt}{dp}\right)_v dp + n\left(\frac{dt}{dv}\right)_p dv + ATdv\right].$$

But

$$dt = \left(\frac{dt}{dp}\right)_v dp + \left(\frac{dt}{dv}\right)_p dv;$$

$$\therefore\ dQ = \left(\frac{dp}{dt}\right)_v [ndt + ATdv].$$

Again, substituting the value for n,

$$dQ = \frac{1}{\left(\frac{dt}{dv}\right)_p}\left[o\left(\frac{dt}{dp}\right)_v dp + o\left(\frac{dt}{dv}\right)_p dv - ATdp\right];$$

$$\therefore\ dQ = \left(\frac{dv}{dt}\right)_p [odt - ATdp].$$

Zeuner gives for his fundamental equations

$$dQ = A(Xdp + Ydv);$$

$$dQ = \frac{A}{\left(\frac{dt}{dp}\right)}\left[Xdt + \left(\frac{1}{\alpha} + t\right)dv\right];$$

$$dQ = \frac{A}{\left(\frac{dt}{dv}\right)}\left[Ydt - \left(\frac{1}{\alpha} + t\right)dp\right];$$

which may readily be deduced from equation (23) and the equations above.

These equations are set down here because they are commonly used by German writers in discussing thermodynamics.

CHAPTER V.

PERFECT GASES.

THE characteristic equation for a perfect gas is derived from a combination of the laws of Boyle and Gay-Lussac, which may be stated as follows:

Boyle's Law.—When a given weight of a perfect gas is compressed (or expanded) at a constant temperature the product of the pressure and the volume is a constant. This law is nearly true at ordinary temperatures and pressures for such gases as oxygen, hydrogen, and nitrogen. Gases which are readily liquefied by pressure at ordinary temperatures, such as ammonia and carbonic acid, show a notable deviation from this law. The law may be expressed by the equation

$$pv = p_1 v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

in which p_1 and v_1 are the initial pressure and volume; p is any pressure and v is the corresponding volume.

Gay-Lussac's Law.—It was found by Guy-Lussac that any volume of gas at freezing-point increases about 0.003665 of its volume for each degree rise of temperature. Gases which are easily liquified deviate from this law as well as from Boyle's law. In the deduction of this law temperatures were measured on or referred to the air-thermometer, and the law therefore asserts that the expansibility or the coefficient of dilatation of perfect gases is the same as that of air. It will be shown in this chapter that the scale of the air-thermometer differs but little from that of the absolute thermodynamic

scale; for practical purposes the difference may be neglected. Gay-Lussac's law may be expressed by the equation

$$v = v_0(1 + \alpha t), \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

in which v_0 is the original volume at freezing-point, α is the coefficient of dilatation or the increase of volume for one degree rise of temperature, and v is the volume corresponding to the temperature t measured from freezing-point.

Coefficient of Dilatation.—Regnault* gives for the dilatation from freezing- to boiling-point at Paris the results:

Hydrogen	0.003667
Atmospheric air	0.003665
Nitrogen	0.003668
Carbonic acid	0.003688

In works on thermodynamics it has been commonly assumed that the coefficient of dilatation for air may be used for all gases, and at all temperatures and pressures, and that, consequently, on the centigrade scale α is 0.003665, or very nearly $\frac{1}{273}$. Professor Holman† suggests that as the pressure approaches zero the coefficient of dilatation of all gases approaches

$$\alpha = \frac{1}{273.7},$$

which agrees with thermodynamic investigations relating to the absolute zero of temperature. On the Fahrenheit scale

$$\alpha = \frac{1}{492.7}.$$

Characteristic Equation.—From equation (57) we may calculate any special volume, such as v_1, getting

$$v_1 = v_0(1 + \alpha t).$$

* *Mémoires de l'Institut de France*, tome xxi.

† *Lecture Notes on Heat*, Mass. Inst. Technology.

Assuming that p_1 in equation (56) is the normal pressure of the atmosphere p_0, and substituting the value just found for v_1, we have for the combination of the laws of Boyle and Gay-Lussac

$$pv = p_0v_0(1 + \alpha t) = p_0v_0\alpha\left(\frac{1}{\alpha} + t\right) \quad . \quad . \quad (58)$$

If now we assume that a perfect gas contracts $\frac{1}{273.7}$ part of its volume at freezing-point for each degree decrease of temperature, then the absolute zero of temperature on the scale of the air-thermometer is 273°.7 C. below freezing-point; and centigrade temperatures can be transformed into absolute temperatures by adding 273°.7. In like manner the zero of the air-thermometer on the Fahrenheit scale is 492°.7 below freezing-point, which on that scale is at 32°; and absolute temperatures will be obtained by adding 460°.7 to the Fahrenheit temperatures.

The usual form of the characteristic equation for perfect gases may be derived from equation (58) by substituting T_0, the absolute temperature of freezing-point, for $\frac{1}{\alpha}$, giving

$$pv = \frac{p_0v_0}{T_0}T = RT, \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where R is a constant representing the quantity

$$\frac{p_0v_0}{T_0}.$$

For solution of examples it is more convenient to write equation (59) in the form

$$\frac{pv}{T} = \frac{p_0v_0}{T_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Absolute Temperatures.—For convenient reference it is desirable to write down the equations

$$T = t + 273°.7 \text{ centigrade scale,}$$
$$T = t + 460°.7 \text{ Fahrenheit scale,}$$

in which t is the ordinary temperature by the thermometer and T is the absolute temperature.

Relation of French and English Units.—Both French and English standard units of length and weight are arbitrary standards represented by the length of bars or the weight of certain pieces of metal. The French metre is the length from end to end of a bar of platinum at freezing-point; the English yard is the length from one line to another on a bronze bar at 62° F.; owing to the different methods of defining the standards, comparisons are very difficult. The ratio of the lengths of the standards used in this book is that given by Professor Rogers,* namely,

$$\text{one metre} = 39.3702 \text{ inches.}$$

The last figure, 2, is probably uncertain to the extent of one unit. For engineering work the metre may be assumed to be equal to 39.37 inches.

The ratio of the standards of weight † is

$$\text{one kilogram} = 2.20462125 \text{ pounds.}$$

Acceleration due to Gravity.—The force with which a body is attracted towards the earth is proportional to the acceleration due to gravity, and varies with the latitude and with the altitude above sea-level, as given by the equation ‡

$$g = 980.6056 - 2.5028 \cos 2\lambda - 0.000003h, \quad . \quad (61)$$

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* *Proc. Am. Acad. of Arts and Sci.*, 1882–83; also additional observations.

† Miller, *Phil. Transactions*, cxlvi, 1856.

‡ Everett's *Units and Phys. Const.*

in which g is the acceleration in centimetres per second, λ is the latitude, and h is the altitude above the sea in centimetres.

Standard Latitude.—It is customary to reduce all physical observations which are affected by the attraction of gravity to the latitude of 45° and to sea-level. This reduction affects the fifth significant figure, and consequently is properly recognized in recording physical constants which may be used by engineers, but it is seldom, if ever, necessary for an engineer to reduce such physical constants to the place where he may be located.

Specific Pressure.—The normal pressure of the atmosphere is assumed to be equivalent to that of a column of mercury 760 mm. high at freezing-point. Now Regnault* gives for the weight of a litre, or one cubic decimetre, of mercury 13.5959 kilograms; consequently the specific pressure of the atmosphere under normal conditions is

$$p_0 = 10333 \text{ kilograms per square metre.}$$

Using the conversion units given above for reducing this specific pressure to the English system of units gives

$$p_0 = 2116.32 \text{ pounds per square foot,}$$

which corresponds to.

14.6967 pounds per square inch,

or to

29.921 inches of mercury.

It is customary and sufficient to use for the pressure of the atmosphere 14.7 pounds to the square inch.

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* *Mémoires de l'Institut de France*, vol. xxi.

Densities of Gases.—The weights in kilograms of one cubic metre of several gases, as determined by Regnault, are given in the following table:

Atmospheric air	1.293187
Nitrogen	1.256167
Oxygen	1.429802
Hydrogen	0.089578
Carbonic acid	1.977414

Now the acceleration due to gravity at Paris, as calculated by equation (61), is 980.9218 centimetres, while the acceleration at 45° latitude and at sea-level is 980.6056 centimetres; consequently the pressure due to 760 mm. of mercury at 45° latitude is equivalent to that of only

$$760 \times \frac{980.6056}{980.9218} = 759.755 \text{ mm.}$$

at Paris, and the weights of one cubic metre of the several gases at the normal pressure of the atmosphere will be smaller in a like proportion. To get the specific volumes we may multiply the weights by the ratio

$$\frac{759.755}{760},$$

and then take the reciprocal of the results, giving results set down in the following paragraph.

Specific Volumes.—The following table gives the specific volumes of several gases in cubic metres per kilogram:

VOLUMES IN CUBIC METRES OF ONE KILOGRAM AT 45° OF LATITUDE.

Atmospheric air	0.7735327
Nitrogen	0.7963291
Oxygen	0.6996231
Hydrogen	11.16705
Carbonic acid	0.5058741

The corresponding quantities for English units are given in the next table:

VOLUMES IN CUBIC FEET OF ONE POUND AT 45° OF LATITUDE.

Atmospheric air	12.3909
Nitrogen	12.7561
Oxygen	11.2070
Hydrogen	178.881
Carbonic acid	8.10324

Value of R.—The constant R which appears in the usual form of the characteristic equation for a gas represents the expression

$$\frac{p_0 v_0}{T_0}.$$

The values for R corresponding to the French and the English system of units may be calculated as follows:

$$\text{French units, } R = \frac{10333 \times 0.77353}{273.7} = 29.20. \quad (62)$$

$$\text{English units, } R = \frac{2116.3 \times 12.391}{492.7} = 53.22. \quad (63)$$

Value of R for other gases may be calculated in a like manner.

Solution of Problems.—Many problems involving the properties of air or other gases may be solved by the characteristic equation

$$pv = RT,$$

or by the equivalent equation

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0},$$

which for general use is the more convenient.

In the first of these two equations the specific pressure and volume to be used for English measures are pounds per square foot, and the volume in cubic feet of one pound.

For example, let it be required to find the volume of 3 pounds of air at 60 pounds gauge-pressure and at 100° F. Assuming a barometric pressure of 14.7 pounds per square inch,

$$v = \frac{53.22(460.7 + 100)}{(14.7 + 60)144} = 2.774 \text{ cubic feet}$$

is the volume of 1 pound of air under the given conditions, and 3 pounds will have a volume of

$$3 \times 2.774 = 8.322 \text{ cubic feet.}$$

The second equation has the advantage that any units may be used, and that it need not be restricted to one unit of weight.

For example, let the volume of a given weight of gas, at 100° C. and at atmospheric pressure, be 2 cubic yards; required the volume at 200° C. and at 10 atmospheres. Here

$$\frac{10v}{473.7} = \frac{1 \times 2}{373.7},$$

$$v = \frac{473.7 \times 2}{10 \times 373.7} = 0.2535 \text{ cubic yards.}$$

Application of the Laws of Thermodynamics.—Equation (55), page 48,

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p \cdot \left(\frac{dp}{dt}\right)_v,$$

which was deduced by applying both laws of thermodynamics to equation (7), may be most conveniently used in this place.

Differentiating the characteristic equation

$$pv = RT,$$

we obtain

$$\left(\frac{dv}{dt}\right)_p = \frac{R}{p}, \qquad \left(\frac{dp}{dt}\right)_v = \frac{R}{v},$$

which substituted in equation (55) gives

$$c_p - c_v = AR. \quad . \quad . \quad . \quad . \quad . \quad (64)$$

Specific Heat at Constant Pressure.—The specific heat for true gases is very nearly constant, and may be considered to be so for thermodynamic equations. Regnault gives for the mean values for specific heat at constant pressure the following results:

Atmospheric air	0.2375
Nitrogen	0.2438
Oxygen	0.2175
Hydrogen	3.409

Specific Heat at Constant Volume.—It is evident from inspection of equation (64) that the specific heat at constant volume is a constant, and that equation also gives one of the best ways of calculating this quantity. Most commonly the ratio of the two specific heats is determined, as follows:

$$\kappa = \frac{c_p}{c_v} = \frac{1}{1 - \frac{AR}{c_p}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

$$\kappa = \frac{1}{1 - \frac{10333 \times 0.77353}{426.9 \times 273.7 \times 0.2375}};$$

$$\kappa = 1.4046 = 1.405, \text{ nearly.}$$

Thermal Capacities.—Substituting the values of the partial differential coefficients as deduced from equation (59), in equations (11), (15), 17), and (18), we have for the values of the thermal capacities for gases

$$l = \frac{p}{R}(c_p - c_v) \quad = \frac{T}{v}(c_p - c_v); \quad . \quad . \quad . \quad (66)$$

$$m = -\frac{v}{R}(c_p - c_v) = -\frac{T}{p}(c_p - c_v); \quad . \quad . \quad (67)$$

$$n = \frac{v}{R}c_v = \frac{T}{p}c_v; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

$$o = \frac{p}{R}c_p = \frac{T}{v}c_p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

General Equations.—To deduce the general equations for gases from equations (5), (6), and (7) it is only necessary to replace the letters l, m, n, and o by their values just obtained, giving

$$dQ = c_v dt + (c_p - c_v)\frac{T}{v}dv; \quad . \quad . \quad . \quad . \quad (70)$$

$$dQ = c_p dt + (c_v - c_p)\frac{T}{p}dp; \quad . \quad . \quad . \quad . \quad (71)$$

$$dQ = c_v\frac{T}{p}dp + c_p\frac{T}{v}dv. \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Isothermal Line.—The equation to the isothermal line for a gas is obtained by making T a constant in the characteristic equation, so that

$$pv = RT_1 = p_1v_1,$$

or

$$pv = p_1v_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

This equation will be recognized as the expression of Boyle's law. It is, of course, the equation to an equilateral hyperbola.

To obtain the external work during an isothermal expansion we may substitute for p in the expression

$$W = \int p dv$$

from the equation to the isothermal line just stated, using for limits the final and initial volumes, v_2 and v_1:

$$W = p_1v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1v_1 \log_e \frac{v_2}{v_1}. \quad . \quad . \quad . \quad (74)$$

If the problem in any case calls for the external work of one unit of weight of a gas, then v_1 and v_2 must be the initial and final specific volumes; but in many cases the initial and final volumes are given without any reference to a weight, and it is then sufficient to find the external work for the given expansion from the initial to the final volume without considering whether or not they are specific volumes.

The pressures must always be specific pressures; in the English system the pressures must be expressed in pounds on the square foot before using them in the equation for external work or, for that matter, in any thermodynamic equation.

For example, the specific volume of air at freezing-point and at 14.7 pounds pressure per square inch is about 12.4 cubic feet; at the same temperature and at 29.4 pounds per square inch the specific volume is 6.2 cubic feet. The external work

during an isothermal expansion of one pound of air from 6.2 to 12.4 cubic feet is

$$W = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e \frac{v_2}{v_1}$$

$$= 29.4 \times 144 \times 6.2 \log_e \frac{12.4}{6.2} = 18190 \text{ foot-pounds.}$$

For example, the external work of isothermal expansion from 3 cubic feet and 60 pounds pressure by the gauge to a volume of 7 cubic feet is

$$W = (60 + 14.7)144 \times 3 \log_e \frac{7}{3} = 27340 \text{ foot-pounds.}$$

In both problems the pressure per square inch is multiplied by 144 to reduce it to the square foot. In the first problem the pressures are absolute, that is, they are measured from zero pressure; in the second problem the pressure by the gauge is 60 pounds above the pressure of the atmosphere, which is here assumed to be 14.7 pounds per square inch, and is added to give the absolute pressure. In careful experimental work the pressure of the atmosphere is measured by a barometer and is added to the gauge-pressure.

Isoenergic Line.—The isothermal line for a perfect gas is also the isoenergic line, a fact that may be proved as follows: The heat applied during an isothermal expansion may be calculated by making T a constant in equation (70) and then integrating; thus:

$$Q = (c_p - c_v) T_1 \int_{v_1}^{v_2} \frac{dv}{v} = (c_p - c_v) T_1 \log_e \frac{v_2}{v_1},$$

or, substituting for $c_p - c_v$ from equation (64),

$$Q = ART_1 \log_e \frac{v_2}{v_1} = Ap_1 v_1 \log_e \frac{v_2}{v_1}. \quad . \quad . \quad (75)$$

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A comparison of equation (75) with equation (74) shows that the heat applied during an isothermal expansion is equivalent to the external work, or we may say that all the heat applied is changed into external work, so that the intrinsic energy is not changed.

An interesting corollary of the discussion just given is that an infinite isothermal expansion gives an infinite amount of work. Thus the area contained between the axis OV (Fig. 26), the ordinate ab, and the isothermal line $a\alpha$ extended without limit is

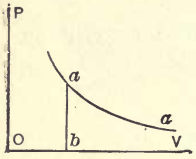

FIG. 26.

$$W = p_0 v_0 \log_e \frac{\infty}{v_0} = \infty.$$

This may also be seen from the consideration that if heat be continually applied, and all changed into work, there will be a limitless supply of work.

Adiabatic Lines.—During an adiabatic change—for example, the expansion of a gas in a non-conducting cylinder—heat is not communicated to, nor abstracted from, the gas; consequently dQ in equations (70), (71), and (72) becomes zero.

From equation (72)

$$0 = dQ = \frac{c_v}{R} v dp + \frac{c_p}{R} p dv;$$

$$\therefore \quad \frac{c_p}{c_v} \frac{dv}{v} = -\frac{dp}{p};$$

$$\therefore \quad \log_e \left(\frac{v}{v_1}\right)^{\frac{c_p}{c_v}} = \log_e \left(\frac{p_1}{p}\right).$$

The ratio $\frac{c_p}{c_v}$ of the specific heats may be represented by κ, and the above equation may be written

$$\left(\frac{v}{v_1}\right)^{\kappa} = \frac{p_1}{p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

$$\therefore \quad v^{\kappa} p = v_1^{\kappa} p_1 = \text{const.} \quad . \quad . \quad . \quad . \quad (77)$$

This is the adiabatic equation for a perfect gas which is most frequently used. If adiabatic equations involving other variables, such as v_1 and T_1, are desired, they may be derived from equation (76) by aid of the characteristic equation, which for this purpose may be written

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1},$$

so that

$$\frac{p_1}{p} = \frac{v T_1}{v_1 T},$$

and

$$\left(\frac{v}{v_1}\right)^{\kappa-1} = \frac{T_1}{T}; \quad \ldots\ldots\ldots \quad (78)$$

$$\therefore \quad T v^{\kappa-1} = T_1 v_1^{\kappa-1}. \quad \ldots\ldots \quad (79)$$

Or equations (78) and (79) may be deduced directly from equation (70) as equations (76) and (77) were from equation (72).

In like manner we may deduce from equation (71)

$$T p^{\frac{1-\kappa}{\kappa}} = T_1 p_1^{\frac{1-\kappa}{\kappa}}, \quad \ldots\ldots \quad (80)$$

or we may derive it from equation (76).

To find the external work the equation

$$W = \int p dv$$

may be used after substituting for p from equation (77):

$$W = \int_{v_1}^{v_2} p dv = v_1^{\kappa} p_1 \int_{v_1}^{v_2} \frac{dv}{v^{\kappa}} = -\frac{p_1 v_1^{\kappa}}{\kappa - 1}\left(\frac{1}{v_2^{\kappa-1}} - \frac{1}{v_1^{\kappa-1}}\right);$$

$$\therefore \quad W = \frac{p_1 v_1}{\kappa - 1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{\kappa-1}\right\}. \quad \ldots\ldots \quad (81)$$

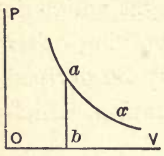

FIG. 27.

In Fig. 27 the area between the axis OV, the ordinate ba, and the adiabatic line $a\alpha$ extended without limit becomes

$$W_1 = \frac{p_1 v_1}{\kappa - 1},$$

and not infinity, as is the case with the isothermal line.

Here, as with the calculation of external work during isothermal expansion, specific volumes should be used when the problem involves a unit of weight; but work may be calculated for any given initial and final volumes without considering whether they are specific volumes or not. The pressures are always pounds on the square foot for the English system.

For example, the external work of adiabatic expansion from 3 cubic feet and 60 pounds pressure by the gauge to the volume of 7 cubic feet is

$$W = \frac{74{\cdot}7 \times 144 \times 3}{1.405 - 1}\left\{1 - \left(\frac{3}{7}\right)^{1.405 - 1}\right\} = 23140 \text{ foot-pounds},$$

which is considerably less than the work for the corresponding isothermal expansion.

Intrinsic Energy.—Since external work during an adiabatic expansion is done at the expense of the intrinsic energy, the work obtainable by an infinite expansion cannot be greater than the intrinsic energy. If it be admitted that such an expansion changes all of the intrinsic energy into external work we shall have

$$E_1 = W_1 = \frac{p_1 v_1}{\kappa - 1}, \quad . \quad . \quad . \quad . \quad (82)$$

which gives a ready way of calculating intrinsic energy. In practice we always deal with differences of intrinsic energy, so that even if there be a residual intrinsic energy after an

infinite adiabatic expansion the error of our method will be eliminated from an equation having the form

$$E_1 - E_2 = \frac{p_1 v_1}{\kappa - 1} - \frac{p_2 v_2}{\kappa - 1}, \quad . \quad . \quad . \quad (83)$$

Exponential Equation.—The expansions and compressions of air and other gases in practice are seldom exactly isothermal or adiabatic; more commonly an actual operation is intermediate between the two. It is convenient and usually sufficient to represent such expansions by an exponential equation,

$$pv^n = p_1 v_1^n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

in which n has a value between unity and 1.405. The form of integration for external work is the same as for that of adiabatic expansion, and the amount of external work is intermediate between that for adiabatic and that for isothermal expansion.

Change of temperature during such an expansion may be calculated by the equations

$$Tv^{n-1} = T_1 v_1^{n-1}, \quad . \quad . \quad . \quad . \quad . \quad (85)$$

$$Tp^{\frac{1-n}{n}} = T_1 p_1^{\frac{1-n}{n}}, \quad . \quad . \quad . \quad . \quad . \quad (86)$$

which may be deduced from equation (82) by aid of the characteristic equation

$$pv = RT$$

as equation (79) is deduced from equation (76).

If it is desired to find the exponent of an equation representing a curve passing through two points, as a_1 and a_2 (Fig. 28), we may proceed by taking logarithms of both sides of the equation

$$p_1 v_1^n = p_2 v_2^n,$$

giving

$$n \log v_1 + \log p_1 = n \log v_2 + \log p_2,$$

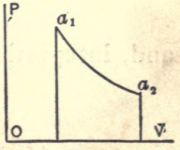

FIG. 28.

so that

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (87)$$

For example, the exponent of an equation to a curve passing through the points

$$p_1 = 74.7, \quad v_1 = 3, \quad \text{and} \quad p_2 = 30, \quad v_2 = 7$$

is

$$n = \frac{\log 74.7 - \log 30}{\log 7 - \log 3} = 1.104.$$

Entropy.—For any reversible process

$$d\phi = \frac{dQ}{T};$$

consequently from equations (70), (71), and (72) we have

$$d\phi = c_v \frac{dt}{T} + (c_p - c_v)\frac{dv}{v},$$

$$d\phi = c_p \frac{dt}{T} + (c_v = c_p)\frac{dp}{p},$$

$$d\phi = c_v \frac{dp}{p} + c_p \frac{dv}{v};$$

and, integrating between limits,

$$\phi_2 - \phi_1 = c_v \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{v_2}{v_1}, \quad . \quad (88)$$

$$\phi_2 - \phi_1 = c_p \log_e \frac{T_2}{T_1} + (c_p - c_v) \log_e \frac{p_1}{p_2}, \quad . \quad (89)$$

$$\phi_2 - \phi_1 = c_v \log_e \frac{p_2}{p_1} + c_p \log_e \frac{v_2}{v_1}, \quad . \quad . \quad . \quad (90)$$

which give ready means of calculating changes of entropy.

For example, the change of entropy in passing from the pressure of 74.7 pounds absolute per square inch and the volume of 3 cubic feet to the pressure of 30 pounds absolute and the volume of 7 cubic feet is

$$\phi_2 - \phi_1 = \frac{0.2375}{1.405} \log_e \frac{30}{74.7} + 0.2375 \log_e \frac{7}{3} = 0.0454.$$

Since the pressures form the numerator and denominator of a fraction, there is no necessity to reduce them to the square foot. In this problem the pressures and volumes are taken at random; they correspond to a temperature of 146° F. at the initial condition.

Comparison of the Air-thermometer with the Absolute Scale.—In connection with the isodynamic line it was shown that the intrinsic energy is a function of the temperature only. This conclusion is deduced from the characteristic equation on the assumption that the scale of the air-thermometer coincides with the thermodynamic scale, and it affords a delicate method of testing the truth of the characteristic equation, and of comparing the two scales.

The most complete experiments for this purpose were made by Joule and Sir William Thomson, who forced air slowly through a porous plug in a tube in such a manner that no heat was transmitted to or from the air during the process. Also the velocity both before and beyond the plug was so small that the work due to the change of velocity could be disregarded. All the work that would be developed in free

expansion from the higher to the lower pressure was used in overcoming the resistance of friction in the plug, and so converted into heat, and as none of this heat escaped it was retained by the air itself, the plug remaining at a constant temperature. It therefore appears that the intrinsic energy remained the same, and that a change of temperature indicated a deviation from the assumptions of the theory of perfect gases.

In the discussion of results given by Joule and Thomson * in 1854 they give for the absolute temperature of freezing-point 273°.7 C. As the result of later † experiments they state that the cooling for a difference of pressure of 100 inches of mercury is represented on the centigrade scale by

$$0°.92\left(\frac{273.7}{T}\right)^2.$$

The following table shows the agreement between this statement and the results of experiment:

FLOW OF AIR THROUGH POROUS PLUG.

Temperature.	Cooling Effect:	
	By Experiment.	By Calculation.
0°	0.92	0.92
7.1	0.88	0.87
39.5	0.75	0.70
92.8	0.51	0.51

From the work of these experiments Rowland ‡ deduced the following comparison of the air-thermometer at constant volume with the absolute thermodynamic scale of temperature:

* *Philosophical Transactions*, vol. 144, p. 349.

† *Ibid.*, vol. 152, p. 579.

‡ *Proceedings of the American Academy*, vol. xv (N. S. viii), p. 75, 1879.

REDUCTION OF THE AIR-THERMOMETER TO THE ABSOLUTE SCALE.

(*Centigrade.*)

Temperature above Freezing.		Correction to Air-thermometer.
Air-thermometer.	Absolute Scale.	
0°	°	°
10	9.9972	− 0.0028
20	19.9952	− 0.0048
30	29.9939	− 0.0061
40	39.9933	− 0.0067
50	49.9932	− 0.0068
60	59.9937	− 0.0063
70	69.9946	− 0.0054
80	79.9956	− 0.0044
90	89.9978	− 0.0022
100	100.000	0.
200	200.037	+ 0.037
300	300.092	+ 0.092
400	400.157	+ 0.157
500	500.228	+ 0.228

Velocity of Sound.—Sound is transmitted through the air in spherical waves, but at a distance from the source of sound the waves are sensibly plane waves, and the progress of the wave is the same as that of a plane wave in a straight tube of uniform section. Let Fig. 29 represent a tube one square metre in section in which a wave moves with a linear velocity u_0 metres per second; that is, a point at a given phase of the wave—for example, C at the greatest condensation—moves at that velocity.

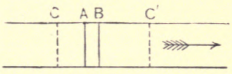

FIG. 29.

Since the wave moves with the velocity u_0, the volume of air disturbed in a unit of time is u_0 cubic metres. If the specific volume in the undisturbed state is v_0, then the weight of air disturbed in a second is

$$w = mg = \frac{u_0}{v_0},$$

m being the mass of air which has the weight w.

Imagine two planes A and B at a small distance apart, which also move with the velocity u_0, so that they remain at the same phase of the wave. Let the absolute velocities of the air at these planes be u_1 and u_2; then the velocities of the air through the planes—that is, the velocities relatively to the planes—is, for A, $u_0 - u_1$, and for B, $u_0 - u_2$. With v_1 and v_2 for the specific volumes at these planes the weights that pass through the planes A and B per second are

$$\frac{u_0 - u_1}{v_1} \quad \text{and} \quad \frac{u_0 - u_2}{v_2}.$$

Since the phase of that portion of the wave between A and B is constant, the weight of the air between them is also constant, and as much air enters per second as leaves during that time. Again: as, on the whole, the air is not transmitted, but only compressed and rarefied, the whole air disturbed per second must pass through the space between the planes. Therefore

$$\frac{u_0 - u_1}{v_1} = \frac{u_0 - u_2}{v_2} = \frac{u_0}{v_0} = mg;$$

$$\therefore \quad u_1 = u_0 - mv_1 g;$$

$$u_2 = u_0 - mv_2 g;$$

$$u_1 - u_2 = mg(v_2 - v_1).$$

Now as the mass m enters the space between the planes with the absolute velocity u_1, and an equal mass leaves with the velocity u_2, consequently there is a change of momentum

$$m(u_1 - u_2);$$

and since this cannot come from the mutual action of the particles, it must come from the difference of pressures at A and B; thus:

$$p_1 - p_2 = m(u_1 - u_2) = m^2 g(v_2 - v_1).$$

As the planes A and B approach each other p_1 and p_2, v_1 and v_2 approach in value, and at the limit

$$dp = -m^2 g dv,$$

$$\frac{dp}{dv} = -m^2 g = -\frac{1}{g}\frac{u_0^2}{v_0^2},$$

the last reduction being obtained by substituting for the value of m from the preceding work. Solving for u_0,

$$u_0^2 = -gv_0^2\frac{dp}{dv}.$$

The vibrations are so rapid that the changes of state may be assumed to be adiabatic; consequently equation (72) gives

$$0 = dQ = \frac{c_v}{R}vdp + \frac{c_p}{R}pdv;$$

$$\therefore \quad \frac{dp}{dv} = -\frac{c_p}{c_v}\cdot\frac{p}{v} = -\kappa\frac{p}{v}.$$

The planes A and B may be taken at any phase of the wave; for example at the phase where the pressure and volume are normal, in which case

$$\frac{dp}{dv} = -\kappa\frac{p_0}{v_0}.$$

Substituting in the equation for u_0, we have

$$u_0^2 = g\kappa v_0 p_0.$$

The equation is commonly given in terms of the density, γ, as follows:

$$u_0 = \sqrt{\kappa g\frac{p}{\gamma}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

The velocity of sound from direct experiment was found by Moll and Van Beek to be 332.26 metres per second; by Regnault to be 330.70 metres per second. Kayser found from Kundt's dust figures the wave length corresponding to a certain tone, and therefrom deduced the velocity of sound, and gives for the velocity 332.50 metres per second. The true value must be nearly 332 metres per second. Solving equation (91) for κ, and inserting the known values of $p_0 v_0$ and g for Paris,

$$\kappa = \frac{u_0^2}{g v_0 p_0} = \frac{\overline{332}^2}{9.8092 \times 0.77328 \times 10333};$$

$$\kappa = 1.4063.$$

This result as calculated compares very well with that already calculated by aid of equation (65), but it depends entirely on the value assigned to the velocity of sound whether the comparison is satisfactory or not. Now that the determination of the mechanical equivalent of heat by Rowland has reduced the probable error of that physical constant to a small amount the calculation by equation (65) is to be preferred, especially as it agrees very well with the results of direct experiments by Röntgen.

Röntgen's Experiments.—Direct experiments to determine κ may be made as follows: Suppose that a vessel is filled with air at a pressure p_1, while the pressure of the atmosphere is p_a. Let a communication be opened with the atmosphere sufficient to suddenly equalize the pressure; then let it be closed, and let the pressure p_2 be observed after the air has again attained the temperature of the atmosphere. If the first operation is sufficiently rapid it may be assumed to be adiabatic, and we may use equation (77), from which

$$\kappa = \frac{\log p_1 - \log p_a}{\log v_a - \log v_1}. \quad . \quad . \quad . \quad . \quad (92)$$

The second operation is at constant volume; consequently the specific volume is the same at the final state as after the adiabatic expansion of the first operation. But the initial and final temperatures are the same; consequently

$$v_1 p_1 = v_a p_2;$$

$$\therefore \quad \log v_a - \log v_1 = \log p_1 - \log p_2,$$

which substituted in equation (92) gives

$$\kappa = \frac{\log p_1 - \log p_0}{\log p_1 - \log p_2}. \quad . \quad . \quad . \quad . \quad . \quad (93)$$

The same experiment may be made by rarefying the air in the vessel, in which case the sign of the second term changes.

Röntgen * employed this method, using a vessel containing 70 litres, and measuring the pressure with a gauge made on the same principle as the aneroid barometer. Instead of assuming the pressure p_0 at the instant of closing the stop-cock to be that of the atmosphere, he measured it with the same instrument. A mean of ten experiments on air gave

$$\kappa = 1.4053.$$

The value which will be used in this book for the ratio of the specific heats of air is

$$\kappa = \frac{c_p}{c_v} = 1.405.$$

EXAMPLES.

1. Find the weight of 4 cubic metres of hydrogen at 30° C and under the pressure of 800 mm. of mercury. Ans. 0.3398 kg.

2. Find the volume of 3 pounds of nitrogen at a pressure of 45 pounds to the square inch by the gauge and at 80° F. Ans. 10.34 cu. ft.

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* Poggendorff's *Annalen*, vol. cxlviii, p. 580.

3. Find the temperature at which one kilogram of air will occupy one cubic metre when at a pressure of 20,000 kilograms per square metre. Ans. 411°.2 C.

4. Find the pressure at which 2 pounds of carbonic acid at freezing-point of water will occupy 3 cubic feet. Ans. 79.4 pounds per sq. in.

5. A gas-receiver having the volume of 3 cubic feet contains half a pound of oxygen at 70° F. What is the pressure? Ans. 29.6 pounds per sq. in.

6. A spherical balloon 20 feet in diameter is to be inflated with hydrogen at 60° F. when the barometer stands at 30.2 inches, so that gas may not be lost on account of expansion when it has risen till the barometer stands at 19.6 inches and the temperature falls to 40° F. How many pounds and how many cubic feet are to be run in? Ans. 15.1 pounds, 2828 cu. ft.

7. A gas-receiver holds 14 ounces of nitrogen at 20° C. and under a pressure of 29.6 inches of mercury. How many will it hold at 32° F. and at the normal pressure of 760 mm.? Ans. 15.18 oz.

8. Two cubic feet of air expand at 50° F. from a pressure of 80 pounds to a pressure of 60 pounds by the gauge. What is the external work? Ans. 6464 ft. lbs.

9. What would have been the external work had the air expanded adiabatically? Ans. 4450 ft.-lbs.

10. Find the external work of 2 pounds of air which expand adiabatically until it doubles its volume, the initial pressure being 100 pounds absolute and the initial temperature 100° F. Ans. 36,100 ft.-lbs.

11. Find the external work of one kilogram of hydrogen which, starting with the pressure of 4 atmospheres and with the temperature of 500° C., expands adiabatically till the temperature becomes 30° C. Ans. 489,280 m.-kg.

12. Find the exponent for an exponential curve passing through the points $p = 30$, $v = 1.9$, and $p_1 = 15$, $v_1 = 9.6$. Ans. 0.4279.

13. Find the exponent for a curve to pass through the points $p = 40$, $v = 2$, and $p_1 = 12$, $v_1 = 6$. Ans. 1.0959.

14. In examples 2 and 3 let p be the pressure in pounds on the square inch and v the volume in cubic feet. Find the external work of expansion in each case. Ans. 21,900 and 12,010 ft.-lbs.

15. Find the intrinsic energy of one pound of nitrogen under the standard pressure of one atmosphere and at freezing-point of water. Ans. 66,655 ft.-lbs.

16. A pound of air has the volume 6 cubic feet under the pressure of 30 pounds absolute to the square inch. Find the intrinsic energy. Ans. 64,000 ft.-lbs.

17. In example 16 find the increase of entropy above that at atmospheric pressure and at freezing-point. Ans. $-$ 0.0516.

18. A kilogram of oxygen at the pressure of 6 atmospheres and at 100° C. expands isothermally till it doubles its volume. Find the change of entropy. Ans. 40.0434.

19. Suppose a hot-air engine, in which the maximum pressure is 100 pounds absolute, and the maximum temperature is 600° F., to work on a Carnot's cycle. Let the initial volume be 2 cubic feet, let the volume after isothermal expansion be 5 cubic feet, and the volume after adiabatic expansion be 8 cubic feet. Find the external work of one cycle; also the horse-power if the engine is double-acting and makes 30 revolutions per minute. Ans. 8966 ft.-lbs., 16.3 H.P.

CHAPTER VI.

SATURATED VAPORS.

OUR knowledge of the properties of saturated vapors is derived mainly from the experiments of Regnault.* In most works on steam and other vapors the results of his experiments are given in the form of empirical equations designed to be used for calculating tables of the properties of vapors. Many such tables have been calculated, and unfortunately they show such diversity of values as to lead to serious inconvenience when applied in practice. It therefore appears advisable to give the original data on which the empirical equations are based, so as to exhibit the limits of their application and the degree of accuracy to be attributed to them. The constants for the equations for calculating the pressures of saturated steam at different temperatures have been recomputed with care and accuracy, because there are minor discrepancies in the equations given by Regnault. Rowland's determinations of the mechanical equivalent of heat and of the specific heat of water have been adopted, and all of Regnault's results depending thereon have been revised and brought into concordance with them.

Pressure of Saturated Vapor.—Regnault's experiments on the temperature of saturated vapor consisted essentially in taking the temperature of the boiling-point of the vapor under varying pressures of the atmosphere, the apparatus being so arranged that the pressure could be varied from a small fraction of an atmosphere to more than twenty atmospheres.

* *Mémoires de l'Institut de France*, etc., tome xxvi.

The temperature was taken with mercurial thermometers, and the pressures were measured by a mercury column, and, after the necessary corrections were applied and temperatures were reduced to the air-thermometer, Regnault selected the results he deemed most trustworthy, and plotted a series of points, and then drew a smooth curve to represent the whole series of experiments.

He then selected points on the experimental curve at regular intervals, and with these points as data he calculated the constants of empirical formulæ for use in calculating the tabular values. The formula selected was of the form

$$\log p = a + b\alpha^n + c\beta^n, \quad . \quad . \quad . \quad . \quad (94)$$

in which p is the pressure, and n is the temperature minus the constant temperature t_0 of the lowest limit of the range of temperature to which the formula applies; i. e.,

$$n = t - t_0.$$

Let the points through which the curve represented by the equation is to pass be (p_0, t_0), (p_1, t_1), (p_2, t_2), (p_3, t_3), and (p_4, t_4), so chosen that

$$t_1 - t_0 = t_2 - t_1 = t_3 - t_2 = t_4 - t_3.$$
$$\therefore\ t_2 - t_0 = 2(t_1 - t_0), \quad (t_3 - t_0) = 3(t_1 - t_0), \quad (t_4 - t_0) = 4(t_1 - t_0).$$

Substituting the five known values of p and t in equation (94),

$$\left.\begin{aligned}
\log p_0 &= a + b + c;\\
\log p_1 &= a + b\alpha^{t_1 - t_0} + c\beta^{t_1 - t_0};\\
\log p_2 &= a + b\alpha^{2(t_1 - t_0)} + c\beta^{2(t_1 - t_0)};\\
\log p_3 &= a + b\alpha^{3(t_1 - t_0)} + c\beta^{3(t_1 - t_0)};\\
\log p_4 &= a + b\alpha^{4(t_1 - t_0)} + c\beta^{4(t_1 - t_0)}.
\end{aligned}\right\} \quad . \quad . \quad . \quad (95)$$

Now subtract each equation, member for member, from the one below it, and for convenience let

$$\log p_1 - \log p_0 = y_0, \text{ etc.}, \quad \alpha^{t_1 - t_0} = m, \quad \beta^{t_1 - t_0} = n.$$

$$\left.\begin{aligned} \therefore\ y_0 &= (m - 1)b + (n - 1)c; \\ y_1 &= (m^2 - m)b + (n^2 - n)c; \\ y_2 &= (m^3 - m^2)b + (n^3 - n^2)c; \\ y_3 &= (m^4 - m^3)b + (n^4 - n^3)c. \end{aligned}\right\} \quad . \quad . \quad . \quad (96)$$

Solving the several equations for c and equating the values,

$$\frac{y_0 - (m-1)b}{n-1} = \frac{y_1 - (m^2 - m)b}{n^2 - n}$$

$$= \frac{y_2 - (m^3 - m^2)b}{n^3 - n^2} = \frac{y_3 - (m^4 - m^3)b}{n^4 - n^3}. \quad . \quad (97)$$

Again, solving for b and equating the values and reducing,

$$\frac{ny_0 - y_1}{n - m} = \frac{ny_1 - y_2}{(n-m)m} = \frac{ny_2 - y_3}{(n-m)m^2}.$$

$$\therefore\ m^2ny_0 - m^2y_1 = mny_1 - my_2 = ny_2 - y_3.$$

$$\therefore\ mny_0 - my_1 = ny_1 - y_2;$$

$$mny_1 - my_2 = ny_2 - y_3.$$

$$\therefore\ mn = \frac{(m+n)y_1 - y_2}{y_0} = \frac{(m+n)y_2 - y_3}{y_1}; \quad . \quad (98)$$

$$m + n = \frac{y_1y_2 - y_0y_3}{y_1^2 - y_0y_2} = M; \quad . \quad . \quad . \quad . \quad . \quad (99)$$

$$mn = \frac{y_2^2 - y_1y_3}{y_1^2 - y_0y_2} = N. \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

Equations (99) and (100) enable us to calculate numerically the values of M and N from the five given values of $\log p$. Then solving for m and n,

$$m = \frac{M}{2} - \left(\frac{M^2}{4} - N\right)^{\frac{1}{2}};$$

$$n = \frac{M}{2} + \left(\frac{M^2}{4} - N\right)^{\frac{1}{2}}.$$

Solving one of the equations (97) for b,

$$b = \frac{ny_0 - y_1}{n(m-1) - (m^2 - m)} = \frac{ny_0 - y_1}{(m-1)(n-m)}. \quad (101)$$

Again, solving the first equation of (96) for c,

$$c = \frac{y_0 - \frac{ny_0 - y_1}{n - m}}{n - 1} = \frac{y_1 - my_0}{(n-1)(n-m)} \quad . \quad . \quad (102)$$

From the first equation of (95)

$$a = \log p_0 - b - c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

Finally, $\alpha = m^{\frac{1}{t_1 - t_0}}$; . . . . . (104)

$$\beta = n^{\frac{1}{t_1 - t_0}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

For temperatures below freezing-point Regnault used the equation

$$p = a + b\alpha^x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

which is an equation to a curve passing through three points at equidistant temperatures, and of which the solution is very simple.

Regnault's Data and Equations for Steam.—For equation (106) the data are:

$t_0 = -32°$ C. $\quad p_0 = 0.32$ mm. of mercury.
$t_1 = -16°$ C. $\quad p_1 = 1.29$ " " "
$t_2 = 0°$ C. $\quad p_2 = 4.60$ " " "

From which Regnault calculated the following equation by aid of seven-place logarithms:

A. For steam from $-32°$ to $0°$ C.,

$$p = a + b\alpha^n;$$
$$a = -0.08038;$$
$$\log b = 9.6024724 - 10;$$
$$\log \alpha = 0.033398;$$
$$n = 32° - t.$$

Regnault uses three equations of the form given by equation (97), for which the following are the data:

B. $t_0 = 0°$ C. $\quad p_0 = 4.60$ mm. of mercury.
$t_1 = 25°$ C. $\quad p_1 = 23.55$ " " "
$t_2 = 50°$ C. $\quad p_2 = 91.98$ " " "
$t_3 = 75°$ C. $\quad p_3 = 288.50$ " " "
$t_4 = 100°$ C. $\quad p_4 = 760$ " " "

C. $t_0 = 100°$ C. $\quad p_0 = 760$ " " "
$t_1 = 130°$ C. $\quad p_1 = 2030$ " " "
$t_2 = 160°$ C. $\quad p_2 = 4651.6$ " " "
$t_3 = 190°$ C. $\quad p_3 = 9426$ " " "
$t_4 = 220°$ C. $\quad p_4 = 17390$ " " "

D. $t_0 = -20°$ C. $\quad p_0 = 0.91$ " " "
$t_1 = +40°$ C. $\quad p_1 = 54.91$ " " "
$t_2 = 100°$ C. $\quad p_2 = 760$ " " "
$t_3 = 160°$ C. $\quad p_3 = 4651.6$ " " "
$t_4 = 220°$ C. $\quad p_4 = 17390$ " " "

And from these data he calculated, by aid of seven-place logarithms, the following equations, which are correct at Paris:

B. For steam from 0° to 100° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 4.7384380;$$
$$\log b = 0.6116485;$$
$$\log c = 8.1340339 - 10;$$
$$\log \alpha = 9.9967249 - 10;$$
$$\log \beta = 0.006865036;$$
$$n = t.$$

C. For steam from 100° to 220° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 5.4583895;$$
$$\log b = 0.4121470;$$
$$\log c = 7.7448901 - 10;$$
$$\log \alpha = 9.997412127 - 10;$$
$$\log \beta = 0.007590697;$$
$$n = t - 100.$$

D. For steam from — 20° to 220° C.,

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 6.2640348;$$
$$\log b = 0.1397743;$$
$$\log c = 0.6924351;$$
$$\log \alpha = 9.994049292 - 10;$$
$$\log \beta = 9.998343862 - 10;$$
$$n = t + 20.$$

Regnault gives complete tables of the properties of saturated steam, calculated by the equations just set down, using equations A and B for their respective ranges of temperature, but he uses equation D (which applies to the entire range from — 20° C. to + 120 C.) instead of equation C. He prob-

ably did this because the constants calculated for equation C with seven-place logarithms are unsatisfactory. None of the constants calculated by seven-place logarithms are quite satisfactory, for two places of significant figures are unavoidably lost in the calculation of tables; one place is lost in calculating the constants, and one is lost in applying the equations, leaving the fifth place, which should appear in the table, subject to error.

Equations for the Pressure of Steam at Paris.—In view of the preceding statements it appeared desirable to recalculate the constants for equations B and C with such a degree of accuracy as to exclude any doubt as to the reliability of the results. Accordingly the logarithms of the five values of p for each equation were taken from Vega's ten-place table, and then the remainder of the calculations were carried on with natural numbers, checking by independent methods, with the following results:

B. For steam from 0° to 100° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 4.7393622142;$$
$$\log b = 0.6117400190;$$
$$\log c = 8.1320378383 - 10;$$
$$\log \alpha = 9.996725532820 - 10;$$
$$\log \beta = 0.006864675924;$$
$$n = t.$$

C. For steam from 100° to 220° C.,

$$\log p = a - b\alpha^n + c\beta^n;$$
$$a = 5.4574301234;$$
$$\log b = 0.4119787931;$$
$$\log c = 7.7417476470 - 10;$$
$$\log \alpha = 9.99741106346 - 10;$$
$$\log \beta = 0.007642489113;$$
$$n = t - 100.$$

Pressure of Steam at Latitude 45°, French System.—It is customary to reduce all measurements to the latitude of 45° and to sea-level. The standard thermometer should then have its boiling- and freezing-points determined under, or

reduced to, such conditions. The value of g, the acceleration due to gravity given by equation (61), is 9.809218 metres at Paris, latitude 48° 50′ 14″, and at an elevation of 60 metres. At 45° and at sea-level $g = 9.806056$; consequently 760 mm. of mercury at 45° latitude give a pressure equal to that of

$$760 \times \frac{980.6056}{980.9218} = 759.755 \text{ mm.}$$

at Paris, and by equation B this corresponds to a temperature of 99°.991 C. In other words, the thermometer which is standard at 45° has each degree 0.99991 of the length of the degree of a thermometer standard at Paris.

Again, the height of a column of mercury at 45° latitude is $\frac{980.9218}{980.6056}$ times the height of a column which will give the same pressure at Paris. Consequently to reduce equation B to 45° latitude we have

$$\log p = a + \log \frac{980.9218}{980.6056} - b\alpha^{0.99991t} + c\beta^{0.99991t},$$

and for equation C

$$\log p = a + \log \frac{980.9218}{980.6056} - b\alpha^{(0.99991\,t\,-\,100)} + c\beta^{(0.99991t\,-\,100)}$$

$$= a + \log \frac{980.9218}{980.6056} - b\alpha^{-\,0.009\alpha^{0.9991(t\,-\,100)}} + c\beta^{-\,0.009\beta^{.99991(t\,-\,100)}}.$$

The resulting equations are:

B. For steam from 0° to 100° C. at 45° latitude,

$$\log p = a_1 - b\alpha_1^n + c\beta_1^n;$$
$$a_1 = 4.7395022;$$
$$\log b = 0.6117400190;$$
$$\log c = 8.1320378383 - 10;$$

$$\log \alpha_1 = 9.996725827522 - 10;$$
$$\log \beta_1 = 0.006864058103;$$
$$n = t.$$

C. For steam from 100° to 220° C. at 45° latitude,

$$\log p = a_1 - b_1\alpha_1^n + c_1\beta_1^n;$$
$$a_1 = 5.4575701;$$
$$\log b_1 = 0.4120020935;$$
$$\log c_1 = 7.7416788646 - 10;$$
$$\log \alpha_1 = 9.997411296464 - 10;$$
$$\log \beta_1 = 0.007641801289;$$
$$n = t - 100.$$

Pressure of Steam at Latitude 45°, English System.— To reduce the equations for the pressure of steam so that they will give the pressures in pounds on the square inch for degrees Fahrenheit there are required the comparison of measures of length and of weight, the comparison of the scales of the thermometers, and the specific gravity of mercury.

Professor Rogers * gives for the length of the metre 39.3702 inches. This differs from the value given by Captain Clarke † by an amount that does not affect the values in the tables, his value being 39.370432 inches.

Professor Miller ‡ gives for the weight of one kilogram 2.20462125 pounds.

Regnault § gives for the weight of one litre of mercury 13.5959 kilograms.

The degree Fahrenheit is $\frac{5}{9}$ of the degree centigrade.

Let $$k = \frac{13.5959 \times 2.204621}{39.3702^3};$$

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* *Proceedings of the Am. Acad. of Arts and Sciences*, 1882–83; also additional observations.

† *Proceedings of the Royal Society*, vol. xv, 1866.

‡ *Philosophical Transactions*, cxlvi, 1856.

§ *Mémoires de l'Institut de France*, vol. xxi.

then equations B and C have for the reduction to degrees Fahrenheit, and pounds on the square inch,

$$\log p = a_1 + \log k - b\alpha_1^{8^n} + c\beta_1^{8^n};$$

$$\log p = a_1 + \log k - b_1\alpha_1^{8^n} + c_1\beta_1^{8^n}.$$

The resulting equations are:

B. For steam from 32° to 212° F. in pounds on the square inch,

$$\log p = a_2 - b\alpha_2^n + c\beta_2^n;$$

$$a_2 = 3.025908;$$

$$\log b = 0.6117400;$$

$$\log c = 8.13204 - 10;$$

$$\log \alpha_2 = 9.998181015 - 10;$$

$$\log \beta_2 = 0.0038134;$$

$$n = t - 32.$$

C. For steam from 212° to 428° F. in pounds on the square inch,

$$\log p = a_2 - b_1\alpha_2^n + c_1\beta_2^n;$$

$$a_2 = 3.743976;$$

$$\log b_1 = 0.4120021;$$

$$\log c_1 = 7.74168 - 10;$$

$$\log \alpha_2 = 9.998561831 - 10;$$

$$\log \beta_2 = 0.0042454;$$

$$n = t - 212.$$

Other Equations for the Pressure of Steam.—A number of other forms have been proposed for the empirical

equations for calculating the pressure of saturated steam; one of the best is that given by Rankine * having the form

$$\log p = A - \frac{B}{T} - \frac{C}{T^2}. \quad . \quad . \quad . \quad (107)$$

Assuming the absolute zero on the Fahrenheit scale to be at $-461°.2$, he computed for pressure on the square inch the following values for his constants:

$$A = 6.1007; \quad \log B = 3.43642; \quad \log C = 5.59873.$$

This equation has the advantage that it may be solved directly for T, a property that Regnault's equations do not have. It gives fairly accurate results, and the greater part of English tables of properties of saturated steam are calculated by its aid.

A number of exponential formulæ have been devised, of which the principal advantage is the facility of application. The following, by Magnus, gives pressures in mm. of mercury for degrees centigrade, and agrees quite well with Regnault's results below 100°, but is not so correct above 100°:

$$p = 4.525 \times 10^{\frac{7.4475t}{234.69+t}}. \quad . \quad . \quad . \quad . \quad (108)$$

Pressure of Other Vapors.—Regnault † determined also the pressure of a large number of saturated vapors at various temperatures, and deduced equations for each in the form of equation (94). The equations and the constants as determined by him for the commoner vapors are given in the following table:

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* *Steam-engine and Other Prime Movers.*

† *Académie des Sciences, Comptes rendus*, tome xxxvi.

PRESSURE OF SATURATED VAPORS.

	$\log p$	a	b	c
Alcohol	$a - b\alpha^n + c\beta^n$	5.4562028	4.9809960	0.0485397
Ether	$a + b\alpha^n - c\beta^n$	5.0286298	0.0002284	3.1906390
Chloroform	$a - b\alpha^n - c\beta^n$	5.2253893	2.9531281	0.0668673
Carbon bisulphide	$a - b\alpha^n - c\beta^n$	5.4011662	3.4405663	0.2857386
Carbon tetrachloride	$a - b\alpha^n - c\beta^n$	12.0962331	9.1375180	1.9674890

	$\log a$	$\log \beta$	n	Limits.
Alcohol	$\bar{1}.99708557$	$\bar{1}.9409485$	$t + 20$	$-20°, +150°$ C.
Ether	0.0145775	$\bar{1}.996877$	$t + 20$	$-20°, +120°$ C.
Chloroform	$\bar{1}.9974144$	$\bar{1}.9868176$	$t - 20$	$+20°, +164°$ C.
Carbon bisulphide	$\bar{1}.9977628$	$\bar{1}.9911997$	$t + 20$	$-20°, +140°$ C.
Carbon tetrachloride	$\bar{1}.9997120$	$\bar{1}.9949780$	$t + 20$	$-20°, +188°$ C.

Zeuner* states that there is a slight error in Regnault's calculation of the constants for aceton, and gives instead

$$\log p = a - b\alpha_n + c\beta^n;$$
$$a = 5.3085419;$$
$$\log b\alpha^n = + 0.5312766 - 0.0026148t;$$
$$\log c\beta^n = - 0.9645222 - 0.0215592t.$$

Differential Coefficient $\frac{dp}{dt}$.—From the general form of the equation (94) we have

$$\log_e p = \frac{1}{M}a + \frac{1}{M}b\alpha^n + \frac{1}{M}c\beta^n, \quad . \quad . \quad (109)$$

M being the modulus of the common system of logarithms. Differentiating,

$$\frac{dp}{pdt} = \frac{1}{M}b \log_e \alpha \cdot \alpha^n + \frac{1}{M}c \log_e \beta \cdot \beta^n;$$

**Mechanische Wärmetheorie.*

or, reducing to common logarithms,

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{M^2}b \log \alpha \cdot \alpha^n + \frac{1}{M^2}c \log \beta \cdot \beta^n;$$

$$\therefore \quad \frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

For saturated steam at 45° latitude the constants to be used with equation (110) are:

French units.

B. For 0° to 100° C., mm. of mercury,

$$\log A = 8.8512729 - 10;$$
$$\log B = 6.69305 - 10;$$
$$\log \alpha_1 = 9.996725828 - 10;$$
$$\log \beta_1 = 0.0068641.$$

C. For 100° to 220° C., mm. of mercury,

$$\log A = 8.5495158 - 10;$$
$$\log B = 6.34931 - 10;$$
$$\log \alpha_1 = 9.997411296 - 10;$$
$$\log \beta_1 = 0.0076418.$$

English units.

B. For 32° to 212° F., pounds on the square inch,

$$\log A = 8.5960005 - 10;$$
$$\log B = 6.43778 - 10;$$
$$\log \alpha_2 = 9.998181015 - 10;$$
$$\log \beta_2 = 0.0038134.$$

C. For 212° to 428° F., pounds on the square inch,

$$\log A = 8.2942434 - 10;$$
$$\log B = 6.09403 - 10;$$
$$\log \alpha_2 = 9.998561831 - 10;$$
$$\log \beta_2 = 0.0042454.$$

It is to be remarked that $\frac{dp}{dt}$ may be found approximately by dividing a small difference of pressure by the corresponding difference of temperature; that is, by calculating $\frac{\Delta p}{\Delta t}$. With a table for even degrees of temperature we may calculate the value approximately for a given temperature by dividing the difference of the pressures corresponding to the next higher and the next lower degrees by two.

The following table of constants for the several vapors named were calculated by Zeuner from the preceding equations for temperature and pressure of the same vapors:

DIFFERENTIAL COEFFICIENT $\frac{1}{p}\frac{dp}{dt}$.

	Sign.		$\log (A\alpha^n)$	$\log (B\beta^n)$
	$A\alpha^n$	$B\beta^n$		
Alcohol	+	−	$-1.1720041 - 0.0029143t$	$-2.9992701 - 0.0590515t$
Ether	+	+	$-1.3396624 - 0.0031223t$	$-4.4616396 + 0.0145775t$
Chloroform	+	+	$-1.3410130 - 0.0025856t$	$-2.0667124 - 0.0131824t$
Carbon bisulphide	+	+	$-1.4339778 - 0.0022372t$	$-2.0511078 - 0.0088003t$
Carbon tetrachloride	+	+	$-1.8611078 - 0.0002880t$	$-1.3812195 - 0.0050220t$
Aceton	+	+	$-1.3268535 - 0.0026148t$	$-1.9064582 - 0.0215592t$

Rowland's Experiments.—The most accurate and reliable determinations of the mechanical equivalent of heat were made by Rowland,* who found that there is a notable variation of the mechanical equivalent at low temperatures. His experiments give a very delicate determination of the specific heat of water at low temperatures, and consequently of the heat of the liquid, i.e., the heat required to raise water from freezing-point to a given temperature.

The apparatus used was similar to that used by Joule, with modifications to give greater certainty of results. The calorimeter was of larger size, and the paddle had the upper vanes curved like the blades of a centrifugal pump, to give a strong

* *Proceedings of the American Academy*, vol. xv (N. S. vii), 1879.

circulation up through the centre, past the thermometer for taking the temperatures, and down at the outside. The paddle was driven by a petroleum-engine, and the power applied was measured by making the calorimeter into a friction-brake, with two arms at which the turning moment was measured. Radiation was made as small as possible, and then was made determinate by use of a water-jacket outside of the calorimeter.

The experiments consisted essentially in delivering a measured amount of work to the water in the calorimeter and in measuring the rise of temperature due to it. The whole range of experiments was from 2° to 41° C. It is, however, convenient to make our calculations of the properties of saturated steam and of water begin at freezing-point; consequently for this purpose it has been assumed that the mechanical equivalent of heat is constant from freezing-point to 3° C. The work required to raise one kilogram of water from 2° to 3° is 430 metre-kilograms, so that our assumption gives for the work required to raise one kilogram of water from freezing-point to 3°, 1290 metre-kilograms. Any error that may come from this assumption is eliminated in practical calculations, for we always deal with differences of the heat of the liquid, and seldom or never have a temperature so low as 2° C. With the assumption just made Rowland's experiments may be expressed by the table on page 95.

In column 2 is given the work in metre-kilograms required to raise one kilogram of water from freezing-point to the temperature given in column 1, while column 3 gives the most probable mechanical equivalent of heat for the several temperatures from 5° to 36° C. The other two columns will be discussed under the head of Specific Heat of Water.

Standard Temperature.—In the beginning of our work it was stated that we should use 62° F. for our standard temperature; and the reasons for so doing may now be shown. We know actually nothing about the specific heat of water from 0° to 2° C.; consequently the commonly accepted value of the thermal unit—i.e., the heat required to raise one unit

ROWLAND'S MECHANICAL EQUIVALENT OF HEAT.

1 Degrees C.	2 Total Number of Meter Kilograms.	3 Mechanical Equivalent of Heat.	4 Heat of the Liquid, Experimental.	5 Heat of the Liquid, Calculated.	1 Degrees C.	2 Total Number of Meter Kilograms.	3 Mechanical Equivalent of Heat.	4 Heat of the Liquid, Experimental.	5 Heat of the Liquid, Calculated.
1	430	.....	1.0068	1.007	22	9424	426.1	22.065	22.063
2	860	.....	2.0135	2.014	23	9850	426.0	23.063	23.061
3	1290	.....	3.0204	3.022	24	10277	425.9	24.062	24.059
4	1721	.....	4.0295	4.029	25	10701	425.8	25.055	25.058
5	2150	429.8	5.0339	5.036	26	11128	425.7	26.054	26.053
6	2580	429.5	6.0408	6.040	27	11553	425.6	27.050	27.048
7	3009	429.3	7.0452	7.045	28	11978	425.6	28.045	28.042
8	3439	429.0	8.0520	8.049	29	12399	425.5	29.031	29.037
9	3868	428.8	9.0564	9.054	30	12828	425.6	30.035	30.032
10	4296	428.5	10.059	10.058	31	13253	425.6	31.030	31.027
11	4723	428.3	11.058	11.060	32	13675	425.6	32.018	32.023
12	5151	428.1	12.061	12.061	33	14101	425.7	33.016	33.018
13	5578	427.9	13.060	13.063	34	14527	425.7	34.011	34.014
14	6006	427.7	14.063	14.064	35	14952	425.8	35.008	35.009
15	6433	427.4	15.065	15.066	36	15379	425.8	36.008	36.007
16	6861	427.2	16.004	16.066	37	15805	.....	37.007	37.005
17	7289	427.0	17.066	17.066	38	16231	.....	38.003	38.004
18	7717	426.8	18.068	18.066	39	16657	.....	39.000	39.002
19	8144	426.6	19.068	19.066	40	17083	.....	39.998	40.000
20	8571	426.4	20.068	20.066	41	17508	.....	40.993	.....
21	8997	426.2	21.065	21.064					

of weight of water from 0° to 1° C., or from 32° to 33° F.—is an ideal quantity inferred from the behavior of water at higher temperatures. It is more scientific to take an easily verified quantity for the standard; and there is a practical convenience in choosing 62° F. for the standard temperature, because it is near the mean temperature of the air during experimental work. Therefore it is near the mean temperature in the calorimeter during ordinary work with that instrument; and the specific heat of water for the range of temperature in the calorimeter may usually be considered to be unity, without error, unless great refinement is desired. Moreover, 62° F. is the temperature at which the English units of weight and measure are standard.

Mechanical Equivalent of Heat.—62° F. corresponds with 16⅔° C., at which the mechanical equivalent of heat given

in the table of Rowland's experiments is 427.1. The value of g at Baltimore, latitude 39° 17′, is 980.05 centimetres; therefore, reducing to 45° of latitude, where $g = 980.6056$ centimetres, the value of J is

$$J = 427.1 \times \frac{980.05}{980.6056} = 426.9 \text{ metre-kilograms.}$$

To reduce to the English system of units it is sufficient to multiply by $\frac{9}{5}$, giving

$$J = 778 \text{ foot-pounds.}$$

Since the value given by Joule is commonly quoted, it will be of interest to make a comparison of his latest work (1873) with Rowland's, as given in the following table:

COMPARISON OF ROWLAND'S AND JOULE'S EXPERIMENTS.

Temperature.	Joule's Value at Manchester, English System.	Reduced to the Air-thermometer and to the Latitude of Baltimore.		Rowland's Value Corresponding.
		English.	French.	
12°.7	774.6	778.5	427.1	428.0
14 .5	767.0	770.5	422.7	427.6
14 .7	772.7	776.1	425.8	427.5
15 .5	773.1	776.4	426.0	427.3
17 .3	774.0	777.0	426.3	426.9

Specific Heat of Water.—From freezing-point to 40° C. the specific heat of water may be determined from Rowland's experiments on the mechanical equivalent of heat as arranged in the table on page 95. For this purpose we may first divide the metre-kilograms required to raise one kilogram of water from freezing-point to a given temperature, as set down in column 2 of that table, by the mechanical equivalent of heat (427.1 kilograms) at the latitude of Baltimore. This gives the experimental values of the heat of the liquid given in column 4. This column shows some accidental experimental irregularities, which were eliminated by drawing a diagram

with temperatures for abscissæ and with heats of the liquid for ordinates. That diagram also made it possible to assign the series of values of the specific heat of water from freezing-point to 40° C. given in the following table:

SPECIFIC HEAT OF WATER.

Range.		Specific Heat.
Centigrade.	Fahrenheit.	
0 to 5	32 to 41	1.0072
5 to 10	41 to 50	1.0044
10 to 15	50 to 59	1.0016
15 to 20	59 to 68	1.
20 to 25	68 to 77	0.9984
25 to 30	77 to 86	0.9948
30 to 35	86 to 95	0.9954
35 to 40	95 to 104	0.9982
40 to 45	104 to 113	1.
45 to 155	113 to 311	1.008
155 to 200	311 to 392	1.046

The calculated heats of the liquid given in column 5 of the table on page 95 were calculated with the specific heats in the above table. They are more regular than the quantities in the 4th column, and differ from them no more than can properly be attributed to accidental irregularities.

For the heat of the liquid and the specific heat of water beyond 40° C. it is necessary to go to Regnault's determinations. Having satisfied himself that the specific heat of water at low temperatures was practically constant, Regnault proceeded to determine the specific heat of water at high temperatures by the method of mixtures, running hot water from the water-space of a boiler into a calorimeter containing cold water. The record of his work gives forty such tests. To prepare these tests for our purposes it was necessary to correct his calculations for the true specific heat of the water in the calorimeter; after this was done the true heat of the liquid was readily determined and was plotted on the same diagram with the heats of the liquid from Rowland's work, and

specific heats for water from 40° to 45°, from 45° to 155°, and from 155° to 200°, centigrade, were assigned as given in the above table of the specific heats of water.

It is interesting to know in this connection that after his investigation of the mechanical equivalent of heat and the consequent determination of the specific heat of water Rowland repeated Regnault's experiments on the specific heat of water at low temperatures by the method of mixtures, and that he found it possible to recognize the very peculiar behavior of water at low temperatures, which is so clearly shown by his experiments recorded in the table on page 95; but his work showed that the unavoidable irregularities of the method of mixtures were a large part of the differences between the true specific heat and unity, which shows why Regnault failed to find any variation in the specific heat of water at low temperatures.

In using the specific heats of water for calculating the heat of the liquid by aid of the table on page 97 it is necessary to proceed step by step. Thus the heat of the liquid at 23° C. is

$$5 \times 1.0072 + 5 \times 1.0044 + 5 \times 1.0016 + 5 \times 1 + 3 \times 0.9984 = 23.061 \text{ B. T. U.}$$

Specific Heat of Other Liquids.—Regnault determined the specific heats and the heats of the liquid of various other liquids besides those for water, using the method of mixtures for all his work. The following table gives his results unaltered:

HEAT OF THE LIQUID.

Alcohol	$q = 0.54754t + 0.0011218t^2 + 0.00000220 6t^3$
Ether	$q = 0.52901t + 0.0002959t^2$
Chloroform	$q = 0.23235t + 0.0000507t^2$
Carbon bisulphide	$q = 0.23523t + 0.0000815t^2$
Carbon tetrachloride	$q = 0.19798t + 0.0000906t^2$
Aceton	$q = 0.50643t + 0.0003965t^2$
Water	$q = t + 0.00002t^2 + 0.0000003t^3$

Regnault's equation for water is included here to make his work complete; it of course gives very different results from those by the table of specific heats on page 97.

The specific heat for any liquid may be determined at any temperature by differentiating the corresponding equation for the heat of the liquid. Thus Regnault's equation for the specific heat of water is

$$c = \frac{dq}{dt} = 1 + 0.00004t + 0.0000009t^2.$$

Total Heat.—This term is defined as the heat required to raise a unit of weight of water from freezing-point to a given temperature, and to entirely evaporate it at that temperature. The experiments made by Regnault were in the reverse order; that is, steam was led from a boiler into the calorimeter and there condensed. Knowing the initial and final weights of the calorimeter, the temperature of the steam, and the initial and final temperatures of the water in the calorimeter, he was able, after applying the necessary corrections, to calculate the total heats for the several experiments.

As a conclusion of the work he gives the following values for the total heats:

10°	610	By equation, 609.6
63°	625	625.2
100°	637	
195°	666	

Assuming an equation of the form

$$\lambda = A + Bt, \quad . \quad . \quad . \quad . \quad . \quad (111)$$

Regnault calculated the constants from the values given for 100° and 195°, and gives the equation

$$\lambda = 606.5 + 0.305t. \quad . \quad . \quad . \quad . \quad (112)$$

An investigation of the original experimental results, allowing for the true specific heat of the water in the calorimeter, showed that the probable errors of the method of determining the total heat were larger than the deviations of the true specific heats from unity, the value assumed by Regnault; and, further, it appeared that his equation represents our best knowledge of the total heat of steam. The probable error appears to lie between $\frac{1}{1000}$ and $\frac{1}{500}$, making the total heat the most uncertain of the experimental properties of steam.

For the Fahrenheit scale the equation becomes

$$\lambda = 1091.7 + 0.305(t - 32). \quad . \quad . \quad . \quad (113)$$

Regnault gives the equations following for other liquids:

Ether.....	$\lambda =$ 94	$+ 0.45t$	$- 0.00055556t^2$
Chloroform	$\lambda =$ 67	$+ 0.1375t$	
Carbon bisulphide...	$\lambda =$ 90	$+ 0.14601t$	$- 0.0004123t^2$
Carbon tetrachloride	$\lambda =$ 52	$+ 0.14625t$	$- 0.000172t^2$
Aceton.....	$\lambda =$ 140.5	$+ 0.36644t$	$- 0.000516t^2$

Heat of Vaporization.—If the heat of the liquid be subtracted from the total heat, the remainder is called the heat of vaporization, and is represented by r, so that

$$r = \lambda - q. \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

Specific Volume of Liquids.—The coefficient of expansion of most liquids is large as compared with that of solids, but it is small as compared with that of gases or vapors. Again, the specific volume of a vapor is large compared with that of the liquid from which it is formed. Consequently the error of neglecting the increase of volume of a liquid with the rise of temperature is small in equations relating to the thermodynamics of a saturated vapor, or of a mixture of a liquid and its vapor when a considerable part by weight of the mixture is vapor. It is therefore customary to consider the specific volume of a liquid σ to be constant.

The following table gives the specific gravities and specific volumes of liquids:

SPECIFIC GRAVITIES AND SPECIFIC VOLUMES OF LIQUIDS.

	Specific Gravity, compared with Water at 4° C.	Specific Volume.	
		Cubic Metres.	Cubic Feet.
Alcohol	0.80625	0.001240	
Ether	0.736	0.001350	
Chloroform	1.527	0.000655	
Carbon bisulphide	1.2922	0.000774	
Carbon tetrachloride	1.6320	0.00613	
Aceton	0.81	0.00123	
Sulphur dioxide	1.4336	0.0006981	0.0112
Ammonia	0.6364	0.001571	0.0252
Water	1	0.001	0.01602

Experiments were made by Hirn* to determine the volumes of liquid at high temperatures compared with the volume at freezing-point, by a method which was essentially to use them for the expansive substance of a thermometer. The results are given in the following equations:

SPECIFIC VOLUMES OF HOT LIQUIDS.

		Logarithms.
Water,	$v = 1 + 0.000108678751$	6.0361445 — 10
100° C. to 200° C.	$+ 0.0000030073653t^2$	4.4781862 — 10
(Vol. at 4° = unity.)	$+ 0.0000000287304223t^3$	1.4583419 — 10
	$- 0.000000000006645703 1t^4$	8.8225409 — 20
Alcohol,	$v = 1 + 0.000738922651$	6.8685991 — 10
30° C. to 160° C.	$+ 0.0000105523512$	3.0233492 — 10
(Vol. at o° = unity.)	$- 0.0000000924808423$	2.9660517 — 10
	$+ 0.0000000004041356714$	0.6065278 — 10
Ether,	$v = 1 + 0.001348905971$	7.1299817 — 10
30° C. to 130° C.	$+ 0.0000065537t^2$	4.8164866 — 10
(Vol. at o° = unity.)	$- 0.0000000344907563$	2.5377028 — 10
	$+ 0.0000000033772062t^4$	0.5285571 — 10
Carbon Bisulphide,	$v = 1 + 0.001168055971$	7.0674636 — 10
30° C. to 160° C.	$+ 0.0000016489598t^2$	4.2172103 — 10
(Vol. at o° = unity.)	$- 0.0000000008111906 2t^3$	0.9091229 — 10
	$+ 0.0000000000609465894t^4$	9.7849494 — 20
Carbon Tetrachloride,	$v = 1 + 0.001067188 31$	7.0282409 — 10
30° C. to 160° C.	$+ 0.0000035651378t^2$	4.5520763 — 10
(Vol. at o° = unity.)	$- 0.0000000149492818t^3$	2.1746202 — 10
	$+ 0.0000000000851823184t^4$	9.9303494 — 20

* *Annales de Chimie et de Physique*, 1867.

Internal and External Latent Heat. — The heat of vaporization overcomes external pressure, and changes the state from liquid to vapor at constant temperature and pressure. Let the specific volume of the saturated vapor be s and that of the liquid be σ; then the change of volume is $s - \sigma = u$ on passing from the liquid to the vaporous state. The external work is

$$p(s - \sigma) = pu, \quad . \quad . \quad . \quad . \quad . \quad (115)$$

and the corresponding amount of heat, or the external latent heat, is

$$Ap(s - \sigma) = Apu. \quad . \quad . \quad . \quad . \quad . \quad (116)$$

The heat required to do the disgregation work, or the internal latent heat, is

$$\rho = r - Apu. \quad . \quad . \quad . \quad . \quad . \quad (117)$$

General Equation.—A pound or a kilogram of a mixture of a liquid and its vapor consists of a certain part, x, of vapor, and the remainder, $1 - x$, of liquid. The specific volume of the mixture is

$$v = xs + (1 - x)\sigma = (s - \sigma)x + \sigma = ux + \sigma, \quad (118)$$

in which s is the specific volume of saturated vapor, σ is the specific volume of the liquid, and u is the increase of volume due to vaporization.

Since the pressure of saturated vapor depends on the temperature only, the variables in the general equation for a mixture of a liquid and its vapor are temperature and volume; and the specific volume as shown by equation (118) depends upon x, the condition of the mixture, and on s or u, which in turn depend on the temperature only. Consequently the general equation may be expressed as a function of t and x.

When a mixture of liquid and its vapor receives heat there is in general an increase in the temperature of the portion x of vapor and in the portion $1 - x$ of liquid, and there

is a vaporization of part of the liquid. Taking c for the specific heat of the liquid and h for the specific heat of the vapor, while r is the heat of vaporization, we shall have for an infinitesimal change

$$dQ = hxdt + c(1 - x)dt + rdx. \quad . \quad . \quad (119)$$

Application of the First Law.—The first law of thermodynamics is applied to equation (119) by combining it with equation (23), so that

$$dQ = A(dE + pdv) = hxdt + c(1 - x)dt + rdx; \quad (120)$$

$$\therefore \quad dE = \frac{1}{A}[hx + c(1 - x)]dt + \frac{r}{A}dx - pdv. \quad . \quad . \quad (121)$$

But $$dv = \left(\frac{dv}{dt}\right)_x dt + \left(\frac{dv}{dx}\right)_t dx;$$

$$\therefore \quad dE = \left\{\frac{1}{A}[hx + c(1 - x)] - p\left(\frac{dv}{dt}\right)_x\right\} dt + \left[\frac{r}{A} - p\left(\frac{dv}{dx}\right)_t\right]dx. \quad (122)$$

Since $\dfrac{d^2E}{dx\,dt} = \dfrac{d^2E}{dt\,dx}$,

$$\frac{d}{dx}\left\{\frac{1}{A}[hx + c(1 - x)] - p\left(\frac{dv}{dt}\right)_x\right\}_t = \frac{d}{dt}\left[\frac{r}{A} - p\left(\frac{dv}{dx}\right)_t\right]_x.$$

Bearing in mind that h, c, and p are functions of t, and not of x, the differentiation gives

$$\frac{1}{A}(h - c) - p\frac{d^2v}{dx\,dt} = \frac{1}{A}\frac{dr}{dt} - \frac{dp}{dt}\left(\frac{dv}{dx}\right)_t - p\frac{d^2v}{dt\,dx}. \quad (123)$$

Since p and r are functions of t, and not of x, the expressions $\dfrac{dr}{dt}$ and $\dfrac{dp}{dt}$ are written instead of $\left(\dfrac{dr}{dt}\right)_x$ and $\left(\dfrac{dp}{dt}\right)_x$. From equation (118) we have (σ being constant)

$$\left(\frac{dv}{dx}\right)_t = u,$$

and, further, we have

$$\frac{d^2v}{dx\,dt} = \frac{d^2v}{dt\,dx},$$

so that equation (123) reduces to

$$\frac{dr}{dt} + c - h = Au\frac{dp}{dt}. \quad . \quad . \quad . \quad . \quad (124)$$

Application of the Second Law.—By this law $\frac{dQ}{T}$ is an exact differential.

$$\therefore \quad \frac{dQ}{T} = \frac{hx + c(1-x)}{T}dt + \frac{r}{T}dx. \quad . \quad . \quad (125)$$

But $$\frac{d^2\phi}{dx\,dt} = \frac{d^2\phi}{dt\,dx} \quad \text{or} \quad \frac{d^2\int\frac{dQ}{T}}{dx\,dt} = \frac{d^2\int\frac{dQ}{T}}{dt\,dx};$$

$$\therefore \quad \frac{d}{dx}\left[\frac{hx + c(1-x)}{T}\right]_t = \frac{d}{dt}\left(\frac{r}{T}\right)_x; \quad . \quad . \quad (126)$$

$$\therefore \quad \frac{h-c}{T} = \frac{T\frac{dr}{dt} - r}{T^2};$$

$$\therefore \quad \frac{dr}{dt} + c - h = \frac{r}{T}. \quad . \quad . \quad . \quad . \quad (127)$$

First and Second Laws Combined.—The combination of equations (124) and (127) gives

$$r = AuT\frac{dp}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

Natures of the Specific Heats. — Both the specific pressure p and the specific volume s of saturated vapor depend on the temperature, so that we can have neither specific heat at constant volume nor specific heat at constant pressure, as we had with perfect gases. The specific heat c for the liquid and the specific heat h for the vapor are the amounts of heat required to raise the temperature of one unit of weight one degree, under the condition that the pressure shall rise with the temperature, according to the law for saturated vapor. The volume of the liquid, indeed, changes so slowly that we can ignore it; but the volume of the vapor changes rapidly. The specific heat of the liquid as determined by Rowland up to 40° C. was at atmospheric pressure, but Regnault's determinations for higher temperatures were under a varying pressure; for our present purpose we may assume the determinations by both experimenters to conform to our definition just given.

Equation (127) gives a ready way of calculating the specific heat for a vapor, for from it

$$h = c + \frac{dr}{dt} - \frac{r}{T}. \quad . \quad . \quad . \quad . \quad . \quad (129)$$

Now r may be readily expressed as a function of t, and then by differentiation $\frac{dr}{dt}$ may be determined. For steam

$$r = \lambda - q = 606.5 + 0.305t + t_1 + c(t - t_1),$$

in which t_1 is the temperature at the beginning of the range, as given by the table on page 97, within which t may fall. Therefore

$$\frac{dr}{dt} = 0.305 - c,$$

and

$$h = 0.305 - \frac{r}{T}. \quad . \quad . \quad . \quad . \quad (130)$$

For other vapors the equations, deduced from the empirical equations for λ and q on pages 98 and 100, are somewhat more complicated, but they involve no especial difficulty.

The following table gives the values of h for steam at several absolute pressures:

SPECIFIC HEAT OF STEAM.

Pressures, lbs. per sq. in., p..	5	50	100	200	300
Temperatures, t° F.....	162.3	280.9	327.6	381.7	417.4
Specific heat, h	−1.607	−1.237	−1.122	−1.001	−0.931

The negative sign shows that heat must be abstracted from saturated steam when the temperature and pressure are increased, otherwise it will become superheated. On the other hand, steam, when it suddenly expands with a loss of temperature and pressure, suffers condensation, and the heat thus liberated supplies that required by the uncondensed portion.

Hirn * verified this conclusion by suddenly expanding steam in a cylinder with glass sides, whereupon the clear saturated steam suffered partial condensation, as indicated by the formation of a cloud of mist. The reverse of this experiment showed that steam does not condense with sudden compression, as shown by Cazin.

Ether has a positive value for h. As the theory indicates, a cloud is formed during sudden compression, but not during sudden expansion.

The table of values of h for steam shows a notable decrease for higher temperatures, which indicates a point of inversion at which h is zero and above which h is positive, but the temperature of that point cannot be determined from our experimental knowledge. For chloroform the point of inversion was calculated by Cazin † to be 123°.48, and determined experimentally by him to be between 125° and 129°. The discrepancy is mostly due to the imperfection of the apparatus

* *Bulletin de la Société Industr. de Mulhouse*, cxxxiii.

† *Comptes rendus de l'Académie des Sciences*, lxii.

used, which substituted finite changes of considerable magnitude for the indefinitely small changes required by the theory.

Specific Volume and Density. — The most important result of the application of the methods of thermodynamics to the properties of saturated vapor is expressed by equation (128), which gives a method of calculating the specific volume; thus:

$$s = u + \sigma = \frac{r}{AT}\frac{1}{\frac{dp}{dt}} + \sigma. \quad . \quad . \quad . \quad (131)$$

The numerical value of σ for water for French units is 0.001, and for English units is $\frac{1}{62.4} = 0.016$, nearly. The density, or weight of a unit of volume, is of course the reciprocal of the specific volume.

It is of interest to consider the degree of accuracy that may be expected from this method of calculating the density of saturated vapor. The value of r depends on λ and q; for the first Regnault gives three figures in the data from which the empirical equation is deduced, and the experimental work does not indicate a greater degree of accuracy. The fourth figure, if stated, is likely to be in error to the extent of five units. The value of T is commonly stated in four figures, of which the last may be in error by two units. A as determined by Rowland has four figures, the last being uncertain to the extent of one or two units. The differential coefficient $\frac{dp}{dt}$ is deduced from the equations for calculating p; and those equations are derived from data having five places of significant figures. Now each of the equations B and C, for steam at 45° latitude for the English system, gives a pressure of 14.6967 pounds on the square inch; but the specific volume calculated by aid of equation B is 26.550 cubic feet, while equation C gives 26.637 cubic feet. The mean, 26.60, differs from either extreme by about one in seven hundred. This discrepancy is due to the fact that the curves represented by

equations B and C meet at the common temperature, 212°, but do not have a common tangent. Since the equations are empirical and not logical, the error or uncertainty is unavoidable, and all calculated specific volumes are affected by a similar uncertainty. The greatest probable error is in determining r, for which it may be about one in one thousand. The error introduced into this equation by using the values of A in common use, that is, 772 instead of 778, is about one in one hundred.

In all recent tables of the properties of saturated vapor the specific volumes are calculated by the method just discussed, on account of the great difficulty of experimental determinations. The error of the calculation is not greater than the errors of some other parts of the table which are determined by direct experiments.

Experimental Determinations of Specific Volume.— The uncertainty of the direct determination of the density of saturated vapor is due to the difficulty of determining when it is dry and saturated; a small quantity of liquid present or a slight degree of superheating will introduce serious errors.

Two series of tests of the specific volume of saturated vapors have been made, by Tate and Fairbairn * and by Perot.† Though made with much skill and ingenuity, the experiments by Tate and Fairbairn are affected by so large experimental errors that they would have very little interest were it not that some tables of the properties of saturated steam now in use depend partly on these tests. The highest pressure used for these tests was 114 inches of mercury, or about 50 pounds absolute per square inch. Within this range the error of Tate and Fairbairn's empirical equation deduced from their experiments is three or four per cent; beyond fifty pounds the errors become very large. The empirical equation just referred to is

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* *Philosophical Transactions*, vol. cl, 1860.

† *Journal Franklin Institute*, vol. cxxxiii, p. 55.

$$V = 25.62 + \frac{49513}{P + 0.72},$$

where V is the volume of steam compared with that of the water from which it is produced, and P is the pressure in inches of mercury.

The principal difficulty in making direct determinations of specific volumes of saturated vapor is to be sure that the vapor is neither moist nor superheated. For this purpose Perot enclosed a glass globe for weighing the vapor in a strong receptacle together with a sealed tube of the liquid to be tested. Both the receptacle and the globe were exhausted by an air-pump, then the temperature was raised and the tube of liquid was broken, and the receptacle and the globe were filled with vapor. The globe was supported on a frame, so as not to be directly in contact with the walls of the recepta-

PEROT'S EXPERIMENTS ON SPECIFIC VOLUMES OF SATURATED VAPOR.

Name of Vapor.	Temperatures.	Specific Volumes, Cubic Metres.		
		Experimental.	By Equation (131).	Difference.
Steam.....	68.20	5.747	5.428	0.329
	88.60	2.531	2.469	0.062
	98.10	1.782	1.768	0.014
	99.60	1.657	1.683	— 0.026
	101.50	1.583	1.583	0.0
	124.10	0.766	0.7804	— 0.14
Bisulphide of Carbon.....	84.60	0.1163	0.121	— 0.005
Ether.....	28.40	0.4262	0.429	— 0.003
	30.00	0.4000	0.4013	— 0.0013
	31.70	0.3751	0.382	— 0.007
	31.90	0.3730	0.380	— 0.007
	57.90	0.1680	0.169	— 0.001
	85.50	0.07777	0.082	— 0.004
	110.50	0.04394	0.0454	— 0.0015

cle, and being filled with vapor and surrounded by vapor at the same pressure it is fair to conclude that it was at the

temperature due to the pressure of the vapor and that it contained only dry saturated vapor. In preparation for the experiment the mouth of the globe was drawn down to a fine tube, around which a platinum wire was wound, so that at the proper time the opening could be closed by passing an electric current through the wire. After the globe was sealed full of dry saturated vapor the receptacle was opened and the globe taken out and weighed.

The results of these tests are shown in the accompanying table, together with values calculated by aid of equation (131).

Zeuner's Equation for Internal Heat.—The following empirical equations were proposed by Zeuner for calculating the heat equivalent of the disgregation work during vaporization. They are most interesting from the light they throw on the critical temperature.

INTERNAL LATENT HEAT.

French Units.

Water	$\rho = 575.40 - 0.791t$
Ether	$\rho = 86.54 - 0.10648t - 0.0007160t^2$
Chloroform	$\rho = 62.44 - 0.11282t - 0.0000140t^2$
Carbon bisulphide	$\rho = 82.79 - 0.11446t - 0.0004020t^2$
Carbon tetrachloride	$\rho = 48.57 - 0.06844t - 0.0002080t^2$
Aceton	$\rho = 131.63 - 0.20184t - 0.0006280t^2$

The following table shows that the equation for water gives a fair degree of approximation:

	0°	50°	100°	150°	200°
By equation (117)	575.5	536.3	496.4	457.4	417.4
By empirical equation	575.4	535.9	496.3	456.8	417.1

Critical Temperature. — The empirical equation for steam, and also the value of ρ in the table above by the exact method, show that the internal latent heat decreases as the temperature rises, and at sufficiently high temperatures it will approach zero. If ρ is made zero in Zeuner's equation for

steam the corresponding temperature is 720° C., which indicates that the true point is much beyond the limits of experiments.

The temperature at which ρ becomes zero for any vapor is called the critical temperature, for at that temperature the distinction between the liquid and its vapor vanishes, and above that temperature the vapor or gas cannot be liquefied by pressure alone. It has been proposed to call a substance which is above the critical temperature a gas, and one which is below a vapor.

Experiments on liquids strongly heated in strong glass tubes show that vaporization proceeds gradually as the temperature rises, until a temperature is reached at which the line of demarcation between the liquid and its vapor becomes indistinct. Above that temperature the liquid all disappears, and the tube is full of gas. This is the critical temperature. Avenarius * by this method determined the critical temperature of four liquids. He also selected from Regnault's experiments the data most applicable, and from them deduced equations like those given by Zeuner for the internal latent heat of vapors, and calculated the critical temperature by their aid. The results are as follows:

	Experimental.	Calculated.
Ether	196°.2 C.	196°.8 C.
Carbon bisulphide.....	276°.1	274°.0
Carbon tetrachloride	292°.5	298°.7
Aceton.....	246°.1	230°.4

Curve of Constant Steam Weight.—It was formerly assumed in the theory of the steam-engine that the interchange of heat between the steam and the iron of the cylinder was by radiation; and, further, that the condensation accompanying adiabatic expansion formed a cloud which instigated a rapid interchange of heat where before little had occurred. The steam-jacket was assumed to impart just heat enough to

* Poggendorff's *Annalen*, cli, 1874.

dissipate this cloud and keep the steam dry. Hence the curve of dry saturated steam was considered to be of great importance in the theory of the steam-engine, and it is sometimes drawn on indicator-cards instead of the hyperbola. The substitution has no good reason, for the curve is not a better approximation to the curve drawn by an indicator, and is more troublesome to construct.

The action of steam in the engine-cylinder has been proved to be quite different, for an interchange of heat is caused by condensation by contact of the steam with the iron, or by evaporation of moisture from it, and the curve of saturated steam no longer plays an important part in the theory of the steam-engine. Still it is of importance as forming the boundary-line between superheated steam and wet steam.

The curve may be represented very closely by the exponential formula

$$pv^n = p_1 v_1^n = \text{const.} \quad . \quad . \quad . \quad . \quad (132)$$

Rankine proposed the value $\frac{17}{16}$ for the exponent n, and Zeuner has found that 1.0646 gives still a closer approximation. The actual curve may be drawn by plotting pressures and volumes from a table of the properties of saturated steam.

Isothermal Lines.—Since the pressure of saturated vapor is a function of the temperature only, the isothermal line of a mixture of a liquid and its vapor is a line of equal pressures, parallel to the axis of volumes. Steam expanding from the boiler into the cylinder of an engine follows such a line; that is, the steam-line of an automatic cut-off engine with ample ports is nearly parallel to the atmospheric line.

The heat required for an increase of volume at constant pressure is

$$Q = r(x_2 - x_1),$$

which may be obtained by integrating equation (119) with the assumption that the temperature is constant; or it may be

written directly, since r is the heat of vaporization, and $x_2 - x_1$ is the weight of liquid vaporized.

The work done by the vapor during such an expansion is

$$W = p(v_2 - v_1) = pu(x_2 - x_1). \quad . \quad . \quad . \quad (133)$$

Isodynamic or Isoenergic Lines.—The following method of treating the isodynamic changes of a mixture of a liquid and its vapor gives the solution of all problems that arise, although it does not give an equation to the curve of pressures and volumes.

The increase of intrinsic energy of the mixture of a liquid and its vapor, above freezing-point, is

$$E = \frac{1}{A}(q + x\rho); \quad . \quad . \quad . \quad . \quad . \quad (134)$$

where q and $x\rho$ are the heat-equivalents of the vibration and disgregation works. The change of intrinsic energy in passing from one condition to another is

$$E_2 - E_1 = \frac{1}{A}(q_2 - q_1 + x_2\rho_2 - x_1\rho_1). \quad . \quad . \quad (135)$$

When the change is isodynamic, the energy remains the same by definition, and

$$q_2 - q_1 + x_2\rho_2 - x_1\rho_1 = 0; \quad . \quad . \quad . \quad (136)$$

which equation, together with the formulæ

$$v_2 = x_2u_2 + \sigma, \quad v_1 = x_1u_1 + \sigma, \quad . \quad . \quad . \quad (137)$$

gives the means of solving all problems.

For example, if a mixture of $\frac{9}{10}$ steam and $\frac{1}{10}$ water expands isoenergically from 100 pounds absolute to 15 pounds absolute the final condition will be

$$x_2 = \frac{q_1 - q_2 + x_1\rho_1}{\rho_2} = \frac{297.9 - 181.8 + 0.9 \times 802.8}{892.6} = 0.9395.$$

If $x = .1$ then $x_2 = .22$ ∴ evaporation
occurs if at start more water than steam

The initial and final specific volumes are

$$v_1 = x_1 u_1 + \sigma = 0.9(4.403 - 0.0160) + 0.0160 = 3.964;$$
$$v_2 = x_2 u_2 + \sigma = 0.9395(26.15 - 0.016) + 0.016 = 24.54.$$

The converse problem requiring the pressure corresponding to a given volume cannot be solved directly. The only method of solving such a problem is to assume a probable final pressure and find the corresponding volume; then, if necessary, assume a new final pressure larger or smaller as may be required, and solve for the volume again; and so on until the desired degree of accuracy is obtained.

The isoenergic line can be well represented by an exponential equation, for which the exponent can be determined by the method given on page 69. This is very fortunate, as there is no ready way of calculating the external work by the aid of the usual tables of the properties of saturated steam. Having given or determined the initial and final volumes, the exponential equation may be determined, and then the external work may be calculated by the equation

$$W = \int p dv = \frac{p_1 v_1}{n - 1} \left\{ 1 - \left(\frac{v_1}{v_2} \right)^{n-1} \right\}. \quad . \quad (18)$$

For example, the exponent for the equation representing the expansion of the problem on page 113 is

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} = \frac{\log 100 - \log 15}{\log 24.54 - \log 3.964} = 1.041,$$

and the external work of expansion is

$$W = \frac{100 \times 144 \times 3.964}{1.041 - 1} \left\{ 1 - \left(\frac{3.964}{24.54} \right)^{0.041} \right\} = 100\,000 \text{ ft.-lbs.}$$

Since there is no change in the intrinsic energy during an isoenergic expansion, the external work is equivalent to the

heat applied. Thus in the example just solved the heat applied is equal to

$$100000 \div 778 = 129 \text{ B. T. U.}$$

Entropy of the Liquid.—Suppose that a unit of weight of a liquid is intimately mingled with its vapor, so that its temperature is always the same as that of the vapor; then if the pressure of the vapor is increased the liquid will be heated, and if the vapor expands the liquid will be cooled. So far as the unit of weight of the liquid under consideration is concerned the processes are reversible, for it will always be at the temperature of the substance from which it receives or to which it imparts heat, i.e., it is always at the temperature of its vapor.

The change of entropy of the liquid can therefore be calculated by equation (36),

$$d\phi = \frac{dQ}{T},$$

which may here be written

$$\theta = \int \frac{dq}{T} = \int \frac{c dt}{T}. \quad . \quad . \quad . \quad . \quad (139)$$

Now the specific heat of water as given in the table on page 97 is constant within certain ranges and varies from one range to the next range of temperature. The calculation, like that for the heat of the liquid, must be made step by step.

For example, the increase of entropy of water from freezing-point to 13° C. is

$$1.0072 \log_e \frac{T_5}{T_0} + 1.0044 \log_e \frac{T_{10}}{T_5} + 1.0016 \log_e \frac{T_{13}}{T_{10}} = 0.04663.$$

For a liquid like ether which has the heat of the liquid represented by an empirical equation,

$$q = 0.529011t + 0.0002959t^2,$$

the specific heat is first obtained by differentiation, giving

$$c = 0.52901 + 0.0005918t.$$

Then the increase of entropy above that for the freezing-point of water may be obtained by aid of equation (139); which gives for ether with the French system of units

$$\theta = \int_{273.7}^{T} \left\{ 0.52901 + 0.0005918(T - 273.7) \right\} \frac{dt}{T};$$

$$\therefore \theta = \int_{273.7}^{T} \left(0.3670 \frac{dt}{T} + 0.0005918 dt \right);$$

$$\therefore \theta = 0.0005918(T - 273.7) + 0.3670 \log_e \frac{T}{273.7};$$

$$\therefore \theta = 0.0005918t + 0.3670 \log_e \frac{T}{273.7}. \quad . \quad . \quad . \quad (140)$$

For temperatures below the freezing-point of water equation (140) gives negative numerical results.

Other liquids for which equations for the heat of the liquid are given on page 98, may be treated in a similar method.

Tables of the properties of saturated vapor should include entropies of the liquid, calculated from freezing-point, by one of the methods just illustrated. If such tables are not at hand, then changes in the entropy of the liquid may be calculated approximately, on the assumption that the specific heat is a constant by the equation

$$\theta_2 - \theta_1 = c \int \frac{dt}{T} = c \log_e \frac{T_2}{T_1}. \quad . \quad . \quad . \quad (141)$$

Entropy due to Vaporization.—When a unit of weight of a liquid is vaporized r thermal units, equal to the heat of vaporization, must be applied at constant temperature. If only the portion x is vaporized, then xr thermal units are applied. Treating such a vaporization as a reversible process, the change of entropy may be calculated by the equation

$$\phi - \phi_1 = \int \frac{dQ}{T} = \frac{1}{T} \int dQ = \frac{xr}{T}. \quad . \quad . \quad (142)$$

Entropy of a Mixture of a Liquid and its Vapor.—The increase in entropy due to heating a unit of weight of a liquid from freezing-point to the temperature t and then vaporizing x portion of it is

$$\theta + \frac{xr}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (143)$$

where θ is the entropy of the liquid and r is the heat of vaporization, both of which are given in tables of the properties of vapors; while T is obtained by adding the absolute temperature of zero to the temperature by the thermometer.

For any other state determined by x_1 and t_1 we shall have, for the increase of entropy above that of liquid at freezing-point,

$$\frac{x_1 r_1}{T_1} + \theta_1.$$

The change of entropy in passing from one state to another is

$$\phi - \phi_1 = \frac{xr}{T} + \theta - \frac{x_1 r_1}{T_1} - \theta_1. \quad . \quad . \quad . \quad (144)$$

When the condition of the mixture of a liquid and its vapor are given by the pressure and value of x, then a table giving the properties at even pressures may be conveniently used for this work.

Adiabatic Equation for a Liquid and its Vapor.—During an adiabatic change the entropy is constant, so that equation (144) gives

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2. \quad . \quad . \quad . \quad . \quad . \quad (145)$$

When the initial state, determined by x_1 and t_1 or p_1, is known and the final temperature t_2, or the final pressure p_2, the final value x_2 may be found by equation (145). The initial and final volumes may be calculated by the equations

$$v_1 = x_1 u_1 + \sigma \quad \text{and} \quad v_2 = x_2 u_2 + \sigma. \quad . \quad . \quad (146)$$

Tables of the properties of saturated vapors commonly give the specific volume s, but

$$s = u + \sigma.$$

Values of σ will be found on page 101.

Problems in which the initial condition and the final temperature or pressure are given may be solved directly by aid of the preceding equations. Those giving the final volume instead of the temperature or pressure can be solved only by approximations. An equation to an adiabatic curve in terms of p and v cannot be given, but such a curve for any particular case may be constructed point by point.

Clausius and Rankine independently and at about the same time deduced equations identical with equations (144) and (145), but by methods each of which differed from that given here.

In the discussion of the specific heat h of a saturated vapor, it appeared that the expansion of dry saturated steam in a non-conducting cylinder would be accompanied by partial condensation. The same fact may be brought out more clearly at this place.

For example, one pound of dry steam at 100 pounds absolute pressure will have the values

$$t_1 = 327°.58 \text{ F.}, \quad r_1 = 884.0, \quad \theta_1 = 0.4733, \quad x_1 = 1.$$

If the final pressure is 15 pounds absolute, we have

$$t_2 = 213°.03 \text{ F.}, \quad r_2 = 965.1, \quad \theta_2 = 0.3143;$$

whence

$$\frac{884.0}{787.3} + 0.4733 = \frac{965.1 x_2}{673.7} + 0.3143;$$

$$\therefore x_2 = 0.8948.$$

On the other hand, h is positive for ether, and partial condensation takes place during compression in a non-conducting cylinder.

For example, let the initial condition be

$$t_1 = 10° \text{ C.}, \quad r_1 = 93.12, \quad \theta_1 = 0.0191, \quad x_1 = 1,$$

and let the final conditions be

$$t_2 = 120° \text{ C.}, \quad r_2 = 72.26, \quad \theta_2 = 0.2045;$$

then

$$\frac{93.12}{283.7} + 0.0191 = \frac{72.26x_2}{393.7} + 0.2045,$$

and

$$x_2 = 0.724.$$

Equation (145) applies to all possible mixtures of a liquid and its vapor, including the case of $x_1 = 0$ or the case of liquid without vapor, but at the pressure corresponding to the temperature according to the law of saturated vapor. When applied to hot water, this equation shows that an expansion in a non-conducting cylinder is accompanied by a partial vaporization.

There is some initial state of the mixture such that the value of x shall be the same at the beginning and at the end, though it may vary at intermediate states. To find that value make $x_2 = x_1$ in equation (145) and solve for x_1, which gives

$$x_1 = \frac{\theta_1 - \theta_2}{\frac{r_2}{T_2} - \frac{r_1}{T_1}}. \quad \ldots\ldots\ (147)$$

The value of x_1 for steam to fulfil the conditions given varies with the initial and final temperatures chosen, but in any case it will not be much different from one half. It may therefore be generally stated that a mixture of steam and water, when expanded in a non-conducting cylinder, will show partial condensation if more than half is steam, and partial evaporation if more than half water. If the mixture is nearly half water and half steam, the change must be investigated to determine whether evaporation or condensation will occur; but in any case the action will be small.

External Work during Adiabatic Expansion.—Since no heat is transmitted during an adiabatic expansion, all of the intrinsic energy lost is changed into external work, so that, by equation (134),

$$W = E_1 - E_2 = \frac{1}{A}(q_1 - q_2 + x_1\rho_1 - x_2\rho_2). \quad . \quad (148)$$

For example, the external work of one pound of dry steam in expanding adiabatically from 100 pounds to 15 pounds absolute is

$$W = 778(297.9 - 181.8 + 1 \times 802.8 - 0.8948 \times 892.6)$$
$$= 93515 \text{ foot-pounds.}$$

The adiabatic curve cannot be well represented by an exponential equation; for if an exponent be determined for such a curve passing through points representing the initial and final states, it will be found that the exponent will vary widely with different ranges of pressure, and still more with different initial values of x; and that, further, the intermediate points will not be well represented by such an exponential curve even though it passes through the initial and final points.

This fact was first pointed out by Zeuner, who found that the most important element in determining n was x_1, the initial condition of the mixture. He gives the following empirical formula for determining n, which gives a fair approximation for ordinary ranges of temperature:

$$n = 1.035 + 0.100x_1. \quad . \quad . \quad . \quad . \quad (149)$$

There does not appear to be any good reason for using an exponential equation in this connection, for all problems can be solved accurately by the method given, and the action of a lagged steam-engine cylinder is far from being adiabatic. An adiabatic line drawn on an indicator-diagram is instructive, since it shows to the eye the difference between the expansion in an actual engine and that of an ideal non-conducting

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cylinder; but it can be intelligently drawn only after an elaborate engine test. For general purposes the hyperbola is the best curve for comparison with the expansion curve of an indicator-diagram, for the reason that it is the conventional curve, and is near enough to the curve of the diagrams from good engines to allow a practical engineer to guess at the probable behavior of an engine, from the diagram alone. It cannot in any sense be considered as the theoretical curve.

EXAMPLES.

1. Calculate the pressure, heat of the liquid, total heat, heat of vaporization, specific volume, etc., at several temperatures for the vapors for which the data and equations are given, and compare with results given in the *Tables of the Properties of Saturated Steam.*

2. Find the external work of expansion of a fluid, following the law given by the equation $pv^{\frac{1}{4}}$, which has the initial volume 3 cubic metres and the initial pressure 4 atmospheres, and which expands till the pressure becomes one atmosphere. Ans. 133660 kgm.

3. A pound of steam and water at 150 pounds pressure is 0.6 steam; what is the increase of entropy above that of water at 32° F.? Ans. 1.1442.

4. A kilogram of chloroform at 100° C. is 0.8 vapor; what is the increase of entropy above that of the liquid at 0° C.? Ans. 0.1959.

5. The initial condition of a mixture of water and steam is $t = 320°$ F., $x = 0.8$; what is the final condition after adiabatic expansion to 212° F.? Ans. 0.7398.

6. The initial condition of a mixture of steam and water is $p = 3000$ mm., $x = 0.9$; find the condition after an adiabatic expansion to 600 mm. Ans. 0.8278.

7. A cubic foot of a mixture of water and steam, $x = 0.8$, is under the pressure of 60 pounds by the gauge. Find its volume after it expands adiabatically till the pressure is

reduced to 10 pounds by the gauge; also the external work of expansion. Ans. 2.6857 cu. ft. and 9981 ft.-lbs.

8. Three pounds of a mixture of steam and water at 120 lbs. absolute pressure occupy 4.5 cu. ft. How much heat must be added to double the volume at the same pressure and what is the change of intrinsic energy ?

Ans. 1065 B.T.U.; 750400 ft.-lbs.

9. A test of an engine with the cut-off at 0.106 of the stroke, and the release at 0.98 of the stroke, and with 4.5 per cent clearance, gave for the pressure at cut-off 62.2 pounds by the indicator, and at release 6.2 pounds; the mixture in the cylinder at cut-off was 0.465 steam, and at release 0.921 steam. Find (1) condition of the mixture in the cylinder at release on the assumption of adiabatic expansion to release; (2) condition of mixture on the assumption of hyperbolic expansion, or that $pv = p_1 v_1$; (3) the exponent of an exponential curve passing through points of cut-off and release; (4) exponent of a curve passing through the initial and final points on the assumption of adiabatic expansion; (5) the piston displacement was 0.7 cubic feet, find the external work under exponential curve passing through the points of cut-off and release; also under the adiabatic curve.

Ans. (1) 0.472; (2) 0.524; (3) $n = 0.6802$; (4) $n = 1.0589$; (5) 3093 and 2120 ft.-lbs.

CHAPTER VII.

SUPERHEATED VAPORS.

A DRY and saturated vapor, not in contact with the liquid from which it is formed, may be heated to a temperature greater than that corresponding to the given pressure for the same vapor when saturated; such a vapor is said to be superheated. When far removed from the temperature of saturation such a vapor follows the laws of perfect gases very nearly, but near the temperature of saturation the departure from those laws is too great to allow of calculations by them for engineering purposes.

In the case of superheated steam various provisional characteristic equations have been proposed for use until the necessary experimental investigation shall give the data for a true theory. The theory given here was proposed by Zeuner. It is convenient for calculation and appears to give good results.

Substituting in the characteristic equation for a gas

$$pv = RT,$$

the value of R from equation (64) gives

$$pv = \frac{c_p - c_v}{A} T = \frac{c_p}{A} \frac{\kappa - 1}{\kappa} T. \quad . \quad . \quad (150)$$

The form of characteristic equation proposed by Zeuner for superheated steam is

$$pv = \frac{c_p}{A} \frac{k - 1}{k} T - Cp^a. \quad . \quad . \quad . \quad . \quad (151)$$

The specific heat at constant pressure c_p is assumed to be constant. k is a constant suggested by the ratio κ of the specific heats of a gas; but it will be shown that the specific heat at constant volume, determined from the equation (151), is a variable; consequently k cannot be the ratio of the specific heats of superheated steam. C and a are constants that are to be determined from the known properties of saturated and superheated steam.

Partial Differential Coefficients.—From the characteristic equation (151) may be deduced the partial differential coefficients

$$\left(\frac{dt}{dv}\right)_p = \frac{Apk}{c_p(k-1)}; \quad \ldots \quad (152)$$

$$\left(\frac{dt}{dp}\right)_v = \frac{Avk}{c_p(k-1)} + \frac{ap^{a-1}AkC}{c_p(k-1)}. \quad \ldots \quad (153)$$

Application of the First Law.—The first law of thermodynamics may be conveniently applied by using equation (48), substituting for n and o their values, in terms of the specific heats, from the proper equations on page 13; thus:

$$\frac{1}{A}\left\{\left(\frac{do}{dp}\right)_v - \left(\frac{dn}{dv}\right)_p\right\} = 1,$$

$$n = c_v\left(\frac{dt}{dp}\right)_v, \qquad o = c_p\left(\frac{dt}{dv}\right)_p;$$

$$\therefore \quad \frac{d\left[c_p\left(\frac{dt}{dv}\right)_p\right]_v}{dp} - \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = A. \quad \ldots \quad (154)$$

In applying this equation it is convenient to substitute for $\left(\frac{dt}{dv}\right)_p$ from equation (152), perform the differentiation indicated, and then simplify the result, giving

$$\frac{Ak}{k-1} - \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = A; \quad . \quad . \quad . \quad (155)$$

$$\therefore \quad \frac{d\left[c_v\left(\frac{dt}{dp}\right)_v\right]_p}{dv} = \frac{A}{k-1}. \quad . \quad . \quad (156)$$

If this expression is integrated we get a new expression for the partial differential coefficient $\left(\frac{dt}{dp}\right)_v$ instead of the somewhat complicated expression of equation (152), namely,

$$\left(\frac{dt}{dp}\right)_v = \frac{Av}{c_v(k-1)}. \quad . \quad . \quad . \quad . \quad . \quad (157)$$

Application of the Second Law.—Equation (55),

$$c_p - c_v = AT\left(\frac{dv}{dt}\right)_p\left(\frac{dp}{dt}\right)_v,$$

deduced by the successive application of the two laws of thermodynamics, can be most conveniently used in this place. Substituting the values of the partial differential coefficients from equations (152) and (157) gives

$$c_p - c_v = c_p c_v \frac{(k-1)^2}{k}\frac{T}{Apv};$$

$$\therefore \quad \frac{c_p}{c_v} = 1 + c_p\frac{(k-1)^2}{k}\cdot\frac{T}{Apv}, \quad . \quad . \quad . \quad (158)$$

which gives the method of calculating the specific heat at constant volume when c_p and k are known.

Exponent a.—Equating the values of the differential coefficient given by equations (152) and (157), we have

$$\frac{Av}{c_v(k-1)} = \frac{Avk}{c_p(k-1)} + \frac{ap^{a-1}AkC}{c_p(k-1)}.$$

Simplifying and solving for a, we have

$$a = \frac{pv}{kCp^a}\left(k - \frac{c_p}{c_v}\right).$$

Substituting the values of $\frac{c_p}{c_v}$ and of Cp^a from equations (158) and (151),

$$a = \frac{pv}{k}\,\frac{k - 1 - c_p\dfrac{(k-1)^2}{k}\dfrac{T}{Apv}}{pv - c_p\dfrac{k-1}{k}\dfrac{T}{A}};$$

$$\therefore\quad a = \frac{k-1}{k}. \qquad \ldots\ldots\ldots\ldots \quad (159)$$

Characteristic Equation for Superheated Vapor.—Substituting the value deduced for a in equation (151) gives, for the characteristic equation for superheated steam,

$$pv = \frac{c_p}{A}\frac{k-1}{k}\,T - Cp^{\frac{k-1}{k}}. \qquad \ldots\ldots \quad (160)$$

Thermal Capacities.—From equations (11), (152), and (158),

$$l = c_v\left(\frac{c_p}{c_v} - 1\right)\left(\frac{dt}{dv}\right)_p = c_v c_p\frac{(k-1)^2}{k}\,\frac{T}{Apv}\,\frac{Apk}{c_p(k-1)};$$

$$\therefore\quad l = c_v(k-1)\frac{T}{v}. \qquad \ldots\ldots \quad (161)$$

From equations (15), (157), and (158),

$$-m = c_v\left(\frac{c_p}{c_v} - 1\right)\left(\frac{dt}{dp}\right)_v = c_v c_p\frac{(k-1)^2}{k}\,\frac{T}{Apv}\,\frac{Av}{c_v(k-1)};$$

$$\therefore\quad -m = c_p\frac{k-1}{k}\frac{T}{p}. \qquad \ldots\ldots \quad (162)$$

From equations (17) and (152),

$$o = c_p\left(\frac{dt}{dv}\right)_p = \frac{Apk}{k-1}. \quad . \quad . \quad . \quad . \quad . \quad (163)$$

From equations (18) and (157),

$$n = c_v\left(\frac{dt}{dp}\right)_v = \frac{Av}{k-1}. \quad . \quad . \quad . \quad . \quad . \quad (164)$$

General Equations.—Substituting the values of l, m, n, and o in equations (5), (6), and (7) gives, for the general equations for superheated steam,

$$dQ = c_v\left\{dt + (k-1)\frac{T}{v}dv\right\}; \quad . \quad . \quad (165)$$

$$dQ = c_p\left\{dt - \frac{k-1}{k}\cdot\frac{T}{p}dp\right\}; \quad . \quad . \quad (166)$$

$$dQ = \frac{A}{k-1}\{vdp + kp\,dv\}. \quad . \quad . \quad . \quad . \quad . \quad (167)$$

It is instructive to compare these equations with the general equations (70), (71), and (72) for perfect gases, which may be written,

$$dQ = c_v\left\{dt + (\kappa-1)\frac{T}{v}dv\right\}; \quad . \quad . \quad (168)$$

$$dQ = c_p\left\{dt - \frac{\kappa-1}{\kappa}\frac{T}{p}dp\right\}; \quad . \quad . \quad (169)$$

$$dQ = \frac{A}{\kappa-1}\{vdp + \kappa p\,dv\}. \quad . \quad . \quad . \quad . \quad . \quad (170)$$

To obtain equation (170), equation (72) may be written,

$$dQ = c_v \frac{v}{R} dp + c_p \frac{p}{R} dv;$$

$$\therefore \quad dQ = \frac{Ac_v v}{c_p - c_v} dp + \frac{Ac_p p}{c_p - c_v} dv.$$

It is to be remarked that equation (165) is not useful in its present form, since c_v is a variable, but it is written for symmetry in comparison with equations (168), (169), and (170).

Entropy.—Equation (166) gives

$$d\phi = \frac{dQ}{T} = c_p \left\{ \frac{dt}{T} - \frac{k-1}{k} \frac{dp}{p} \right\}; \quad . \quad . \quad . \quad . \quad (171)$$

$$\therefore \quad \phi - \phi_1 = c_p \left\{ \log_e \frac{T}{T_1} - \frac{k-1}{k} \log_e \frac{p}{p_1} \right\}, \quad . \quad (172)$$

which is to be compared with equation (89), page 71, for gases. Equations (165) and (167) cannot conveniently be used for calculating change of entropy.

Value of k.—The characteristic equation for superheated vapor is intended to apply to all degrees of superheating, approaching at one limit the condition of a gas, and at the other that of saturated vapor. For a mixture of a liquid and its vapor we have, from equation (144),

$$d\phi = d\left(\frac{xr}{T}\right) + d\theta = d\left(\frac{xr}{T}\right) + \frac{c}{T} dt,$$

or for saturated steam with $x = 1$

$$d\phi = \frac{c}{T} dt + d\left(\frac{r}{T}\right) = \frac{1}{T}\left(c dt + dr - \frac{r}{T} dt\right). \quad . \quad (173)$$

Equations (171) and (173) should both be true for dry saturated vapor, whence

$$c_p\left(1 - \frac{k-1}{k}\frac{T}{p}\frac{dp}{dt}\right) = c + \frac{dr}{dt} - \frac{r}{T}. \quad . \quad . \quad (174)$$

By equation (129) the right-hand member of equation (174) is equal to h, the specific heat of saturated vapor; consequently

$$\frac{k-1}{k} = \frac{c_p - h}{\frac{T}{p}\frac{dp}{dt}c_p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (175)$$

Superheated Steam.—Regnault gives as the results of three experiments on the specific heat of superheated steam at constant pressure

0.48111, 0.48080, 0.47963,

and for the mean value

$$c_p = 0.4805.$$

With this value of c_p and the known values of the other factors, determined from the properties of saturated steam, the following values of k were calculated:

Pressure, pounds on the sq. in.	5	50	100	200	300
k	1.33	1.332	1.330	1.324	1.316

Zeuner assumed for the constant k the value

$$k = \tfrac{4}{3} = 1.333 +,$$

which may be compared with the ratio of the specific heats of air,

$$\kappa = 1.405.$$

With this assumed value of k and the known values of A and c_p the coefficient of T in the characteristic equation (160) becomes:

French system, $\frac{c_p}{A}\frac{k-1}{k} = B = 51.28;$

English system, $\frac{c_p}{A}\frac{k-1}{k} = B = 93.46.$

The specific volume of saturated steam under atmospheric pressure and at boiling-point is 26.60 cubic feet or 1.661 cubic metres. Solving equation (160) for C,

$$C = \frac{\frac{c_p}{A}\cdot\frac{k-1}{k}\cdot T - pv}{p^{\frac{k-1}{k}}};$$

and therefore we have:

French system,

$$C = \frac{51.28 \times 373.7 - 10333 \times 1.661}{10333^{\frac{1}{7}}} = 198.4;$$

English system,

$$C = \frac{93.46 \times 672.7 - 2116.32 \times 26.60}{2116.32^{\frac{1}{7}}} = 971.$$

Substituting the constants in the characteristic equation gives:

French system, $pv = 51.3T - 198p^{\frac{1}{7}}.$ . . . . . (176)

English system, $pv = 93.5T - 971p^{\frac{1}{7}}.$ . . . . . (177)

Zeuner's constants for equation (176) differ from those given, since he used 424 for the mechanical equivalent of one calorie, and 273 for the absolute temperature of freezing-point.

In using these equations for superheated steam it is to be remembered that the pressures are specific pressures—i.e., kilograms per square metre or pounds per square foot—whereas the pressures of saturated steam are commonly stated in millimetres of mercury or in pounds on the square inch.

The form of the equation lends itself to the ready calculation of volume or temperature; but the calculation of pressure can be made only by successive approximations.

For example, the specific volume of steam having the pressure of 100 pounds by the gauge and a temperature of 400° F. is

$$v = \frac{93.5T - 971p^{\frac{1}{4}}}{p} = \frac{93.5 \times 860.7 - 971(144 \times 114.7)^{\frac{1}{4}}}{144 \times 114.7};$$

$$\therefore \quad v = 4.20 \text{ cubic feet.}$$

For example, the pressure of superheated steam having a temperature of 400° F. and a specific volume of 5 cubic feet is approximately

$$p = \frac{93.5T}{v} = \frac{93.5 \times 860.7}{5} = 16100;$$

more accurately it is

$$p = \frac{93.5 \times 860.7 - 971 \times \overline{16095}^{\frac{1}{4}}}{5} = 13900;$$

and a third approximation is

$$p = \frac{93.5 \times 860.7 - 971 \times \overline{13900}^{\frac{1}{4}}}{5} = 13990,$$

$p = 97.1$ pounds per square inch absolute.

Specific Heat at Constant Volume.—The specific heat of superheated steam at constant volume may be calculated by applying equation (158) to the case of saturated steam. The following table gives the values obtained at several pressures:

SPECIFIC HEAT OF SUPERHEATED STEAM.

Pressures, pounds per square inch,	5	50	100	200	300
Specific heat, c_v,	0.351	0.348	0.346	0.344	0.341

This table develops the fact already mentioned that the specific heat of superheated steam at constant volume, deduced from the form of the characteristic equation (160) and the known properties of saturated and superheated steam, is a variable. This conclusion applies properly to steam that is only slightly superheated, whereas our experimental knowledge of the properties of superheated steam relates to steam that is superheated to a marked degree.

Intrinsic Energy.—The combination of the equation

$$dQ = A(dE + pdv)$$

with equation (167) gives

$$dE = \frac{1}{k-1}(vdp + pdv) = \frac{1}{k-1}d(pv);$$

$$\therefore \quad E - E_1 = \frac{pv}{k-1} - \frac{p_1v_1}{k-1}, \quad . \quad . \quad . \quad (178)$$

an equation identical in form with that for a perfect gas.

It is convenient to calculate the intrinsic energy from the freezing-point of water, using a combination of equation (178) and equation (134),

$$E = \frac{1}{A}(q + x\rho),$$

for saturated vapor. The increase of intrinsic energy due to heating a liquid from freezing-point to the temperature t and entirely vaporizing it is

$$\frac{1}{A}(q + \rho).$$

The increase of energy due to superheating the vapor under the constant pressure p so that the specific volume increases from that for saturated steam to that for superheated steam is

$$\frac{pv}{k-1} - \frac{ps}{k-1} = \frac{p(v-s)}{k-1}$$

The total increase of energy is

$$E = \frac{p(v-s)}{k-1} + \frac{1}{A}(q + \rho). \quad . \quad . \quad . \quad (179)$$

Total Heat of Superheated Vapor.—By the total heat of superheated vapor is meant the heat required to change one unit of weight of the liquid at freezing-point into superheated vapor having a given temperature. It may be separated into three parts: the heat of the liquid q, the heat of vaporization r, and the heat required to superheat the steam,

$$c_p(t_s - t),$$

in which t_s is the temperature of the superheated steam and t is the temperature of saturated steam at the same pressure. The total heat is consequently

$$q + r + c_p(t_s - t). \quad . \quad . \quad . \quad . \quad . \quad (180)$$

Comparison with Experiments.—Experiments on the specific volume of superheated steam were made by Hirn,*

* *Théorie Mécanique de la Chaleur.*

from the report of which Zeuner selected the experimental data in the following table. The specific volume has been calculated by aid of equation (176), and placed in the table opposite the experimental results to show the comparison of the characteristic equation with experiments.

SPECIFIC VOLUME OF SUPERHEATED STEAM.

Pressure in atmospheres.	Temperature. Centigrade.	Specific Volume. Cubic meters.	
		Hirn's experiments.	Equation (176).
1	118.5	1.74	1.75
1	141	1.85	1.87
3	200	0.697	0.699
4	165	0.4822	0.476
4	200	0.522	0.520
4	246	0.5752	0.577
5	162.5	0.3758	0.376
5	205	0.414	0.418

The table on the next page shows that the characteristic equation for superheated steam applies fairly well to the limiting case of saturated steam. The values in column 2 were taken directly from the table of the properties of saturated steam, and the corresponding quantities in column 3 were calculated by aid of equation (177). The entropies in column 4 are calculated by the expression

$$\frac{r}{T} + \theta.$$

Column 5 is obtained by calculating by equation (172) the change of entropy from freezing-point to the given pressure and corresponding temperature, and adding it to the entropy at freezing-point; the change of entropy is negative and when added gives a decreasing value to the entropy as the pressure rises.

APPLICATION TO SATURATED STEAM.

Absolute Pressure, Pounds per Square Inch. 1	Specific Volumes, Cubic Feet.		Entropy.	
	Tabular Value. 2	Equation (177). 3	Equation (143). 4	Equation (172). 5
14.7	26.60	29.6	1.7484	1.752
30	13.59	13.7	1.6891	1.704
60	7.096	7.12	1.6340	1.641
100	4.403	4.38	1.5945	1.598
150	3.011	3.00	1.5649	1.568
200	2.294	2.30	1.5446	1.546
300	1.554	1.57	1.5262	1.517

Adiabatic Line.—During an adiabatic change the entropy remains constant; consequently from the general equation (165) we have

$$0 = \int \frac{dQ}{T} = c_v \int \frac{dt}{T} + c_v(k-1) \int \frac{dv}{v};$$

$$\therefore \quad \log_e \frac{T}{T_1} = -(k-1) \log_e \frac{v}{v_1};$$

$$\therefore \quad T v^{k-1} = T_1 v_1^{k-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (181)$$

which is deduced in the same way as the corresponding equation for a perfect gas, but differs in that k is an arbitrary constant, while the equation for perfect gases has in the exponent k the ratio of the specific heats.

From equation (166) we may deduce in a similar manner the equation

$$T p^{\frac{1-\kappa}{\kappa}} = T_1 p_1^{\frac{1-\kappa}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (182)$$

Equation (167) is not in convenient form for treatment in

the manner used for deducing the two preceding equations, but from those equations we may readily deduce

$$pv^k = p_1 v_1^k, \quad . \quad . \quad . \quad . \quad . \quad . \quad (183)$$

which may be considered to be the typical adiabatic equation.

The external work during an adiabatic expansion may be calculated by an equation having the same form as that deduced for perfect gases, i e.,

$$W = \frac{p_1 v_1}{k-1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{k-1}\right\}, \quad . \quad . \quad . \quad (184)$$

provided that the vapor remains superheated to the end of the expansion.

If a vapor is not very strongly superheated it is liable to become saturated and moist during an adiabatic expansion, and in that case an extension of the method used for a mixture of a liquid and its vapor must be employed. The increase of entropy of superheated vapor above that of the liquid at freezing-point may be divided into three parts: 1st, that due to heating the liquid; 2d, that due to vaporizing the liquid; and 3d, that due to superheating the steam. The first two parts have already been discussed in the chapter on saturated vapors; the third part is represented by

$$c_p \int \frac{dt}{T} = c_p \log_e \frac{T_s}{T_1},$$

where c is the specific heat of the vapor at constant pressure and T_s is the temperature of the superheated vapor, while T_1 is the temperature of saturated vapor at the given pressure. Assuming that the steam is moist at the final pressure p_2 and temperature T_2, we may calculate the condition x_2 by aid of the equation

$$\frac{r_1}{T_1} + \theta_1 + c_p \log_e \frac{T_s}{T_1} = \frac{x_2 r_2}{T_2} + \theta_2. \quad . \quad . \quad (185)$$

The specific heat of superheated steam is

$$c_p = 0.4805.$$

For example, let the initial pressure be 100 pounds absolute per square inch and the initial temperature be 400° F.; required the condition of the steam after an adiabatic expansion to 15 pounds absolute. Here we have

$$t_1 = 327°.6, \qquad r_1 = 884.0, \qquad \theta_1 = 0.4733,$$

$$t_2 = 213°.0, \qquad r_2 = 965.1, \qquad \theta_2 = 0.3143;$$

$$\therefore \frac{884.0}{788.3} + 0.4733 + 0.4805 \log_e \frac{860.7}{788.3} = \frac{965.1 x_2}{673.7} + 0.3143;$$

$$\therefore \quad x = 0.923.$$

The external work for such an adiabatic expansion is obtained by taking the difference of the initial and final intrinsic energies, which may be calculated individually by equations (179) and (134).

For example, with the conditions of the preceding problem we have

$$v_1 = \frac{93.5\,T_1 - 971 p_1^{\frac{1}{4}}}{p_1} = \frac{93.5 \times 860.7 - 971 \times \overline{14400}^{\frac{1}{4}}}{14400}$$

$$= 4.85 \text{ cubic feet.}$$

Consequently the intrinsic energy is

$$E_1 = \frac{p_1(v_1 - s_1)}{k - 1} + 778(q_1 + \rho_1)$$

$$= \frac{14400(4.85 - 4.403)}{\frac{4}{3} - 1} + 778(297.9 + 802.8) = 875700 \text{ ft.-lbs.}$$

The final intrinsic energy is

$$E_2 = 778(q_2 + x_2\rho_2) = 778(181.8 + 0.923 \times 892.6)$$
$$= 782400 \text{ foot-pounds.}$$

so that the external work is

$$E_1 - E_2 = 875700 - 782400 = 93300 \text{ foot-pounds.}$$

Isoenergic Line.—The equation to this line is obtained from equation (178) by making E equal to E_1, so that

$$pv = p_1v_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (186)$$

which, like the isoenergic line for a perfect gas, is the equation to a rectangular hyperbola.

The external work during an isoenergic expansion is

$$W = \int pdv = p_1v \log_e \frac{v_2}{v_1} = p_1v_1 \log_e \frac{p_1}{p_2}. \quad . \quad (187)$$

Since all the heat applied is expended in external work,

$$Q = AW. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

Isothermal Line.—The equation to the isothermal line for a superheated vapor is obtained by making T a constant in the characteristic equation

$$pv = \frac{c_p}{A}\frac{k-1}{k}T - Cp^{\frac{k-1}{k}},$$

so that

$$pv = p_1v_1 - C\left(p^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}}\right). \quad . \quad . \quad . \quad (189)$$

The heat applied during an isothermal change is obtained by integrating equation (166) with T constant, giving

$$Q = \frac{k-1}{k} c_p T \log_e \frac{p_1}{p_2}. \quad . \quad . \quad . \quad . \quad . \quad (190)$$

But we have in general

$$Q = A(E_2 - E_1 + W),$$

so that the external work is

$$W = \frac{Q}{A} + E_1 - E_2,$$

which may be reduced by equations (190) and (178) to

$$W = \frac{k-1}{k} \frac{c_p T_1}{A} \log_e \frac{p_1}{p_2} + \frac{p_1 v_1}{k-1} - \frac{p_2 v_2}{k-1}. \quad . \quad (191)$$

Properties of Sulphur Dioxide.—One of the most interesting and important applications of the theory of superheated vapors is found in the approximate calculation of properties of certain volatile liquids which are used in refrigerating-machines, and for which we have not sufficient experimental data to construct tables in the manner explained in the chapter on saturated vapors.

For example, Regnault made experiments on the pressures of saturated sulphur dioxide, ammonia, and carbon dioxide, but did not determine the heat of the liquid nor the total heat. He did, however, determine some of the properties of these substances in the gaseous or superheated condition, from which it is possible to construct the characteristic equations for the superheated vapors. These equations can then be used to make approximate calculations of the saturated vapors, for such equations are assumed to be applicable down

to the saturated condition. Of course such calculations are subject to a considerable unknown error, since the experimental data are barely sufficient to establish the equations for the superheated vapors.

The specific heat of gaseous sulphur dioxide is given by Regnault * as 0.15438, and the coefficient of dilatation as 0.0039028. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Börnstein † as 2.21295. Gmelin ‡ gives the following experimental determinations: by Thomson, 2.222; by Berzelius, 2.247. The figure 2.23 will be assumed in this work, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7735327}{2.23} = 0.347 \text{ cubic metres.}$$

The corresponding pressure and temperature are 10333 and 273°.7 C.

Now the coefficient of dilatation is the ratio of the increase of volume at constant pressure, for one degree increase of temperature, to the original volume. Writing the equation (160) in the following form,

$$pv = \frac{c_p}{A} aT - Cp^a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

and applying it at 0° C. and 1° C., we have

$$p_0 v_0 = \frac{c_p}{A} aT_0 - Cp_0^a;$$

$$p_0 v_1 = \frac{c_p}{A} aT_1 - Cp_0^a;$$

* *Mémoires de l'Institut de France*, tome xxi, xxvi.

† *Physikalische-chemische Tabellen.*

‡ Watt's translation, p. 280.

$$\therefore \frac{v_1 - v_0}{v_0} = \frac{c_p}{A}\frac{a}{p_0 v_0}.$$

Substituting the known values and solving for a, we obtain 0.212; but the equation obtained from the equation (192) with this figure does not agree well with Regnault's experiments on the compressibility of sulphur dioxide. If, instead, we make

$$a = 0.22,$$

then by equation (192) the coefficient of dilatation becomes 0.00404, and it will be shown later that the equation deduced with this value agrees quite well with the experiments on compressibility.

The coefficient of T in equation (192) is therefore

$$0.15438 \times 426.9 \times 0.22 = 14.5,$$

and the coefficient of p^a is

$$\frac{14.5 \times 273.7 - 10333 \times 0.347}{10333^{0.22}} = 48 \text{ nearly};$$

so that the equation becomes

$$pv = 14.5T - 48p^{0.22}. \quad . \quad . \quad . \quad . \quad . \quad (193)$$

Regnault found for the pressures

$$p_1 = 697.83 \text{ mm. of mercury},$$
$$p_2 = 1341.58 \quad " \quad " \quad "$$

and at 7°.7 C. the ratio

$$\frac{p_1 v_1}{p_2 v_2} = 1.02088.$$

Reducing the given pressures to kilograms on the square metre, and the temperature to the absolute scale, and applying to equation (192), we obtain 1.016 instead of the experimental value for the above ratio.

Regnault gives for the pressure of saturated sulphur dioxide, in mm. of mercury, the equation

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 5.6663790;$$
$$\log b = 0.4792425;$$
$$\log c = 9.1659562 - 10;$$
$$\log \alpha = 9.9972989 - 10;$$
$$\log \beta = 9.98729002 - 10;$$
$$n = t + 28^\circ \text{ C.}$$

Applying equation (110), page 92, to this case,

$$\frac{1}{p}\frac{dp}{dt} = A\alpha^n + B\beta^n;$$

$$\log \alpha = 9.9972989;$$
$$\log \beta = 9.98729002;$$
$$\log A = 8.6352146;$$
$$\log B = 7.9945332;$$
$$n = t + 28^\circ \text{ C.}$$

The specific volume of saturated sulphur dioxide may be calculated by inserting in equation (193) for the superheated vapor the pressures calculated by aid of the above equation. The results at several temperatures are as follows:

t	-30° C.	0	$+30^\circ$ C.
s	0.8292	0.2256	0.0825

Andréeff * gives for the specific gravity of fluid sulphur

* *Ann. Chem. Pharm.*, 1859.

dioxide 1.4336; consequently the specific volume of the liquid is

$$\sigma = 0.0007.$$

The value of r, the heat of vaporization, may now be calculated at the given temperatures by equation (128), page 104,

$$r = AuT\frac{dp}{dt},$$

in which $$u = s - \sigma.$$

The results are

t	$-30°$ C.	0	$+30°$ C.
r	106.9	97.60	90.54

Within the limits of error of our method of calculation, the value of r may be found by the equation

$$r = 98 - 0.27t. \quad . \quad . \quad . \quad . \quad . \quad . \quad (194)$$

To find the specific heat of the liquid we may use equation (174), page 129,

$$c_p\left(1 - a\frac{T}{p}\cdot\frac{dp}{dt}\right) = c + \frac{dr}{dt} - \frac{r}{T}.$$

At 0° C. the specific heat is approximately

$$c = 0.4.$$

In English units we have for superheated sulphur dioxide

$$pv = 26.4T - 184p^{0.22}, \quad . \quad . \quad . \quad . \quad . \quad (195)$$

the pressures being in pounds on the square foot, the volumes in cubic feet, and the temperatures in Fahrenheit degrees absolute.

For pressures in pounds on the square inch at temperatures on the Fahrenheit scale,

$$\log p = a - b\alpha^n - c\beta^n;$$
$$a = 3.9527847;$$
$$\log b = 0.4792425;$$
$$\log c = 9.1659562 - 10;$$
$$\log \alpha = 9.9984994 - 10;$$
$$\log \beta = 9.99293890 - 10;$$
$$n = t + 18°.4 \text{ F.}$$

For the heat of vaporization

$$r = 176 - 0.27(t - 32), \quad . \quad . \quad . \quad . \quad (196)$$

and for the specific heat of the liquid

$$c = 0.4.$$

In applying these equations to the calculation of a table of the properties of saturated sulphur dioxide the pressures corresponding to the temperatures are calculated as usual. Then the heat of the liquid is calculated by aid of the constant specific heat. The heat of vaporization is calculated by aid of equation (196). Next the specific volume is calculated by inserting the given temperature and the corresponding pressure for the saturated vapor in the characteristic equation (193) or (195). Having the specific volume of the vapor and that of the liquid, the heat equivalent (Apu) of the external work is readily found. Finally, the entropy of the liquid is calculated by the equation

$$\theta = c \log_e \frac{T}{T_0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

Properties of Ammonia.—The specific heat of gaseous ammonia, determined by Regnault, is 0.50836. The theoretical specific gravity compared with air, calculated from the chemical composition, is given by Landolt and Börnstein as 0.58890. Gmelin gives the following experimental determinations: by Thomson, 0.5931; by Biot and Arago, 0.5967. For this work the figure 0.597 will be assumed, which gives for the specific volume at freezing-point and at atmospheric pressure

$$v_0 = \frac{0.7735327}{0.597} = 1.30 \text{ cubic metres.}$$

The coefficient of dilatation has not been determined, and consequently cannot be used to determine the value of a in equation (192). It, however, appears that very consistent results are obtained if a is assumed to be $\frac{1}{4}$, as for superheated steam. The coefficient of T then becomes

$$0.50836 \times 426.9 \times \tfrac{1}{4} = 54.3,$$

and the coefficient of $p^{\frac{1}{4}}$ is

$$\frac{54.3 \times 273.7 - 10333 \times 1.30}{10333^{\frac{1}{4}}} = 142;$$

so that the equation becomes

$$pv = 54.3T - 142p^{\frac{1}{4}}. \quad . \quad . \quad . \quad . \quad . \quad (198)$$

The coefficient of dilatation, calculated by the same process as was used in determining a for sulphur dioxide, is 0.00404, which may be compared with that for sulphur dioxide.

Regnault found for the pressures

$$p_1 = 703.50 \text{ mm. of mercury,}$$
$$p_2 = 1435.3 \quad \text{“} \quad \text{“} \quad \text{“}$$

and at 8°.1 C. the ratio

$$\frac{p_1 v_1}{p_2 v_2} = 1.0188,$$

while equation (198) gives under the same conditions 1.0200.

For saturated ammonia Regnault gives the equation

$$\begin{aligned} \log p &= a - b\alpha^n - c\beta^n; \\ a &= 11.5043330; \\ \log b &= 0.8721769; \\ \log c &= 9.9777087 - 10; \\ \log \alpha &= 9.9996014 - 10; \\ \log \beta &= 9.9939729 - 10; \\ n &= t + 22° \text{ C.}; \end{aligned}$$

by aid of which the pressures in mm. of mercury may be calculated for temperatures on the centigrade scale. The differential coefficient may be calculated by aid of the equation

$$\begin{aligned} \frac{1}{p}\frac{dp}{dt} &= A\alpha^n + B\beta^n; \\ \log A &= 8.1635170 - 10; \\ \log B &= 8.4822485 - 10; \\ \log \alpha &= 9.9996014 - 10; \\ \log \beta &= 9.9939729 - 10; \\ n &= t + 22° \text{ C.} \end{aligned}$$

The specific volume of saturated ammonia calculated by equation (198) at several temperatures are

t	$-30°$ C.	0	$+30°$ C.
s	0.9982	0.2961	0.1167

Andréeff gives for the specific gravity of liquid ammonia at 0° C. 0.6364, so that the specific volume of the liquid is

$$\sigma = 0.0016.$$

The values of r at the several given temperatures, calculated by equation (128), are

t	$-30°$ C.	0	$+30°$ C.
r	325.7	300.15	277.5

which may be represented by the equation

$$r = 300 - 0.8t. \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

The specific heat of the liquid, calculated by aid of equation (129), is

$$c = 1.1.$$

In English units the properties o superheated or gaseous ammonia may be represented by the equation

$$pv = 99T - 540p^{\frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

in which the pressures are taken in pounds on the square foot and volumes in cubic feet, while T represents the absolute temperature in Fahrenheit degrees.

The pressure in pounds on the square inch may be calculated by the equation

$$\begin{aligned} \log p &= a - b\alpha^n - c\beta^n; \\ a &= 9.7907380; \\ \log b &= 0.8721769 - 10; \\ \log c &= 9.9777087 - 10; \\ \log \alpha &= 9.9997786 - 10; \\ \log \beta &= 9.9966516 - 10; \\ n &= t + 7°.6 \text{ F.} \end{aligned}$$

The heat of vaporization may be calculated by the equation

$$r = 540 - 0.8(t - 32), \quad . \quad . \quad . \quad . \quad . \quad (201)$$

and the specific heat of the liquid is

$$c = 1.1.$$

EXAMPLES.

1. What is the weight of one cubic foot of superheated steam at 500° F. and at 60 pounds pressure absolute?

Ans. 0.107 lb.

2. Superheated steam at 50 pounds absolute has half the density of saturated steam at the same pressure. What is the temperature? Ans. 930°.8 F.

3. Find the increase of intrinsic energy and increase of entropy of a pound of superheated steam, at 100 pounds absolute and at 400° F., above the values at 32° for water.

Ans. 876400 ft.-lbs. and 1.6369.

4. Find the external work of one kilogram of steam in expanding adiabatically from the pressure of 3000 mm. of mercury and the temperature 300° C. to the pressure of 2000 mm. Find also the final temperature and volume.

Ans. (1) 7690 m-kgs.; (2) 244°.7 C.; (3) 0.8842 cu. ft.

5. In example 4 find the external work for an isothermal expansion from the initial condition to the final volume as determined in that example. Ans. 8120 m.-kgs.

6. Let the initial temperature of superheated steam be 380° F. at the pressure of 150 pounds absolute. Find the condition after an adiabatic expansion to 20 pounds absolute. Determine also the initial and final volumes.

Ans. (1) 0.8953; (2) 3.094 cu. ft.; (3) 17.83 cu. ft.

7. In example 9, page 122, suppose that the steam at cut-off were superheated 10° F. above the temperature of saturated steam at the given pressure, and solve the example.

Ans. (1) 0.8865; (2) 79°.5 superheating; (3) same as before; (4) $n = 1.137$; (5) 1972 and 1950 ft.-lbs.

CHAPTER VIII.

FLOW OF FLUIDS.

ONE of the most important problems in thermodynamics is to find the amount of a gas or a vapor which will be discharged through a given orifice in a unit of time. To make the statement of the problem more concrete it will be supposed that the fluid passes from the large cylinder A (Fig. 30) into the smaller cylinder B through a well-rounded orifice at C. The cylinders will be supposed each to be at the same temperature as the fluid in it, so that there will be no communication of heat to or from the walls of the cylinders.

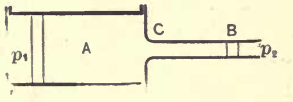

FIG. 30.

The process is clearly non-reversible, so that only the first law of thermodynamics can be used, and, as in equation (43),

$$dQ = A(dE + dW + dK),$$

a term must be added to represent the kinetic energy due to ordinary motion of translation.

Let it be supposed that there is a frictionless piston in each cylinder; the piston in A exerts the pressure p_1 on the fluid in front of it, and the piston in B has on it the fluid pressure p_2. Each unit of weight of fluid passing from A through the orifice has the work p_1v_1 done on it, while each pound entering the cylinder B does the work p_2v_2. The assumption of pistons is merely a matter of convenience, and if they are suppressed the same conditions with regard to external work will hold.

If the velocity in A is V_1 the kinetic energy of one unit of weight in that cylinder is $\frac{V_1^2}{2g}$; the kinetic energy in B is $\frac{V_2}{2g}$ for a velocity V_2.

The intrinsic energies in A and B are E_1 and E_2. If there is no heat communicated to or from the fluid the sum of the intrinsic energy, external work, and kinetic energy must remain constant, so that

$$E_1 + p_1v_1 + \frac{V_1^2}{2g} = E_2 + p_2v_2 + \frac{V_2^2}{2g}; \quad . \quad . \quad (202)$$

this is the fundamental equation for the flow of a fluid.

It is proper to include a term for the gain or loss of heat at the orifice, which is commonly made of metal and is in metallic contact with the walls of the cylinders, and will not have the temperature of the fluid in it; but the amount of heat that can be communicated to the fluid at the orifice is too small to take account of, consequently the term is not put into the fundamental equation.

Usually the velocity in the large cylinder A is small and the term depending on it may be neglected. Solving for the term depending on the velocity in B and dropping the subscript, we have

$$\frac{V^2}{2g} = E_1 - E_2 + p_1v_1 - p_2v_2. \quad . \quad . \quad . \quad (203)$$

Incompressible Fluids.—There is little if any change of volume or of intrinsic energy in a liquid in passing through an orifice under pressure, so that the equation of flow becomes in this case

$$\frac{V^2}{2g} = (p_1 - p_2)v_1. \quad . \quad . \quad . \quad . \quad . \quad (204)$$

If the difference of pressure is due to a difference of level or head, h, we have

$$p_1 - p_2 = h\gamma,$$

where γ is the density, or weight of a unit of volume, and is the reciprocal of the specific volume; consequently equation (204) reduces to

$$\frac{V^2}{2g} = h, \quad \ldots\ldots\ldots \quad (205)$$

which is the usual equation for the flow of a liquid through a small orifice.

Flow of Gases.—The intrinsic energy of a unit of weight of a gas is

$$E = \frac{pv}{\kappa - 1},$$

so that the equation for the flow of a gas is

$$\frac{V^2}{2g} = \frac{p_1v_1}{\kappa - 1} - \frac{p_2v_2}{\kappa - 1} + p_1v_1 - p_2v_2;$$

$$\therefore \quad \frac{V^2}{2g} = \frac{\kappa}{\kappa - 1}(p_1v_1 - p_2v_2). \quad \ldots\ldots\ldots \quad (206)$$

For an adiabatic transformation

$$p_2v_2^{\kappa} = p_1v_1^{\kappa}; \quad \ldots\ldots\ldots \quad (207)$$

$$\therefore \quad p_2v_2 = p_1v_1\left(\frac{v_1}{v_2}\right)^{\kappa - 1} = p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}};$$

so that equation (206) may be reduced to

$$\frac{V^2}{2g} = p_1v_1\frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right]. \quad \ldots\ldots \quad (208)$$

If the area of the orifice is a, then the volume discharged per second is

$$aV,$$

and the weight discharged per second is

$$w = \frac{aV}{v_2},$$

where v_2 is the specific volume at the lower pressure p_2. Substituting for V from equation (208) and for v_2 from (207),

$$w = \frac{a}{v_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}}} \left\{ 2gp_1v_1 \frac{\kappa}{\kappa - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} \right] \right\}^{\frac{1}{2}};$$

$$\therefore \quad w = a\left\{ \frac{2gp_1}{v_1} \frac{\kappa}{\kappa - 1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}} \right] \right\}^{\frac{1}{2}}. \quad . \quad (209)$$

But from the characteristic equation

$$v_1 = \frac{RT_1}{p_1},$$

so that

$$w = a\frac{p_1}{\sqrt{T_1}}\sqrt{\frac{2g}{R}}\left\{ \frac{\kappa}{\kappa - 1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}} \right] \right\}^{\frac{1}{2}}. \quad (210)$$

The equations deduced for the flow of air apply to the flow from a large cylinder or reservoir into a small straight tube through a rounded orifice. The lower pressure is the pressure in the small tube and differs materially from the pressure of the space into which the tube may deliver. In order that the flow shall not be affected by friction against the sides of the tube it should be short—not more than once

or twice its diameter. The flow does not appear to be affected by making the tube very short, and the degree of rounding is not important; the equations for the flow of both air and steam may be applied with a fair degree of approximation to orifices in thin plates and to irregular orifices.

Professor Fliegner* made a large number of experiments on the flow of air from a reservoir into the atmosphere, with pressures in the reservoir varying from 808 mm. of mercury to 3366 mm. He used two different orifices, one 4.085 and the other 7.314 mm. in diameter, both well rounded at the entrance.

He found that the pressure in the orifice, taken by means of a small side orifice, was 0.5767 of the absolute pressure in the reservoir so long as that pressure was more than twice the atmospheric pressure; under such conditions the pressure in the orifice is independent of the pressure of the atmosphere.

If the ratio $\frac{p_2}{p_1}$ is replaced by the number 0.5767 and if κ is replaced by its value 1.405 in equation (210) we shall have for the equation for the flow of a gas

$$w = 0.4822a\sqrt{\frac{2g}{R}}\frac{p_1}{\sqrt{T_1}}. \quad . \quad . \quad . \quad . \quad (211)$$

For the flow into the atmosphere from a reservoir having a pressure less than twice the atmospheric pressure Fliegner found the empirical equation

$$w = 0.9644a\sqrt{\frac{2g}{R}}\cdot\sqrt{\frac{p_a(p_1 - p_a)}{T_1}}, \quad . \quad . \quad (212)$$

where p_a is the pressure of the atmosphere.

These equations were found to be justified by a comparison with experiments on the flow of air, made by Fliegner himself, by Zeuner, and by Weisbach.

* *Der Civilingenieur*, vol. xx, p. 14, 1874.

Although these equations were deduced from experiments made on the flow of air into the atmosphere, it is probable that they may be used for the flow of air from one reservoir into another reservoir having a pressure differing from the pressure of the atmosphere.

Fliegner's Equations for Flow of Air.—Introducing the values for g and R in the equations deduced by Fliegner, we have the following equations for the French and English systems of units:

French units.

$$p_1 > 2p_a, \quad w = 0.395a\frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \quad w = 0.790a\sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

English units.

$$p_1 > 2p_a, \quad w = 0.530a\frac{p_1}{\sqrt{T_1}};$$

$$p_1 < 2p_a, \quad w = 1.060a\sqrt{\frac{p_a(p_1 - p_a)}{T_1}}.$$

p_1 = pressure in reservoir;
p_a = pressure of atmosphere;
T_1 = absolute temperature of air in reservoir (degrees centigrade, French units; degrees Fahrenheit, English units).

In the English system p_1 and p_a are pounds per square inch, and a is the area of the orifice in square inches, while w is the flow of air through the orifice in pounds per second. If desired, the area may be given in square feet and the pressures in pounds on the square foot, as is the common convention in thermodyanmics.

In the French system w is the flow in kilograms per second. The pressures may be given in kilograms per square metre and the area a in square metres; or the area may be given in square decimetres or square centimetres, and the pressures in kilograms on the same unit of area used in connection therewith. If the pressures are in millimetres of mercury, multiply by 13.5959; if in atmospheres multiply by 10333.

Theoretical Maxima.—It is interesting to investigate the conditions that give the maximum discharge of air as calculated by equation (210), neglecting for the moment Fliegner's experimental limit of the ratio of pressures. For this purpose we may equate to zero the first differential coefficient of the weight with regard to the pressure p_2, assuming p_1 to be constant. The variable term is

$$\left(\frac{p_2}{p_1}\right)^{\frac{2}{\kappa}} - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa+1}{\kappa}},$$

and the result of equating to zero the differential coefficient of this expression with regard to p_2 is

$$\frac{p_2}{p_1} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} = 0.5274,$$

a number which is somewhat less than Fliegner's experimental ratio.

There is no algebraic maximum to the velocity of flow as calculated by equation (208), but the kinetic theory of gases gives a theoretical limit to the velocity, for which the following statement is given by Joule.*

Maximum Velocity of Flow.—According to the kinetic theory of gases, the pressure of a gas on the walls of the containing vessel is due to the impact of the molecules of the gas. To estimate the mean velocity of the molecules Joule

* *Memoir Phil. Soc.*, vol. ix, p. 107.

proceeds in the following manner: The weight of one cubic metre of gas is $\frac{1}{v}$, and the pressure which it exerts on each of the six sides of a cubical vessel containing it is p. Suppose that the weight $\frac{1}{v}$ of the gas to be divided into three equal portions, one of which oscillates between each pair of faces of the cube and produces the pressure by impact, first on one and then on the other of the pair. Now if a body have a velocity equal to g it will be brought to rest by a force equal to its weight acting on it for one second; and that force acting for two seconds will bring it to rest and then impart to it the same velocity in the opposite direction. In two seconds there will be g impacts on each of the pair of faces, and it will be assumed that the effect of the impacts is equal to that of a pressure equal to $\frac{1}{3gv}$ kilograms on each face; that is, on one square metre. The pressure will vary as the square of the velocity, since both the force required to reverse the velocity and the number of impacts increase with the velocity. Finally, Joule makes

$$u^2 = p \div \frac{1}{3gv},$$

in which u is the mean velocity of the molecules of the gas. This may be written

$$\frac{u}{\sqrt{3}} = \sqrt{gpv} = \sqrt{gRT}.$$

From this discussion Fliegner assumes that the maximum velocity of flow of gas through an orifice by the kinetic theory of gases is

$$V_{\max} = \sqrt{gRT} = 16.9\sqrt{T_1}$$

for French units. But his limiting ratio of pressures $\frac{p_2}{p_1} = 0.5767$ inserted in equation (208) gives

$$V_{max} = 17.1 \sqrt{T_1}.$$

Flow of Saturated Vapor.—For a mixture of a liquid and its vapor equation (134) gives

$$E = \frac{1}{A}(q + x\rho),$$

so that equation (202) gives for the adiabatic flow from a receptacle in which the initial velocity is zero

$$\frac{V^2}{2g} = \frac{1}{A}(q_1 - q_2 + x_1\rho_1 - x_2\rho_2) + p_1v_1 - p_2v_2. \quad . \quad (213)$$

Substituting for v_1 and v_2 from

$$v = xu + \sigma,$$

$$A\frac{V^2}{2g} = q_1 - q_2 + x_1\rho_1 - x_2\rho_2 + Ap_1x_1u_1 - Ap_2x_2u_2 + A\sigma(p_1 - p_2).$$

But

$$\rho + Apu = r;$$

$$\therefore \quad A\frac{V^2}{2g} = x_1r_1 - x_2r_2 + q_1 - q_2 + A\sigma(p_1 - p_2). \quad (214)$$

The last term of the right-hand member is small, and frequently can be omitted.

The value of x_2 can be determined by the equation

$$\frac{x_1r_1}{T_1} + \theta_1 = \frac{x_2r_2}{T_2} + \theta_2;$$

or, if the proper tables are lacking, we may use the approximate form

$$\frac{x_2r_2}{T_2} = \frac{x_1r_1}{T_1} + c \log_e \frac{T_1}{T_2}.$$

It is necessary to remember that while the tables commonly give the pressure in pounds on the square inch, or in atmospheres, etc., p_1 and p_2 in the last term of equation (214) are the specific pressures; that is, the pressures in pounds on the square foot, or kilograms on the square metre.

The weight of fluid that will pass through an orifice having an area of a square metres or square feet may be calculated by the formula

$$w = \frac{aV}{x_2 u_2 + \sigma}. \quad \ldots\ldots\ldots \quad (215)$$

The equations deduced are applicable to all possible mixtures of liquid and vapor, including dry saturated steam and pure hot water. In the first place steam will be condensed in the tube, and in the second water will be evaporated.

If steam blows out of an orifice into the air, or into a large receptacle, and comes to rest, the energy of motion will be turned into heat and will superheat the steam. Steam blowing into the air will be wet near the orifice, superheated at a little distance, and if the air is cool will show as a cloud of mist further from the orifice.

Rankine's Equations. — After an investigation of the experiments made by Mr. R. D. Napier on the flow of steam Rankine concludes that the pressure in the orifice is never less than the pressure which gives the maximum weight of discharge, and that the discharge in pounds per second may be calculated by the following empirical equations:

$$p_1 = \text{or} > \frac{5}{3} p_a, \qquad w = a\frac{p_1}{70};$$

$$p_1 < \frac{5}{3} p_a, \qquad w = a\,\frac{p_a}{42}\left\{\frac{3(p_1 - p_a)}{2p_a}\right\}^{\frac{1}{2}};$$

in which p_1 is the pressure in the reservoir, p_a is the pressure of the atmosphere, both in pounds on the square inch, and a is the area in square inches.

The error of these equations is liable to be about two per cent; but the flow through a given orifice may be known more closely if tests are made on it at or near the pressure during the flow, and a special constant is found for that orifice.

Experiments on Flow of Steam.—The results of tests made on the flow of steam through orifices or short tubes with well-rounded entrances, by Mr. W. H. Kunhardt* in the laboratories of the Massachusetts Institute of Technology, are given in the following table:

FLOW OF STEAM THROUGH SHORT TUBES WITH ROUNDED ENTRANCES.

Diameters 0.25 of an inch.

	Length of Tube, Inches.	Duration, Minutes.	Pressure above Atmosphere, Pounds per Square Inch.			Barometer, Pounds per Square Inch.	Ratio of Absolute Pressures.		Temperature of Steam below Tube. Fahrenheit.	Per Cent of Moisture in Steam above Tube.	Flow in Pounds per Hour.			Coefficient of Flow, Equation (215).
			Above the Tube.	Below the Tube.	At Small Orifice in Side of Tube.		Pressure above Tube to Pressure below.	Pressure at Side Orifice to Pressure above Tube.			By Experiment Weighed.	Calculated by Thermodynamic Equation (215).	Calculated by Rankine's Equation.	
1	1.5	30	74.1	14.8	41.2	14.7	0.332	0.630	126.2	1.2	221.0	217.0	224	1.018
2	"	30	71.0	13.2	39.6	14.8	0.326	0.634	138.7	1.5	213.0	207.8	215	1.025
3	"	20	72.6	19.7	40.6	14.7	0.394	0.634	141.4	0.5	216.0	211.4	220	1.022
4	"	20	75.9	20.4	42.6	14.7	0.387	0.632	139.8	0.7	228.0	219.3	227	1.040
5	"	20	71.9	24.5	40.6	14.7	0.454	0.638	140.6	0.7	213.0	209.7	218	1.016
6	0.5	30	72.8	14.8	39.0	14.8	0.338	0.614	138.7	0.3	225.0	213.6	221	1.053
7	"	20	72.1	20.4	38.8	14.8	0.405	0.617	142.2	0.5	223.5	211.7	219	0.056
8	"	30	72.6	24.7	39.0	14.8	0.452	0.616	144.0	0.5	223.0	213.1	220	1.046
9	"	30	73.1	29.9	39.2	14.8	0.509	0.615	145.2	0.5	225.5	213.0	222	1.054
10	0.25	30	72.6	24.8	36.1	14.9	0.454	0.583	143.8	0.4	225.0	213.5	220	1.054
11	"	30	72.6	19.9	36.1	14.9	0.398	0.583	141.6	0.4	225.0	213.5	220	1.054
12	"	30	72.7	14.9	36.2	14.8	0.339	0.583	140.5	0.4	227.0	213.0	220	1.066
13	"	30	126.3	27.8	69.0	14.7	0.295	0.594	155.0	0.5	358.8	338.9	355	1.058
14	"	30	125.0	40.8	67.9	14.7	0.398	0.598	157.0	0.2	355.0	334.8	352	1.060

The following table gives the results of some experiments on the flow of steam through an orifice 0.25 of an inch in diameter, in a thin plate, made by Mr. G. P. Aborn† in the laboratories of the Massachusetts Institute of Technology:

* *Transactions Am. Soc. Mech. Engs.*, vol. xi, p. 187.

† Thesis, 1886.

FLOW OF STEAM THROUGH AN ORIFICE.

Number of Experiment.	Higher Pressure.	Difference of Pressure.	Flow in Pounds per Hour by Experiment.
1	71.8	0.92	29.7
2	71.5	1.85	43.1
3	71.9	2.79	52.6
4	71.6	3.89	67.6
5	71.9	5.55	77.6
6	71.8	6.50	84.2
7	71.7	8.07	91.8
8	72.9	9.23	93.9
9	72.5	12.8	110.3
10	73.7	15.9	124.9
11	72.7	21.1	141.5
12	74.2	27.0	156.8
13	71.9	33.7	166.3
14	74.3	41.0	180.7
15	72.7	49.2	187.7
16	72.9	57.0	195.8
17	73.7	64.4	196.9
18	72.0	68.4	197.8

Flow of Superheated Steam.—The form of the equation for the change of intrinsic energy of superheated steam is the same as for a perfect gas, i.e.,

$$E_1 - E_2 = \frac{p_1 v_1}{k - 1} - \frac{p_2 v_2}{k - 1},$$

and consequently the equations for the velocity of flow and the weight of the discharge can be deduced in much the same form as for a perfect gas, provided that the steam remains superheated. Though there are no experiments on the flow of superheated steam under such conditions, it is probable that the ratio of the pressures in the orifice and in the reservoir is something between 0.57 and 0.6.

But steam is seldom sufficiently superheated to avoid condensation during adiabatic flow through an orifice. If the steam becomes moist in the orifice, then the intrinsic energy at the initial condition must be found from equation (179), which may be written

$$E_1 = p_1\left(\frac{v_1 - s_1}{k - 1}\right) + \frac{1}{A}(q_1 + \rho_1), \quad . \quad . \quad (216)$$

while the intrinsic energy at the pressure in the orifice is given by the equation for saturated steam,

$$E_2 = \frac{1}{A}(q_2 + x_2\rho_2), \quad . \quad . \quad . \quad . \quad . \quad (217)$$

so that the equation for the velocity of flow becomes

$$\frac{V^2}{2g} = \frac{p_1(v_1 - s_1)}{k-1} + \frac{1}{A}(q_1 - q_2 + \rho_1 - x_2\rho_2) + p_1v_1 - p_2v_2, \quad (218)$$

in which v_1 is to be calculated from the temperature T_1 and the pressure p_1 by aid of equation (176) or (177), while v_2 is to be found from the equation

$$v_2 = x_2u_2 + \sigma.$$

The value of x_2 for the final condition can be determined by aid of equation (185),

$$\frac{r_1}{T_1} + \theta_1 + c_p \log_e \frac{T_2}{T_1} = \frac{x_2r_2}{T_2} + \theta_2.$$

Finally, the weight discharged per second may be found by the equation

$$w = \frac{aV}{v_2}.$$

In this work the most expeditious way is to make numerical calculations by the several equations without attempting any further algebraic reduction. The pressure in the orifice will be very nearly 0.6 of the pressure p_1 in the reservoir, provided that p_1 is more than $\frac{5}{8}$ the pressure of the atmosphere into which discharge takes place.

EXAMPLES.

1. Find the velocity of flow of air from the pressure of 6 atmospheres in a reservoir to the pressure of 5 atmospheres in the throat of the orifice; also from 5 to 4 atmospheres, from 4 to 3, and from 3 to 2, the initial temperature in each case being 30° C. Ans. 175.8, 193.9, 219.2, 258.0 metres per sec.

2. Find the weight of air per second that will be discharged from an orifice $\frac{1}{10}$ inch in diameter, from a reservoir having the temperature 60° F. and a pressure of 150 pounds per square inch, into the atmosphere. Ans. 2.736 lbs.

3. Find the weight of saturated steam per second discharged through an orifice 1 inch in diameter, from a boiler having the gauge-pressure 60 pounds, into the atmosphere. Find also for the following values of x, 0.9, 0.8, 0.6, 0.5, 0.4, 0.2, and for hot water. Ans. 0.850 lbs.; 0.893, 0.943, 1.077, 1.169, 1.289, 1.710, 3.064.

4. Find the velocity of flow of superheated steam with the initial temperature 360° F. and initial pressure 100 pounds absolute, when the pressure in the throat of the orifice is 60 pounds absolute. Ans. 1400 ft. per sec.

5. In example 4 find the weight per second discharged through an orifice 1 inch in diameter. Ans. 1.09 lbs.

CHAPTER IX.

INJECTORS.

AN injector is an instrument by means of which a jet of steam acting on a stream of water with which it mingles, and by which it is condensed, can impart to the resultant jet of water a sufficient velocity to overcome a pressure that may be equal to or greater than the initial pressure of the steam. Thus, steam from a boiler may force feed-water into the same boiler, or into a boiler having a higher pressure. The mechanical energy of the jet of water is derived from the heat energy yielded by the condensation of the steam-jet. There is no reason why an injector cannot be made to work with any volatile liquid and its vapor, if occasion may arise for doing so; but in practice it is used only for forcing water. An essential feature in the action of an injector is the condensation of the steam by the water forced; other instruments using jets without condensation, like the water-injector in which a small stream at high velocity forces a large stream with a low velocity, differ essentially from the steam-injector.

Method of Working.—A very simple form of injector is shown by Fig. 31, consisting of three essential parts, *a* the *steam-nozzle*, *b* the *combining-tube*, and *c* the *delivery-tube*. Steam is supplied to the injector through a pipe connected at *d*; water is supplied through a pipe at *f*, and the injector forces water out through the pipe at *e*. The steam-pipe must have on it a valve for starting and regulating the injector, and the delivery-pipe leading to the boiler must have on it a

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check-valve to prevent water from the boiler from flowing back through the injector when it is not working. The water-supply pipe commonly has a valve for regulating the flow of water into the injector.

This injector, known as a *non-lifting* injector, has the water-reservoir set high enough so that water will flow into the injector through the influence of gravity. A *lifting*

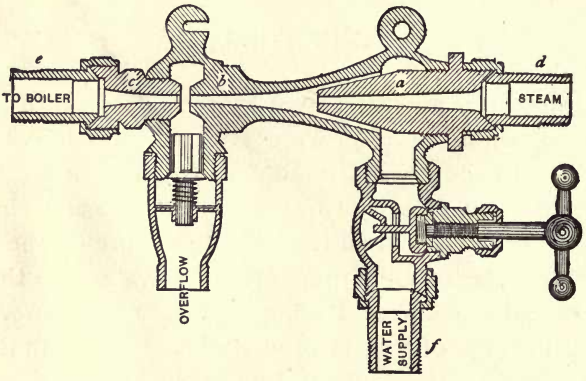

FIG. 31.

injector has a special device for making a vacuum to draw water from a reservoir below the injector, which will be described later.

To start the injector shown by Fig. 31, the steam-valve is first opened slightly to blow out any water that may have gathered above the valve, through the overflow, since it is essential to have dry steam for starting. The steam-valve is then closed, and the water-valve is opened wide. As soon as water appears at the overflow between the combining-tube and the delivery-tube the steam-valve is opened wide, and the jet of steam from the steam-nozzle mingles with and is condensed by the water and imparts to it a high velocity, so that it passes across the overflow space between the combining-tube and the delivery-tube and passes into the boiler. When the injector is working a vacuum is formed at the space

between the combining and delivery tubes, and the valve at the overflow then closes and excludes air which would mingle with the water and might interfere with the action of the injector.

Theory of the Injector.—The two fundamental equations of the theory of the injector are deduced from the principles of the conservation of energy and the conservation of momenta.

The heat energy in one pound of steam at the absolute pressure p_1 in the steam-pipe is

$$\frac{1}{A}(x_1 r_1 + q_1),$$

where r_1 and q_1 are the heat of vaporization and heat of the liquid corresponding to the pressure p_1; $\frac{1}{A}$ is the mechanical equivalent of heat (778 foot-pounds), and x_1 is the quality of the steam; if there is two per cent of moisture in the steam, then x_1 is 0.98.

Suppose that the water entering the injector has the temperature t_3, and that its velocity where it mingles with the steam is V_w'; then its heat energy per pound is

$$\frac{1}{A} q_3,$$

and its kinetic energy is

$$\frac{V_w'^2}{2g},$$

where q_3 is the heat of the liquid at t_3, and g is the acceleration due to gravity (32.2 feet).

If the water forced by the injector has the temperature t_4,

and if the velocity of the water in the smallest section of the delivery-tube is V_w, then the heat energy per pound is

$$\frac{1}{A}q_4,$$

and the kinetic energy is

$$\frac{V_w^2}{2g}.$$

Let each pound of steam draw into the injector y pounds of water; then, since the steam is condensed and forced through the delivery-tube with the water, there will be $1+y$ pounds delivered for each pound of steam. Equating the sum of the heat and kinetic energies of the entering steam and water to the sum of the energies in the water forced from the injector, we have

$$\frac{1}{A}(x_1r_1+q_1)+y\left(\frac{1}{A}q_2+\frac{V_w'^2}{2g}\right)=(1+y)\left(\frac{1}{A}q_4+\frac{V_w^2}{2g}\right). \quad (219)$$

The terms depending on the velocities V_w' and V_w are never large and can commonly be neglected. Thus, for a lifting-injector the pressure causing water to enter the injector is always less than the pressure of the atmosphere and for a non-lifting injector it is only a little more. If the pressure of the atmosphere is 14.7 pounds per square inch and if there is a perfect vacuum in the injector, then by equation (204), the heat equivalent of the term depending on V_w', assuming y to be 15, will be

$$y\frac{AV_w'^2}{2g}=15\times\frac{1}{778}\times 144(14.7-0)\frac{1}{62.4}=0.6 \text{ B. T. U.}$$

If the injector delivers water against a boiler-pressure of 150 pounds by the gauge or 164.7 pounds absolute, then the

heat equivalent of the term depending on V_w will be

$$(y+1)\frac{AV_w^2}{2g} = \frac{16 \times 144 \times 164.7}{778 \times 62.4} = 7.8 \text{ B. T. U.}$$

In both calculations the pressure of the water-jet in the smallest section of the combining tube is assumed to be small enough to be neglected. The determination of the term depending on V_w comes from the consideration that V_w must be greater than the velocity with which cold water would flow out under the influence of the boiler-pressure.

Now since r_1 is always greater than 800, the term depending on V_w is about one per cent of the total left-hand member of equation (219), and the term depending on V_w' is less than one tenth of a per cent. For practical calculations we may neglect both, reducing equation (219) to

$$y = \frac{x_1 r_1 + q_1 - q_4}{q_4 - q_3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

Equation (220) may be applied to any kind of injector, including double injectors which have two steam-nozzles.

For example, if dry steam is supplied to the injector at 120 pounds by the gauge or 134.7 pounds absolute, if the supply-temperature of the water is 65° F., and if the delivery-temperature is 165° F., then the water pumped per pound of steam is

$$y = \frac{r_1 + q_1 - q_4}{q_4 - q_3} = \frac{867.4 + 321.2 - 133.4}{133.4 - 33.12} = 10.5 \text{ pounds.}$$

The momentum of one pound of steam issuing from the steam-nozzle with the velocity V_s is $V_s \div g$; the momentum of y pounds of water entering the combining-tube with the velocity V_w' is $yV_w' \div g$; and the momentum of $1 + y$ pounds of water at the smallest section of the delivery-tube is

$(1+y)V_w \div g$. Equating the sum of the momenta of water and steam before mingling to the momentum of the combined water and steam in the delivery-tube,

$$V_s + yV_w' = (1+y)V_w. \quad . \quad . \quad . \quad . \quad . \quad (221)$$

This equation can be used to calculate any one of the velocities provided the other two can be determined independently. Unfortunately there is much uncertainty about all of the velocities so that the proper sizes of the orifices and of the forms and proportions of the several members of an injector have been determined mainly by experiment. The best exposition of this matter is given by Mr. Strickland Kneass,* who has made many experiments for Wm. Sellers & Co. The practical part of what follows is largely drawn from his work.

Velocity of the Steam-jet.—Equation (214) gives

$$V_s = \left\{ \frac{2g}{A}(x_1 r_1 - x_2 r_2 + q_1 - q_2) \right\}^{\frac{1}{2}}, \quad . \quad . \quad (222)$$

where r_1 and q_1 are the heat of vaporization and the heat of the liquid of the supply of steam at the pressure p_1, and r_2 and q_2 are corresponding quantities at the pressure p_2 for that section of the tube for which the velocity is calculated; x_1 is the quality of the steam at the pressure p_1 (usually 0.98 to unity) and x_2 is the quality at the pressure p_2 to be calculated by aid of the equation

$$\frac{x_1 r_1}{T_1} + \theta_1 = \frac{x_2 r_2}{T_2} + \theta_2.$$

Here T_1 and T_2 are the absolute temperatures corresponding to the pressures p_1 and p_2, and θ_1 and θ_2 are the entropies of

* *Practice and Theory of the Injector*, J. Wiley & Sons.

the liquid at the same pressures. Of course $\frac{1}{A}$ is the mechanical equivalent of heat and g is the acceleration due to gravity.

In the discussion of the flow of steam it appeared that the pressure in an orifice with rounded approach is about $\frac{6}{10}$ of the absolute pressure of the steam in the pipe leading to the orifice, and that the quantity of steam discharged is not affected by the pressure against which the discharge takes place, provided the latter is not more than half the pressure causing the flow. This principle may be applied with fair approximation to the injector, so far as the calculation of the amount of steam used by an injector is concerned; conversely the same principle can be used for calculating the size of the orifice in the steam-nozzle. If preferred, Rankine's equation (page 158) can be used for this purpose. But it is pointed out by Mr. Kneass that by flaring or expanding the steam-nozzle the pressure in it may be reduced, and consequently the velocity of the steam discharged may be very much increased. Experiments on a nozzle of a Sellers' injector showed that the pressure at the exit when the injector was working was a little less than the pressure of the atmosphere for various pressures from 60 to 120 pounds by the gauge. Applying to a special case, we have,

For example, the velocity of discharge from a straight orifice under the pressure of 120 pounds by the gauge or 134.7 pounds absolute is

$$V_2 = \left\{ \frac{2g}{A}(x_1 r_1 + x_2 r_2 + q_1 - q_2) \right\}^{\frac{1}{2}}$$

$$= \{2 \times 32.2 \times 778(867.4 - 0.967 \times 895.1 + 321.2 - 282.1)\}^{\frac{1}{2}}$$

$$= 1430 \text{ feet per second,}$$

having for x_2

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right) = \frac{773.2}{895.1}\left(\frac{867.4}{810.6} + 0.5025 - 0.4533\right)$$

$$= 0.9670,$$

provided that $p_2 = 0.6p_1 = 80.8$ pounds absolute.

If, however, the pressure at the exit of an expanded nozzle is 14.7 pounds absolute, then

$$x_2 = \frac{672.7}{965.8}\left(\frac{867.4}{810.7} + 0.5025 - 0.3127\right) = 0.8775,$$

and

$$V_2 = \{2 \times 32.2 \times 778(867.4 - 0.8711 \times 965.8 + 321.2 - 180.8)\}^{\frac{1}{2}}$$

$$= 2830 \text{ feet per second,}$$

which is nearly twice that just calculated for the velocity at the smallest section of the steam-nozzle.

Velocity of Entering Water.—The velocity of the water in the combining-tube where it mingles with the steam depends on (*a*) the lift or head from the reservoir to the injector, (*b*) the pressure (or vacuum) in the combining-tube, and (*c*) on the resistance which the water experiences from friction and eddies in the pipe, valves, and passages of the injector. The first of these can be measured directly for any given case; for example, where a test is made on an injector. In determining the proportions of an injector it is safe to assume that there is neither lift nor head for a non-lifting injector, and that the lift for a lifting-injector is as large as can be obtained with certainty in practice. The lift for an injector is usually moderate, and seldom if ever exceeds 20 feet.

The vacuum in the combining-tube may amount to 22 or 24 inches of mercury, corresponding to 25 or 27 feet of water; that is, the absolute pressure may be 3 or 4 pounds per square

inch. The vacuum after the steam and water are combined appears to be limited by the temperature of the water; thus, if the temperature is 165° F., the absolute pressure cannot be less than 5.3 pounds. But the final temperature is taken in the delivery-pipe after the water and condensed steam are well mixed and are moving with a moderate velocity.

The resistance of friction in the pipes, valves, and passages of injectors has never been determined; since the velocity is high the resistance must be considerable.

If we assume the greatest vacuum to correspond to 27 feet of water, the maximum velocity of the water entering the combining-tube will not exceed

$$\sqrt{2gh} = \sqrt{2 \times 32.2 \times 27} = 42 \text{ feet.}$$

If, on the contrary, the effective head producing velocity is as small as 5 feet, the corresponding velocity will be

$$\sqrt{2 \times 32.2 \times 5} = 18 \text{ feet.}$$

It cannot be far from the truth to assume that the velocity of the water entering the combining-tube is between 20 and 40 feet per second.

Velocity in the Delivery-tube. — The velocity of the water in the smallest section of the delivery-tube may be estimated in two ways: in the first place it must be greater than the velocity of cold water flowing out under the pressure in the boiler, and in the second place it may be calculated by aid of equation (221), provided that the velocities of the entering steam and water are determined or assumed.

For example, let it be assumed that the pressure of the steam in the boiler is 120 pounds by the gauge, and that, as calculated on page 167, each pound of steam delivers 10.5 pounds of water from the reservoir to the boiler. As there is a good vacuum in the injector we may assume that the

pressure to be overcome is 132 pounds per square inch, corresponding to a head of

$$\frac{132 \times 144}{62.4} = 305 \text{ feet.}$$

Now the velocity of water flowing under the head of 305 feet is

$$\sqrt{2gh} = \sqrt{2 \times 32.2 \times 305} = 140 \text{ feet per second.}$$

The velocity of steam flowing from a pressure of 120 pounds by the gauge through a diverging-tube with the pressure equal to that of the atmosphere at the exit has been calculated to be 2830 feet per second. Assuming the velocity of the water entering the combining-tube to be 20 feet, then by equation (221) we have in this case

$$V_w = \frac{V_s + yV_w'}{1 + y} = \frac{2830 + 10.5 \times 20}{1 + 10.5} = 266 \text{ feet.}$$

If the steam is assumed to flow through a converging-nozzle then the velocity at 120 pounds boiler-pressure will be 1430 feet per second, and the velocity of the water-jet becomes

$$V_w = \frac{1430 + 10.5 \times 20}{1 + 10.5} = 143 \text{ feet.}$$

Since there is considerable resistance due to friction and to sudden change of velocity in leaving the delivery-tube, this last velocity is probably insufficient. But if the entrance velocity is assumed to be 30 feet, then the calculation gives 151 feet for the velocity in the delivery-tube, which is probably enough to make the injector work.

Sizes of the Orifices.—From direct experiments on injectors it appears that the quantity of steam delivered by the steam-nozzle can be calculated in all cases by the method for

the flow of steam through a straight orifice, assuming the pressure in the orifice to be $\frac{6}{10}$ of the absolute pressure above the orifice.

Now each pound of steam forces y pounds of water from the reservoir to the boiler; consequently if w pounds are drawn from the reservoir per second the injector will use $w \div y$ pounds of steam per second.

The specific volume of the mixture of water and steam in the smallest section of the steam-nozzle is

$$v_2 = x_2 u_2 + \sigma,$$

where x_2 is the quality, u_2 is the increase of volume due to vaporization, and σ is the specific volume of the water. The volume of steam discharged per second is

$$\frac{wv_2}{y},$$

and the area of the orifice is

$$a_s = \frac{wv_2}{yV_s}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

where V_s is the velocity at the smallest section.

For example, for a flow from 134.7 pounds absolute to 80.8 pounds absolute x_2 is 0.9670 and V_s is 1430 feet, as found on page 169. Again, for an increase of temperature from 65° F. to 165° F. the water per pound of steam is 10.5. Calculating the specific volume at 80.8 pounds, we have

$$v_2 = x_2 u_2 + \sigma = 0.9670(5.375 - 0.016) + 0.016 = 5.198 \text{ cubic feet.}$$

If the injector is required to deliver 1200 gallons an hour, or

$$\frac{1200 \times 231 \times 62.4}{1728 \times 60 \times 60} = 2.78$$

pounds per second, the area of the steam-nozzle must be

$$a_s = \frac{wv_2}{yV_s} = \frac{2.78 \times 5.198}{10.5 \times 1430} = 0.000962 \text{ square feet.}$$

The corresponding diameter is 0.420 of an inch, or 10.6 millimetres.

In trying to determine the size of the orifice in the delivery-tube we meet with two serious difficulties: we do not know the velocity of the steam in the smallest section of the delivery-tube, and we do not know the condition of the fluid at that place. It has been assumed that the steam is entirely condensed by the water in the combining-tube before reaching the delivery-tube, but usually there are small bubbles of uncondensed steam still mingled with the water, so that the actual density of the heterogeneous mixture is from 0.6 to 0.9 that of water. Since the pressure at the entrance to the delivery-tube is small, the specific volume of the steam is very large, and a fraction of a per cent of steam is enough to reduce the density of the steam to one-half. Even if the steam is entirely condensed, the air carried by the water from the reservoir is enough to sensibly reduce the density at the low pressure (or vacuum) found at the entrance to the delivery-tube.

If V_w is the probable velocity of the jet at the smallest section of the delivery-tube, and if δ is the density of the fluid, then the area of the orifice in square feet is

$$a_w = \frac{w(1+y)}{V_w \delta y}, \quad \ldots\ldots \quad (224)$$

for each pound of steam mingles with and is condensed by y pounds of water and passes with that water through the delivery-tube; w, as before, is the number of pounds of water drawn from the reservoir per second.

For example, let it be assumed that the actual velocity in

the delivery-tube to overcome a boiler-pressure of 120 pounds by the gauge is 150 feet per second, and that the density of the jet is about 0.9 that of water; then with the value of $w = 2.78$ and $y = 10.5$, we have

$$a_w = \frac{w(1+y)}{V_w \delta y} = \frac{2.78 \times 11.5}{150 \times 0.9 \times 62.4 \times 10.5} = 0.000361 \text{ sq. ft.}$$

The corresponding diameter is 0.257 of an inch, or 6.5 millimetres.

Steam-nozzle.—The entrance to the steam-nozzle should be well rounded to avoid eddies or reduction of pressure as the steam approaches; in some injectors, as the Sellers' injector, Fig. 32, the valve controlling the steam supply is placed near the entrance to the nozzle, but the bevelled valve-seat will not interfere with the flow when the valve is open.

It has already been pointed out that the steam-nozzle may advantageously be made to expand or flare from the smallest section to the exit. The length from that section to the end may be between two and three times the diameter at that section.

Consider the case of a steam-nozzle supplied with steam at 120 pounds boiler-pressure: it has been found that the velocity at the smallest section, on the assumption that the pressure is then 80.8 pounds, is 1430 feet per second, and that the specific volume is 5.198 cubic feet. If the pressure is reduced to 14.7 pounds, that is, to atmospheric pressure, at the exit, the velocity becomes 2830 feet per second, the quality being $x_2 = 0.8775$. The specific volume is consequently

$$v_2 = x_2 u_2 + \sigma = 0.8775(26.60 - 0.016) + 0.016 = 23.34 \text{ cu. ft.}$$

The areas will be directly as the specific volumes and inversely as the velocities, so that for this case we shall have

the ratio of the areas

$$\left.\begin{matrix} 5.198 : 23.34 \\ 2830 : 1430 \end{matrix}\right\} = 1 : 2.27;$$

and the ratio of the diameters will be

$$\sqrt{1} \cdot \sqrt{2.27} = 1 : 1.5.$$

Combining-tube.—There is great diversity with different injectors in the form and proportions of the combining-tube. It is always made in the form of a hollow converging cone, straight or curved. The overflow is commonly connected to a space between the combining-tube and the delivery-tube; it is, however, sometimes placed beyond the delivery-tube, as in the Sellers' injector, Fig. 32. In the latter case the combining and delivery tubes may form one continuous piece, as is seen in the double injector shown by Fig. 33.

The Delivery-tube.—This tube should be gradually enlarged from its smallest diameter to the exit in order that the water in it may gradually lose velocity and be less affected by the sudden change of velocity where this tube connects to the pipe leading to the boiler.

It is the custom to rate injectors by the size of the delivery-tube; thus a No. 6 injector may have a diameter of 6 mm. at the smallest section of the delivery-tube.

Mr. Kneass found that a delivery-tube cut off short at the smallest section would deliver water against 35 pounds pressure only, without overflowing; the steam-pressure being 65 pounds. A cylindrical tube four times as long as the internal diameter, under the same conditions would deliver only against 24 pounds. A tube with a rapid flare delivered against 62 pounds, and a gradually enlarged tube delivered against 93 pounds.

If the delivery-tube is assumed to be filled with water

without any admixture of steam or air, then the relative velocities at different sections may be assumed to be proportional to the corresponding areas. This gives a method of tracing the change of velocity of the water in the tube from its smallest diameter to the exit.

A sudden change in the velocity is very undesirable, as at the point where the change occurs the tube is worn and roughened, especially if there are solid impurities in the water. It has been proposed to make the form of the tube such that the change of velocity shall be uniform until the pressure has fallen to that in the delivery-pipe; but this idea is found to be impracticable, as it leads to very long tubes with a very wide flare at the end.

Efficiency of the Injector.—The injector is used for feeding boilers, and for little else. Since the heat drawn from the boiler is returned to the boiler again, save the very small part which is changed into mechanical energy, it appears as though the efficiency was perfect, and that one injector is as good as another provided that it works with certainty. We may almost consider the injector to act as a feed-water heater, treating the pumping in of feed-water as incidental. It has already been pointed out on page 167 that the kinetic energy of the jet in the delivery-tube is less than one per cent of the energy due to the condensation of the steam. On this account the injector is used wherever cold water must be forced into a boiler, as on a locomotive, or when sea-water is supplied to a marine boiler. Considering only the advantage of supplying hot water to the boiler, it almost seems as though the more steam an injector uses the better it is. Such a view is erroneous, as it is often desirable to supply water without immediately reducing the steam-pressure and then it is necessary to use as little steam as may be. It is, however, true that simplicity of construction and certainty of action are of the first importance in injectors.

Lifting Injector.—The injector described at the begin-

ing of this chapter was placed so that water from the reservoir would run in under the influence of gravity. When the

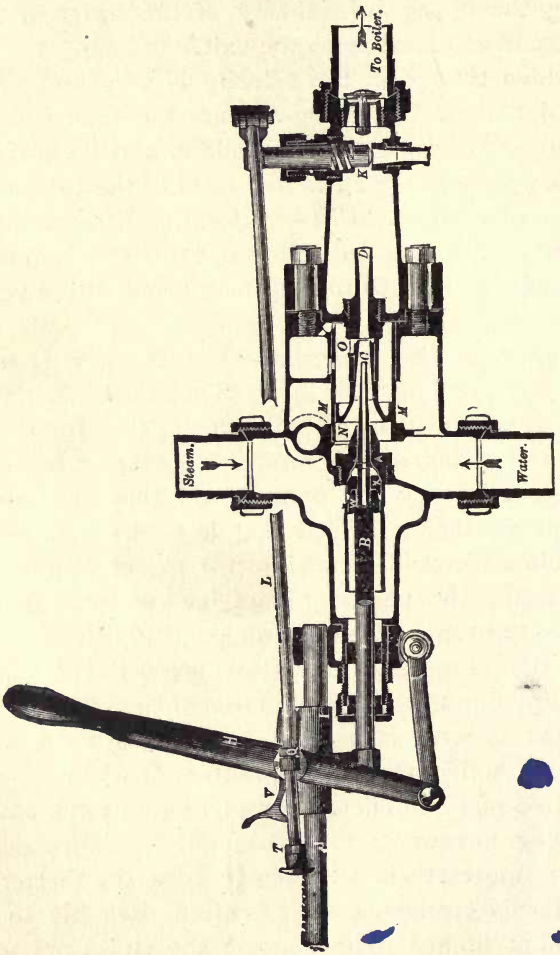

FIG. 32.

injector is placed higher than the reservoir a special device is provided for lifting the water to start the injector. Thus in the Sellers' injector, Fig. 32, there is a long tube which pro-

trudes well into the combining-tube when the valves w and x are both closed. When the rod B is drawn back a little by aid of the lever H the valve w is opened, admitting steam through a side orifice to the tube mentioned. Steam from this tube drives out the air in the injector through the overflow, and water flows up into the vacuum thus formed, and is itself forced out at the overflow. The starting-lever H is then drawn as far back as it will go, opening the valve x and supplying steam to the steam-nozzle. This steam mingles with and is condensed by the water and imparts to the water sufficient velocity to overcome the boiler-pressure. Just as the lever H reaches its extreme position it closes the overflow valve K through the rod L and the crank at R.

Since lifting-injectors may be supplied with water under a head, and since a non-lifting injector when started will lift water from a reservoir below it, or may even start with a small lift, the distinction between them is not fundamental.

Double Injectors.—The double injector illustrated by Fig. 33, which represents the Korting injector, consists of two complete injectors, one of which draws water from the reservoir and delivers it to the second, which in turn delivers the water to the boiler. To start this injector the handle A is drawn back to the position B and opens the valve supplying steam to the lifting-injector. The proper sequence in opening the valves is secured by the simple device of using a loose lever for joining both to the valve-spindle; for under steam-pressure the smaller will open first, and when it is open the larger will move. The steam-nozzle of the lifter has a good deal of flare, which tends to form a good vacuum. The lifter first delivers water out at the overflow with the starting-lever at B; then that lever is pulled as far as it will go, opening the valve for the second injector or forcer, and closing both overflow valves.

Automatic Injectors.—In the discussions of injectors thus far given it has been assumed that they work at full capacity, but as an injector must be able to bring the water-

level in a boiler up promptly to the proper height, it will have much more than the capacity needed for feeding the boiler steadily. Any injector may be made to work at a reduced capacity by simply reducing the opening of the steam-valve, but the limit of its action is soon reached. The

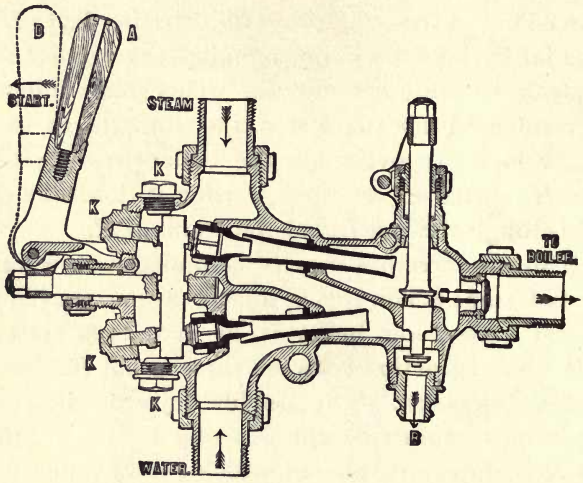

FIG. 33.

limit may be extended somewhat by partially closing the water-supply valve and so limiting the water-supply.

The original Giffard injector had a movable steam-nozzle to control the thickness of the sheet of water mingling with the steam, and also had a long conical valve thrust into the steam-nozzle by which the effective area of the steam-jet could be regulated. Thus both water and steam passages could be controlled without changing the pressures under which they were supplied, and the injector could be regulated to work through a wide range of pressures and capacities. The main objection was that the injector was regulated by hand and required almost constant attention.

In the Sellers injector, Fig. 32, the regulation of the steam-supply by a long cone thrust through the steam-nozzle is retained, but the supply of water is regulated by a movable

combining-tube, which is guided at each end and is free to move forwards and backwards. At the rear the combining-tube is affected by the pressure of the entering water, and in front it is subjected to the pressure in the closed space *O*, which is in communication with the overflow space between the combining-tube and the delivery-tube, in this injector the space is only for producing the regulation of the water-supply by the motion of the combining-tube, as the actual overflow is beyond the delivery-tube at *K*. When the injector is running at any regular rate the pressures on the front and the rear of the combining-tube are nearly equal, and it remains at rest. If the supply of steam is decreased by partially closing the steam-valve or by reducing the boiler-pressure, the velocity of the water in the combining-tube is diminished and the pressure in the chamber *O* increased, so that the combining-tube is forced back and the supply of water is regulated to correspond with the steam-supply. A contrary action ensues if the supply of steam is increased either by raising the boiler-pressure or by opening the steam-valve wider. The injector is always started at full capacity by pulling the steam-valve wide open, as already described; after it is started the steam-supply is regulated at will by the engineer or boiler attendant, and the water is automatically adjusted by the movable combining-tube, and the injector will require attention only when a change of the rate of feeding the boiler is required on account of either a change in the draught of steam from the boiler, or a change of steam-pressure, for the capacity of the injector increases with a rise of pressure.

The action of this injector is well represented by the following table of experimental results, furnished by the makers:

For each pressure of steam noted in column 1, the water was delivered by the injector into the boiler under approximately the same pressure. The delivery was measured by observing the indications of a water-meter. The pressures in column 8 were obtained by throttling the steam supplied to the injector, and observing the pressure at which it ceased to

EXPERIMENTS ON A SELLERS INJECTOR.

(Diam. Water-orifice 6 mm.)

Pressure of Steam supplied to Injector, and Pressure against which Water is delivered. *Lbs. per Sq. Inch.*	Delivery in Cubic Feet per Hour.			Temperature, Fahrenheit Degrees.			Pressure of Steam required to deliver Water against Pressure in Column 1.	Highest Temperature admissible of Feed-water, Fahr. Degrees.
	Maximum.	Minimum.	Ratio of Minimum to Maximum Delivery.	Feed-water.	Delivered Water.			
					At Maximum Delivery.	At Minimum Delivery.		
1	2	3	4	5	6	7	8	9
10	75.3	63.6	0.845	66	100	94	3	132
20	82.4	61.2	0.743	66	108	104	9	134
30	94.2	56.5	0.600	66	114	116	16	134
40	100.1	60.0	0.599	66	120	123	22	132
50	108.3	64.7	0.597	66	124	125	27	131
60	116.5	63.6	0.546	66	127	133	34	130
70	124.8	63.6	0.510	67	130	142	40	130
80	133.0	67.1	0.505	66	134	144	46	131
90	141.3	69.5	0.492	67	136	148	52	132
100	147.2	64.7	0.456	66	140	159	58	132
110	153.0	67.1	0.439	67	144	162	63	132
120	156.6	73.0	0.466	67	148	162	69	134
130	161.2	74.2	0.460	66	150	165	75	130
140	166.0	78.9	0.476	66	153	166	81	126
150	170.7	70.6	0.414	66	157	167	88	121

work, each experiment being repeated several times with precisely the same results. The temperatures in column 9 were obtained by gradually heating the water supplied to the injector, and noting the temperature at which it ceased to operate, each temperature recorded being checked by several repetitions of the experiment.

A double injector, such as that represented by Fig. 33, is to a certain extent automatic, since an increase of steam-pressure causes at once an increase in the amount of water drawn in by the lifter and an increase in the flow of steam through the steam-nozzle of the forcer. Such injectors have a wide range of action and can be controlled by regulating the valve on the steam-pipe.

Restarting Injectors.—If the action of any of the injectors thus far described is interrupted for any reason, it is necessary to shut off steam and start the injector anew; sometimes the injector has become heated while its action is interrupted, and there may be difficulty in starting. To overcome this difficulty various forms of restarting injectors have been devised, such as the Gresham represented by Fig. 34. This injector has four jets arranged in line, as follows: the steam-nozzle, the draught-tube, the movable combining-tube, and the delivery-tube. When the injector is not working the movable combining-tube rests on the upper end of the tube in which it slides, and leaves a free passage to the overflow. When steam is admitted through the steam-nozzle it expels the air in the injector, draws in water from the reservoir and throws it out at the overflow. As soon as the supply of water is established so that it can condense the steam, sufficient velocity is imparted to cause the water to pass through the delivery-tube, and a vacuum is produced above the combining-tube which draws it up and shuts off the passage to the overflow. If the action of the injector is temporarily interrupted the combining-tube drops down and the instrument is ready to start as soon as water is supplied to it.

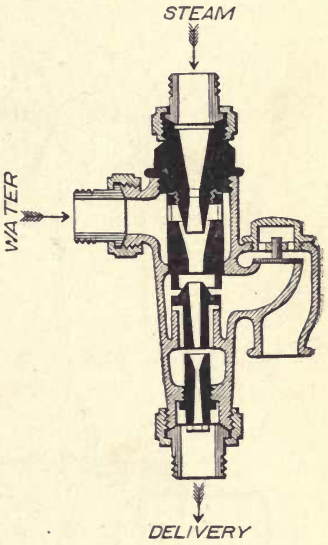

FIG. 34.

Sellers' self-acting injector, represented by Fig. 35, is both an automatic and a restarting injector. It is a double injector with all the jets in one line; *a*, *b*, and *c* are the steam-nozzle, the combining-tube, and the delivery-tube of the forcer; the lifter is composed of the annular steam-nozzle *d*, and the annular delivery-tube *e*, surrounding the nozzle *a*.

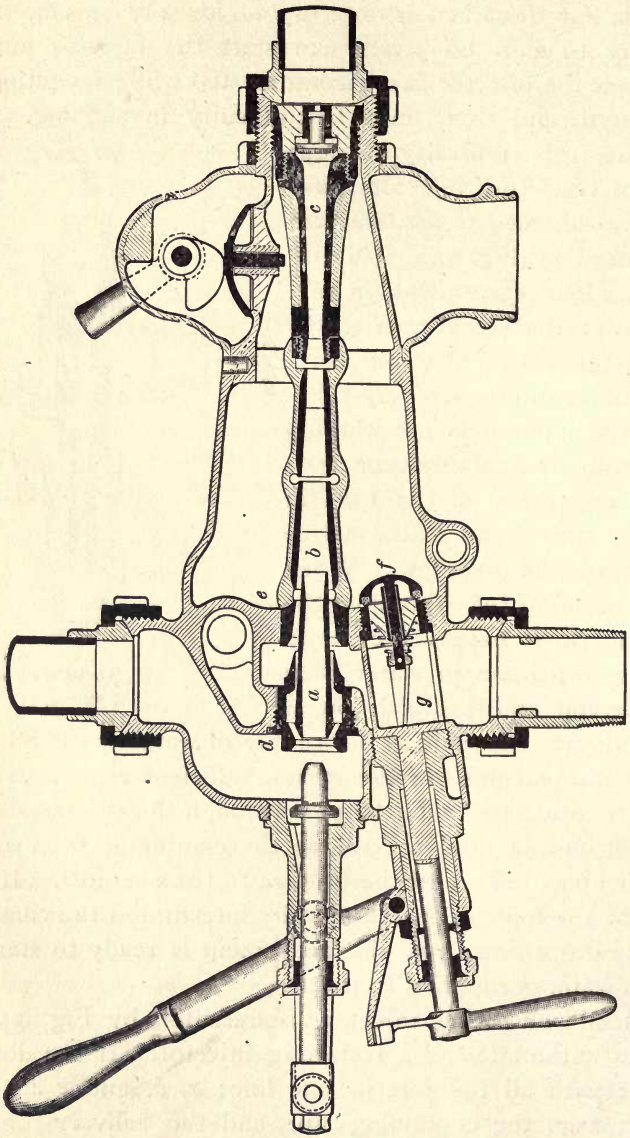

FIG. 35.

The proportions are such that the lifter can always produce a suction in the feed-pipe even when there is a discharge from the main steam-nozzle, and it is this fact that establishes the restarting feature. When the feed-water rises to the tubes it meets the steam from the lifter-nozzle and is forced in a thin sheet and with high velocity into the combining-tube of the forcer, where it comes in contact with the main steam-jet, and mingling with and condensing it, receives a high velocity which enables it to pass the overflow orifices and proceed through the delivery-tube to the boiler.

Like any double injector, the lifter and forcer have a considerable range of action through which the water is adjusted to the steam-supply; but there is a further adjustment in this injector, for when a good vacuum is established in the space surrounding the combining-tube, water can enter through the check-valve *f*, and flowing through the orifices in the combining-tube mingles with the jet in it, and is forced with that jet into the boiler.

The steam-valve is seated on the end of the lifter-nozzle, and it has a protruding plug which enters the forcer-nozzle. When the valve is opened to start the injector, steam is supplied first to the starter, and soon after, by withdrawing the plug, to the forcer. If the steam is dry the starting-lever may be moved back promptly; if there is condensed water in the steam-pipe, the starting-handle should be moved a little way to first open the valve of the lifter, and then it is drawn as far back as it will go, as soon as water appears at the overflow. The water-supply may be regulated by the valve *g*, which can be rotated a part of a turn. The minimum delivery of the injector is obtained by closing this valve till puffs of steam appear at the overflow, and then opening it enough to stop the escape of steam.

When supplied with cold water this injector wastes very little in starting. If the injector is hot or is filled with hot water when started, it will waste hot water till the injector is cooled by the water from the feed-supply, and will then work

as usual. If air leaks into the suction-pipe or if there is any other interference with the normal action, the injector wastes water or steam till normal conditions are restored, when it starts automatically.

The following table gives the results of tests made on an injector of this type:*

TEST OF A SELLERS IMPROVED INJECTOR OF 1887, SIZE 10½, LIFT 4 FEET, SUPPLY-WATER WEIGHED.

MAXIMUM CAPACITY.					
Mean steam-pressure.....	30	60	122	151	200.5
1. Gallons of water per hour.....	1912	2535	3517	3765	4005
2. Temperature of supply-water...	67.0	67.0	54.0	50.0	50.5
3. Temp. of delivered water.....	113.25	125.0	133.4	135.7	154.0
4. Weight of water delivered per pound of steam used...	25.90	19.10	13.60	12.60	10.34
MINIMUM CAPACITY.					
Mean steam-pressure.....	30	60	120	148	200.6
5. Gallons of water per hour.....	765	937	1290	1432	1732
6. Temperature of supply-water...	67.0	67.0	54.5	55.0	50.0
7. Temp. of delivered water.....	171	212	238	250	263
RANGE.					
Mean steam-pressure.....	30	60	121	149.5	200.5
8. Per cent min. capacity of max..	40.0	37.0	36.6	38.0	43.2
9. Actual range in gals. per hr ...	1147	1598	2227	2333	2273
10. Per cent range of max. capacity.	60.0	63.0	63.3	62.0	56.8
LIMITING TEMPERATURE OF WATER-SUPPLY.					
Mean steam-pressure.....	30	60	120	150	
11. Limiting re-starting temp.....	130	135	122	120	
12. Limiting operating temp.....	139	144	137	133	

In these tests the amount of water supplied to the injector was measured; the amount of water delivered per pound of steam was calculated by aid of equation (220).

Exhaust-steam Injectors.—Injectors supplied with exhaust-steam from a non-condensing engine can be used to feed boilers up to a pressure of about 80 pounds. Above this pressure a supplemental jet of steam from the boiler must

* *The Railroad Gazette*, Dec. 11, 1896.

be used. Such an injector, as made by Schäffer and Budenberg, is represented by Fig. 36; when used with low boiler-pressure this injector has a solid cone or spindle instead of the live-steam nozzle. To provide a very free overflow the combining-tube is divided, and one side is hung on a hinge and can open to give free exit to the overflow when the injector is started. When the injector is working it closes down into place. The calculation for an exhaust-steam injector shows that enough velocity may be imparted to the water in the delivery-tube to overcome a moderate boiler-pressure.

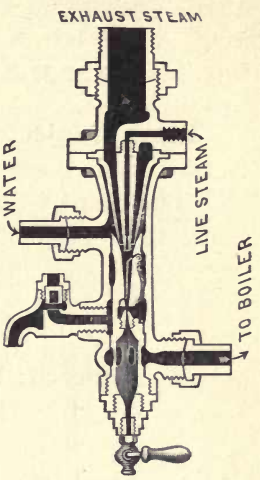

FIG. 36.

For example, an injector supplied with steam at atmospheric pressure, and raising the feed-water from 40° F. to 100° F., will draw from the reservoir

$$\frac{1146.6 - 68.01}{68.01 - 8.06} = 18.0$$

pounds of water per pound of steam. Adiabatic expansion to the pressure of 7.8 pounds will reduce the steam to the quality

$$x_2 = \frac{642.7}{986.9}\left(\frac{965.8}{672.7} + 0.3127 - 0.2667\right) = 0.9649.$$

The velocity of the steam will be

$$V_s = \{2 \times 32 \times 778(1146.6 - 0.9649 \times 986.9 - 150.6)\}^{\frac{1}{2}} = 1480 \text{ feet.}$$

Assuming the water to enter the combining-tube with a velocity of 30 feet, the velocity of the jet in the delivery-tube will be

$$V_w = \frac{1480 + 18.0 \times 30}{1 + 18.0} = 106 \text{ feet,}$$

and it can overcome a pressure of

$$\frac{\overline{106}^2 \times 62.4}{2 \times 32.2 \times 144} = 75.6 \text{ pounds absolute.}$$

Tests of Injectors.—The table opposite gives the results of tests made on several different injectors by Messrs. Bradlee and Blanchard,* in the laboratories of the Massachusetts Institute of Technology:

Sizes of Orifices.

Hancock:	lifter, steam	0.114
"	forcer, steam	0.206
"	" water	0.149
Lombard:	steam	0.224
"	water	0.164
Dodge:	steam	0.161
"	water	0.131

The experiments 15 to 21, inclusive, were made with improved methods of reducing the evaporation of the hot water delivered by the injector, and the results are more consistent and reliable than the preceding ones. It is apparent that the weight of steam used, which is obtained by taking the difference between the weights of water supplied and delivered, is diminished by the evaporation, and that consequently the experimental quantity of water delivered by one pound of steam is made too large thereby: this explains part of the discrepancy of the first fourteen experiments.

The calculation for Experiment 17 on the Dodge injector, on the assumption that the pressure in the steam-orifice is 0.6 of the absolute boiler-pressure, and the steam in the supply-pipe contains three per cent of moisture, gives for the quality in the orifice 0.94, for the velocity of the steam-jet

* Thesis, 1888.

TESTS OF INJECTORS.

Number.	Name of Injector.	Gauge Pressures. Pounds per Sq. Inch.			Barometer, Pounds per Sq. Inch.	Lift in Feet.	Suction in Inches of Mercury.	Temperatures, Fahrenheit.			Water supplied per Hour, Pounds.	Water delivered per Hour, Pounds.	Steam used per Hour, Pounds.	Water per Pound of Steam		
		Boiler.	Intermediate.	Delivery.				Supply.	Intermediate.	Delivery.				By Experiment.	Calculated Steam Saturated.	Calculated 3 per cent Priming.
1	Hancock....	72.0	11.5	76	14.7	5.4	4.8	50.5	73.4	133.8	2111.4	2268.2	156.8	13.5	....	12.6
2	"	71.3	12.3	73	14.7	5.0	4.4	44.7	68.5	134.4	2013.4	2172.5	159.1	12.7	....	11.6
3	"	72.5	11.5	74	14.8	6.7	5.9	49.4	73.7	134.2	2170.0	2332.7	162.7	13.3	....	12.4
4	"	71.0	10.8	72	14.8	7.6	6.7	50.1	72.7	134.1	2126.4	2289.7	163.3	13.0	....	12.5
5	"	71.2	10.8	75	14.7	10.2	9.0	51.4	74.7	138.0	2000.0	2160.0	160.0	12.5	....	12.0
6	"	69.6	10.4	71	14.7	10.1	8.9	48.3	71.6	134.4	1983.4	2141.1	157.7	12.6	.. .	12.2
7	"	69.5	10.5	73	14.7	8.5	7.5	46.9	69.7	132.0	2000.0	2159.0	159.0	12.6	....	12.3
8	"	71.5	11.6	75	15.0	8.5	7.5	56.9	79.8	143.6	2022.5	2181.2	158.7	12.7	....	12.0
9	"	69.1	11.0	73	14.6	8.9	7.9	62.8	86.7	149 7	1962.9	2120.4	157.5	12.5	....	11.9
10	"	23.7	4.4	1.7	14.9	5.2	4.6	54.5	67.7	105.0	1607.8	1677.0	69.2	23.3	21.5	....
11	"	28.2	2.2	2.5	15.0	4.2	3.7	55.2	69.0	109.5	1661.3	1741.8	80.5	20.6	20.0	...
12	Lombard....	71.3	....	72	14.8	7.9	....	60.8	....	190.2	1424.6	1607.7	183.1	7.8	....	7.6
13	"	71.2	....	74	14.8	4.5	....	56.1	....	169.8	1661.3	1840.3	179.0	9.3	....	8.9
14	"	71.8	....	74	14.8	6.7	5.9	58.0	....	180.3	1536.5	1714.3	177.8	8.6	...	8.2
15	Dodge.....	70.3	....	75	14.7	5.4	4.8	61.9	....	144.1	1146.5	1235.0	89.5	13.0	12.9	12.6
16	"	69.8	....	73	14.8	7.8	6.9	61.4	....	150.7	1038.0	1132.2	94.2	11.0	....	11.5
17	"	71.3	....	74	14.9	10.4	9.2	60.0	....	161.3	939.5	1032.6	93.1	10.1	....	10.1
18	"	70.5	..	74	15.0	12.9	11.4	56.8	....	169.5	835.3	925.2	89.9	9.3	....	9.0
19	Hancock....	71.4	11.3	74	14.8	9.9	8.7	69.9	94.0	159.0	1937.6	2101.7	164.1	11.8	....	11.5
20	"	71.0	10.6	75	14.8	12.7	11.2	63.1	88.0	154.2	1871.1	2040.7	169.6	11.0	....	11.3
21	" ...	70.4	9.4	72	14.7	14.3	12.6	56.8	81.4	149.3	1857.6	2022.4	164.8	11.3	...	11.2

1398 feet, and for the specific volume in the orifice 7.76 cubic feet. The discharge through the steam-nozzle by calculation is 91.7 pounds, while the experiment showed 93.1 pounds; which is fairly satisfactory, as the steam is determined by taking the difference between the water supplied to and that delivered by the injector. If the velocity of the water entering the combining-tube is assumed to be 20 feet, then the velocity of the jet in the delivery-tube appears to be 144 feet, while the velocity required to overcome the delivery-pressure of 74 pounds by the gauge or 88.7 pounds absolute is 115 feet. But if the jet is assumed to be clear water with a density of 62.4 pounds, the velocity calculated from the area of the steam-orifice and the quantity of water delivered will be 78 feet per second; consequently the density of the jet will be about 0.60 that of water.

Water-injector.—Fig. 37 represents a device called a water-injector, in which a small stream of water in the pipe

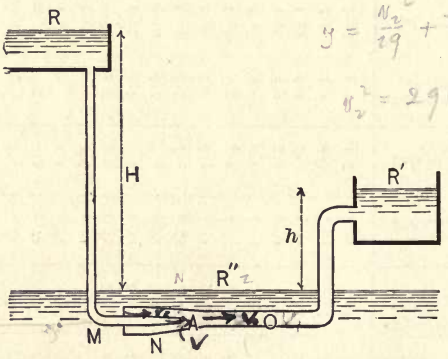

FIG. 37.

M flowing from the reservoir R raises water from the reservoir R'' to the reservoir R'.

Let one pound of water from the reservoir R draw y pounds from R'', and deliver $1 + y$ pounds to R'. Let the velocity of the water issuing from A be v; that of the water

entering from R'' be v_2 at N; and that of the water in the pipe O be v_1. The equality of momenta gives

$$v + yv_2 = (1 + y)v_1. \quad . \quad . \quad . \quad . \quad . \quad (225)$$

Let x be the excess of pressure at M above that at N expressed in feet of water; then

$$v_2^2 = 2gx; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$
$$v^2 = 2g(H + x); \quad . \quad . \quad . \quad . \quad . \quad . \quad (227)$$
$$v_1^2 = 2g(h + x). \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

Substituting in equation (225),

$$\sqrt{H + x} + y\sqrt{x} = (1 + y)\sqrt{h + x};$$

$$\therefore \quad y = \frac{\sqrt{H + x} - \sqrt{h + x}}{\sqrt{h + x} - \sqrt{x}}. \quad . \quad . \quad . \quad (229)$$

It is evident from inspection of the equation (229) that y may be increased by increasing x, for example, by placing the injector above the level of the reservoir so that there may be a vacuum in front of the orifice A.

If the weight G of water is to be lifted per second, then $\frac{G}{y}$ pounds per second must pass the orifice A, G pounds the space at N, and $\left(1 + \frac{1}{y}\right)G$ pounds through the section at O; which, with the several velocities v, v_2, and v_1, give the data for the calculation of the required areas.

PROBLEM.—Required the calculation for a water-injector to raise 1200 gallons of water an hour, $H = 96$ ft., $h = 12$ ft., $x = 4$ ft.

$$\sqrt{x} = \sqrt{4} = 2; \quad \sqrt{H + x} = \sqrt{100} = 10; \quad \sqrt{h + x} = \sqrt{16} = 4;$$

$$y = \frac{10 - 4}{4 - 2} = 3.$$

The velocities are

$$v = \sqrt{2 \times 32.2 \times 100} = 80.25 \text{ feet per second};$$
$$v_1 = \sqrt{2 \times 32.2 \times 16} = 32.10 \text{ feet per second};$$
$$v_2 = \sqrt{2 \times 32.2 \times 4} = 16.05 \text{ feet per second.}$$

1200 gallons an hour = 0.04452 cubic feet per second.

The areas are

$$a = \frac{0.04452}{3 \times 80.25} = 0.000185 \text{ square feet};$$

$$a_1 = \frac{4 \times 0.04452}{3 \times 32.10} = 0.06185 \text{ square feet};$$

$$a_2 = \frac{0.04452}{16.05} = 0.00277 \text{ square feet.}$$

The diameters corresponding to the velocities v and v_1 are

$$d = 0.18 \text{ of an inch};$$
$$d_1 = 0.58 \text{ of an inch.}$$

The area a_2 is of annular form, having the area 0.4 of a square inch.

Ejector.—When the injector is used for raising water where there is no advantage in heating the water, it is a very wasteful instrument. The efficiency is much improved by arranging the instrument as in Fig. 38, so that the steam-nozzle A shall deliver a small stream of water at a high velocity, which, as in the water-injector, delivers a larger stream at a less velocity. Each additional conical nozzle increases the quantity at the expense of the velocity, so that a large quantity of water may be lifted a small height.

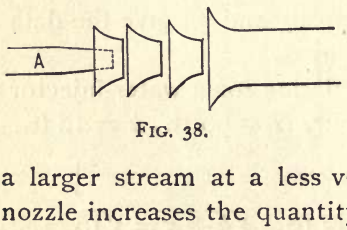

FIG. 38.

Ejectors are commonly fitted in steamships as auxiliary pumps in case of leakage, a service for which they are well

fitted, since they are compact, cheap, and powerful, and are used only in emergency, when economy is of small consequence.

Ejector-condensers.—When there is a good supply of cold condensing water, an exhaust-steam injector, using all the steam from the engine, may be arranged to take the place of the air-pump of a jet-condensing engine. The energy of the exhaust-steam flowing from the cylinder of the engine to the combining-tube, where the absolute pressure is less and where the steam is condensed, is sufficient to eject the water and the air mingled with it against the pressure of the atmosphere, and thus to maintain the vacuum.

For example, if the absolute pressure in the exhaust-pipe is 4 pounds, and if the temperatures of the injector and the delivery are 60° F. and 100° F., then the velocity of the steam-jet will be 1226 feet, and each pound of steam will draw into the injector 24 pounds of water. If the injection water enters with a velocity of 20 feet, the velocity of the water-jet will be 68.2 feet per second, which can overcome the pressure of 31 pounds absolute.

CHAPTER X

HOT-AIR ENGINES AND GAS-ENGINES.

Engines of Maximum Efficiency.—In order to have the maximum efficiency, an engine must work on such a cycle that its working substance shall always have the temperature of the source of heat when acquiring heat, and the temperature of the refrigerator when rejecting heat; that is, the engine must be reversible.

The older forms of hot-air engines all had the source of heat at one constant temperature and the refrigerator at another lower constant temperature. To have the maximum efficiency it was required that the working substance should receive heat from external sources at one temperature, and reject heat to external sources at one temperature only.

Carnot's engine is the only simple engine which can fulfil these conditions when air is the working substance. The cycle of that engine has never been adopted in practice, since it involves incompatible requirements; that is, the isothermal changes should be very slow and the adiabatic changes should be very rapid, to make the cycle of an actual engine approximate to the ideal cycle. And, further, Carnot's cycle for a perfect gas is so attenuated from the fact that the angle between an isothermal and an adiabatic where they cross is very acute, that the mean effective pressure is small, and the cylinder of the engine to work on such a cycle would be very large for the power developed.

By aid of a device called a regenerator or economizer, actual engines have been made which have an ideal cycle of maximum efficiency. Such a cycle is represented by Fig. 39.

The curves DC and AB are isothermals, which form those parts of the cycle during which heat is received from the source and rejected to the refrigerator. The curves BC and DA correspond to the adiabatic lines of Carnot's cycle, and must fulfil the condition that the heat given to the regenerator during one operation, as that represented by BC, must be equal to the heat received from the regenerator during the converse operation DA.

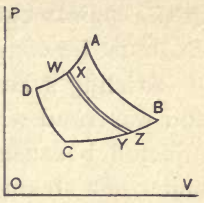

FIG. 39.

To obtain the relation between the curves BC and DA, draw the intermediate isothermals XZ and WY with a difference of temperature dt. The heat received by one unit of weight of the working substance in passing from W to X is

$$dQ = c_p dt + m dp, \quad . \quad . \quad . \quad . \quad . \quad (230)$$

and that rejected from Z to Y is

$$dQ = c_p dt + m' dp'. \quad . \quad . \quad . \quad . \quad (231)$$

The required condition will be fulfilled by making equation (230) equal to equation (231), so that

$$m dp = m' dp'.$$

Substituting for m from equation (53) gives

$$\left(\frac{dv}{dt}\right)_p dp = \left(\frac{dv'}{dt}\right)_p dp'.$$

Deducing the values of the partial differential coefficients from the characteristic equation for a gas, and substituting, we have

$$\frac{R}{p} dp = \frac{R}{p'} dp';$$

$$\therefore \quad \frac{dp}{p} = \frac{dp'}{p'};$$

$$\therefore \quad p = Cp'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (232)$$

That is, the required relation is that the ratio of the pressures at the points cut by any isothermal from the paths DA and BC must be constant.

Stirling's Engine.—This engine was invented in 1816, and was used with good economy for a few years, and then rejected because the heaters, which took the place of the boiler of a steam-engine, burned out rapidly. It is described and its performance given in detail by Rankine in his *Steam-engine*. An ideal sketch is given by Fig. 40. E is a displacer piston filled with non-conducting material, and working freely in an inner cylinder. Between this cylinder and an outer one from A to C is placed a regenerator made of plates of metal, wire screens, or other material, so arranged that it will readily take heat from or yield heat to air passing through it. At the lower end both cylinders have a hemispherical head; that of the outer cylinder is exposed to the fire of the furnace, and that of the inner is pierced with holes through which the air streams when displaced by the plunger. At the upper end there is a coil of pipe through which cold water flows. The working cylinder H has free communication with the upper end of the displacer cylinder, and consequently it can be oiled and the piston may be packed in the usual manner, since only cool air enters it.

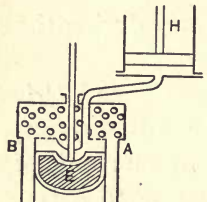

FIG. 40.

In the actual engine the cylinder H is double-acting, and there are two displacer cylinders, one for each end of the working cylinder.

If we neglect the action of the air in the clearance of the cylinder H and the communicating pipe, we have the following ideal cycle. Suppose the working piston to be at the beginning of the forward stroke, and the displacer piston at the bottom of its cylinder, so that we may assume that the air is all in the upper part of that cylinder or in the refrigerator, and at the lowest temperature T_2, the condition of one

pound of air being represented by the point *D* of Fig. 39. The displacer piston is then moved quickly by a cam to the upper end of the stroke; while the working piston moves so little that it may be considered to be at rest. The air is thus all driven from the upper end of the displacer cylinder through the regenerator, from which it takes up heat abandoned during the preceding return stroke, thereby acquiring the temperature T_1, and enters the lower end of that cylinder. During this process the line *AD* of constant volume is described on Fig. 41. When this process is complete, the working cylinder makes the forward stroke, and the air expands at constant temperature, this part of the cycle being represented by the isothermal *AB* of Fig. 41. At the end of the forward stroke the displacer piston is quickly moved down, thereby driving the air through the regenerator, during which process heat is given up by the air, into the upper part of the displacer cylinder; this is accompanied by a cooling at constant volume, represented by the line *BC*. The working piston then makes the return stroke, compressing the air at constant temperature, as represented by the isothermal line *CD*, and completing the cycle.

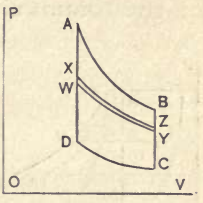

FIG. 41.

To construct the diagram drawn by an indicator, we may assume that in the clearance of the cylinder *H*, the communicating pipe, and refrigerator there is a volume of air which flows back and forth and changes pressure, but remains at the temperature T_2. If we choose, we may also make allowance for a similar volume which remains in the waste spaces at the lower end of the displacer cylinder, at a constant temperature T_1.

In Fig. 42, let *ABCD* represent the cycle of operations, without any allowance for clearance or waste spaces; the minimum volume will be that displaced by the displacer piston, while the maximum volume is larger by the volume displaced by the working piston. Let the point *E* represent

the maximum pressure, the same as that at A; and the united volumes of the clearance at one end of the working cylinder, of the communicating pipe, of the clearance at the top and bottom of the displacer cylinder, and the volume in the refrigerator and regenerator. Each part of this combined volume will have a constant temperature, so that the volume at different pressures will be represented by the hyperbola EF. To find the actual diagram $A'B'C'D'$, draw any horizontal line, as sy, cutting the true diagram at u and v, and the hyperbola EF at t; make ux and vy equal to st; then x and y are points of the actual diagram. The indicator will draw an oval similar to $A'B'C'D'$ with the corners rounded.

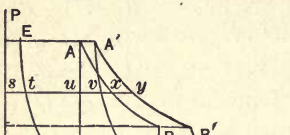

FIG. 42.

To show that the diagram, Fig. 41, fulfils the condition for maximum efficiency, drawn an intermediate isothermal XY. Since DA and BC are lines of constant volume,

$$\frac{p_x}{p_d} = \frac{T_x}{T_a} = \frac{T_y}{T_c} = \frac{p_y}{p_c};$$

$$\therefore \quad p_x = \frac{p_d}{p_c} p_y = C p_y.$$

The diagram in Fig. 43 was reduced from an indicator-diagram from a recent hot-air engine made on the same principle as Stirling's hot-air engine. To avoid destruction of the lubricant in the working cylinder Stirling found it advisable to connect only the cool end of the displacer cylinder with the working cylinder, and had two displacer cylinders for one working cylinder.

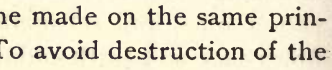
FIG. 43.

It has been found that a good mineral oil can be used to lubricate the displacer piston, and that the hot end also of the displacer cylinder can be advantageously connected with the

working cylinders, of which there are two. Thus each working cylinder is connected with the hot end of one displacer cylinder and with the cool end of the other displacer cylinder.

The distortion of the diagram Fig. 43 is due in part to the large clearance and waste space, and partly to the fact that the displacer pistons are moved by a crank at about 70° with the working crank.

A test on the engine mentioned by Messrs. Underhill and Johnson * showed a consumption of 1.66 of a pound of anthracite coal per horse-power per hour; but the friction of the engine is large, so that the consumption per brake horse-power is 2.37 pounds. This engine, like the original Stirling engine appears to have given much difficulty from the burning of the heaters. The difficulty is likely to be more serious with large than with small engines, as the volume of the displacer cylinders increases more rapidly than the heating surface.

Ericsson's Engine.—This engine consists essentially of a working cylinder, a compressing-pump, and a reservoir. To give a perfect ideal cycle, a regenerator and a refrigerator are required. The pump, which must have a water-jacket which acts as a refrigerator, draws air from the atmosphere at constant pressure, compresses it at constant temperature, and forces it into the reservoir under constant pressure. The pump cycle is represented by the diagram *EDAF* (Fig. 44). The engine draws air from the reservoir through the regenerator, during which process it is heated from the temperature T_2 to T_1; the supply is then cut off by a slide-valve, and the air in the cylinder expands at constant temperature down to the atmospheric pressure. On the return stroke the air is forced from the cylinder at constant pressure through the regenerator, being thereby cooled to the

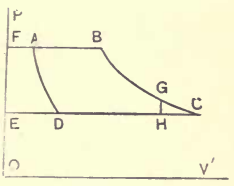

FIG. 44.

* Thesis, 1889.

temperature T_2. The engine cycle is represented by the diagram *FBCE*. The diagram of effective work is *ABCD*, which fulfils the condition of maximum efficiency, since *AD* and *BC* are isothermals, and *AB* and *CD* are lines of constant pressure.

The actual engine does not expand down to the atmospheric pressure, so that the diagram is cut short by a line like *GH*. Also, the clearances of the two cylinders introduce irregularities and modifications of the diagram.

An engine of 300 horse-power designed by Ericsson for a ship named after himself used about 2 pounds of coal per indicated horse-power per hour; but its friction was excessive, and it was finally replaced by a steam-engine. Even when this type of engine is provided with a regenerator and is made to work on a closed cycle with two reservoirs, one at a high pressure from which air is supplied to the working cylinder, and one at a lower pressure from which the pump is supplied, it may be expected to give a poorer economy than an engine working on the Stirling cycle.

Gas-engines.—The chief difficulty with hot-air engines is to transmit heat to and from the working substance. In gas-engines this difficulty is removed by mixing the fuel with the air (so that heat is developed in the working substance itself), and by rejecting the hot gases after they have done their work. The fuel may be illuminating-gas, fuel-gas, or vapor of a volatile liquid like gasoline. It will be shown that the specific volume and the specific heat of the mixture of air and gas, both before and after the heat is developed by combustion, are not very different from the same properties of air. The general theory of gas-engines may therefore be developed on the assumption that the working substance is air, which is heated and cooled in such a manner as to produce the ideal cycles to be discussed, as is done by Clerk.*

Experience has shown that in order to work efficiently, the mixture of gas and air supplied to a gas-engine must be

* *Gas-engines*, Dugald Clerk.

compressed to a considerable pressure before it is ignited. This may be done either by a separate compressor or in the cylinder of the engine itself; the second type of engines, of which the Otto engine is an example, is the only successful type at the present time; the other type has some advantages which may lead to its development.

Gas-engine with Separate Compressor.—This engine has a compressor, a reservoir, and a working cylinder. When run as a gas-engine a mixture of gas and air is drawn into a pump or compressor, compressed to several atmospheres, and forced into a receiver. On the way from the receiver to the working cylinder the mixture is ignited and burned so that the temperature and volume are much increased. After expansion in the working cylinder the spent gases are exhausted at atmospheric pressure.

The ideal diagram, represented by Fig. 45, has a general resemblance to that of Ericsson's engine, but it differs essentially because the expansion and compression curves are adiabatics instead of isothermal lines. *ED* represents the supply of the combustible mixture to the compressor, *DA* is the adiabatic compression, and *AF* represents the forcing into the receiver. *FB* represents the supply of burning gas to the working cylinder, *BC* represents the expansion and *CE* the exhaust. In practice this type of engine always has a release, represented by *GH*, before the expansion has reduced the pressure of the working substance to that of the atmosphere.

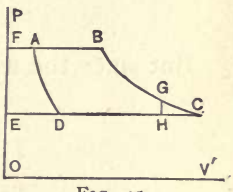

FIG. 45.

This type of engine has been used as an oil-engine by supplying the fuel in the form of a film of oil to the air after it has been compressed. In such case the compressor draws in air only, and there is not an explosive mixture in the receiver. The Brayton engine when run in this way could burn crude petroleum or, after it was started, could burn refined kerosene. Its chief defect appears to have been in-

complete combustion and consequent fouling of the cylinder with carbon.

The effective cycle may be considered to be represented by the diagram $ABCD$ (Fig. 45), and may be assumed to be produced in one cylinder by heating the air from A to B, by cooling it from C to D, and by the adiabatic expansion and compression from B to C and from D to A. If T_a and T_b are the absolute temperatures corresponding to the points A and B, then the heat added from A to B is

$$c_p(T_b - T_a),$$

and the heat withdrawn from C to D is

$$c_p(T_c - T_d),$$

so that the efficiency of the ideal cycle is

$$\eta = \frac{c_p(T_b - T_a) - c_p(T_c - T_d)}{c_p(T_b - T_a)} = 1 - \frac{T_c - T_d}{T_b - T_a}. \quad (233)$$

But since the expansion and compression are adiabatic,

$$\frac{T_c}{T_b} = \left(\frac{p_c}{p_b}\right)^{\frac{\kappa-1}{\kappa}}, \quad \frac{T_d}{T_a} = \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}};$$

but $p_c = p_d$ and $p_b = p_a$, therefore

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} \quad \text{and} \quad \frac{T_c - T_d}{T_b - T_a} = \frac{T_d}{T_a} = \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}};$$

so that the equation for efficiency becomes

$$\eta = 1 - \left(\frac{p_d}{p_a}\right)^{\frac{\kappa-1}{\kappa}}. \quad (234)$$

For example, with the pressure in the reservoir at 90 pounds above the atmosphere the efficiency is

$$\eta = 1 - \left(\frac{14.7}{14.7 + 90}\right)^{\frac{1.405-1}{2.405}} = 0.43.$$

When the cycle is incomplete the expression for the efficiency is not so simple, for it is necessary to assume cooling at constant volume from G to H (Fig. 45), and cooling at constant pressure from H to D; so that the heat rejected is

$$c_v(T_g - T_h) + c_p(T_h - T_d),$$

and the efficiency becomes

$$\eta = 1 - \frac{\frac{1}{\kappa}(T_g - T_h) + (T_h - T_d)}{T_b - T_a}. \quad . \quad . \quad (235)$$

For example, let it be assumed that the pressure at A is 90 pounds above the atmosphere, that the temperature at B is 2500° F., and that the volume at G is three times the volume at B.

First, the temperature at A is

$$T_a = T_d\left(\frac{p_a}{p_d}\right)^{\frac{\kappa-1}{\kappa}} = (60 + 460.7)\left(\frac{14.7 + 90}{14.7}\right)^{\frac{0.405}{1.405}} = 917,$$

provided that the temperature of the atmosphere is 60° F.

The temperature at G is

$$T_g = T_b\left(\frac{v_b}{v_g}\right)^{\kappa-1} = 2960.7\left(\frac{1}{3}\right)^{0.405} = 1897,$$

and the pressure at G is

$$p_g = p_b\left(\frac{v_b}{v_g}\right)^{\kappa} = (14.7 + 90)\left(\frac{1}{3}\right)^{1.405} = 22.4 \text{ pounds},$$

so that the temperature at H is

$$T_h = T_g \frac{p_h}{p_g} = 1897 \times \frac{14.7}{22.4} = 1247,$$

and finally the efficiency is

$$\eta = 1 - \frac{\frac{1}{1.405}(1897 - 1247) + 1247 - 520.7}{2960.7 - 917} = 0.42.$$

Gas-engine with Compression in Cylinder.—All successful gas-engines of the present day compress the explosive mixture of gas and air in the working-cylinder, and as they take gas at one end of the cylinder only they must make four strokes for each explosion. The first forward stroke of the piston from the head of the cylinder draws in the mixture of gas and air, which is compressed on the return stroke; at the completion of this return stroke the mixture is ignited and the pressure rises very rapidly; the next forward stroke is the working stroke, which is succeeded by an exhaust stroke to expel the spent gases. In almost all engines these four strokes are of equal length, for the advantage of making them of unequal length, as required for the best ideal cycle, is more than counterbalanced by the mechanical difficulty of producing unequal strokes.

The most perfect ideal cycle, represented by Fig. 46, has four strokes of unequal length so arranged that the piston starts from the head of the cylinder when gas is drawn in, and the pressure in the cylinder is reduced to that of the atmosphere before the exhaust stroke. Thus there is the filling stroke, represented by EC; the compression stroke, represented by CD; the working stroke, represented by AB; and the exhaust stroke, represented by BE.

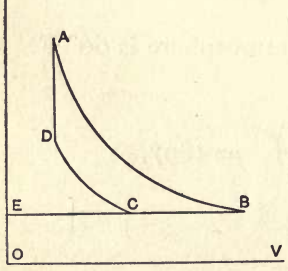

FIG. 46.

The effective cycle is $ABCD$, which may be considered to

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be performed by adding heat at constant volume from D to A, and withdrawing heat at constant pressure from B to C, together with the adiabatic expansion and compression AB and CD.

The heat added under this assumption is

$$c_v(T_a - T_d),$$

and the heat rejected is

$$c_p(T_b - T_c),$$

so that the efficiency is

$$\eta = \frac{c_v(T_a - T_d) - c_p(T_b - T_c)}{c_v(T_a - T_d)} = 1 - \kappa\frac{T_b - T_c}{T_a - T_d}. \quad (236)$$

If the temperature at A and the pressure at D are assumed, then it is necessary to make preliminary calculations of the temperatures at D and at B before using equation (236) by the equations

$$T_d = T_c\left(\frac{p_d}{p_c}\right)^{\frac{\kappa-1}{\kappa}}; \quad \ldots\ldots\ldots (237)$$

$$p_a = p_d\frac{T_a}{T_d}; \quad \ldots\ldots\ldots (238)$$

$$T_b = T_a\left(\frac{p_b}{p_a}\right)^{\frac{\kappa-1}{\kappa}}. \quad \ldots\ldots\ldots (239)$$

For example, if the pressure at the end of compression is 90 pounds above the atmosphere, and the temperature at the end of the explosion is 2500° F., then

$$T_d = (60 + 460.7)\left(\frac{14.7 + 90}{14.7}\right)^{\frac{0.405}{1.405}} = 917,$$

provided that the temperature of the atmosphere is 60° F.

$$p_a = 104.7\frac{2500 + 460.7}{917} = 338 \text{ pounds};$$

$$T_b = (2500 + 460.7)\left(\frac{14.7}{340}\right)^{\frac{0.405}{1.405}} = 1199;$$

$$\eta = 1 - 1.405\frac{1199 - 520.7}{2960.7 - 917} = 0.55.$$

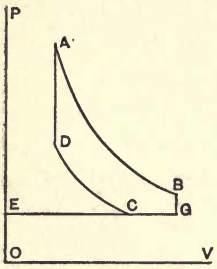

FIG. 47.

If the expansion is not carried to the atmospheric pressure, then the diagram shows a release at the end of the stroke, as in Fig. 47, and the cycle must be considered to be formed by adding heat as before at constant volume, but by withdrawing heat at constant volume to cause a loss of pressure from B to G, and by withdrawing heat at constant pressure during the process represented by GC. The heat rejected becomes, therefore,

$$c_v(T_b - T_g) + c_p(T_g - T_c),$$

and the efficiency is

$$\eta = \frac{c_v(T_a - T_d) - c_v(T_b - T_g) - c_p(T_g - T_c)}{c_v(T_a - T_d)}$$

$$= 1 - \frac{T_b - T_g + \kappa(T_g - T_c)}{T_a - T_d}. \quad . \quad . \quad . \quad . \quad . \quad (240)$$

Assuming, as before, the pressure at D and the temperature at A, it becomes necessary to find the temperatures at B and at G as well as the temperature at D; this last may of course be found by equation (237). If the pressure at B is assumed also, then equations (238) and (239) may be used as before to find T_b, and T_g may be found by the equation

$$T_g = T_b\frac{p_b}{p_g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (241)$$

For example, let it be assumed that the expansion ceases when the pressure becomes 20 pounds above the atmosphere,

the other conditions being as in the previous example. Then

$$T_b = (2500 + 460.7)\left(\frac{14.7 + 20}{338}\right)^{\frac{0.405}{1.405}} = 1536;$$

$$T_c = 1536\frac{14.7}{34.7} = 650;$$

and

$$\eta = 1 - \frac{1536 - 650 + 1.405(650 - 520.7)}{2960.7 - 917} = 0.48.$$

Though not essential to the solution of the example, it is interesting to know that the volume at C is

$$\left(\frac{90 + 14.7}{14.7}\right)^{\frac{1}{\kappa}} = 4 +$$

times the volume at D, and that the volume at B is

$$\left(\frac{338}{34.7}\right)^{\frac{1}{\kappa}} = 5 +$$

times the volume at A.

When, as in common practice, the four strokes of the piston are of equal length the diagram takes the form shown by Fig. 48; the effective cycle may be considered to be equivalent to heating at constant volume from D to A and cooling at constant volume from B to C, together with adiabatic expansion and compression from A to B and from C to D.

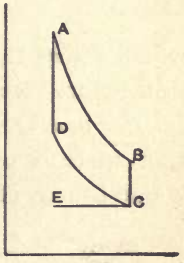

FIG. 48.

The heat applied is

$$c_v(T_a - T_d),$$

and the heat rejected is

$$c_v(T_b - T_c),$$

so that the efficiency is

$$\eta = \frac{c_v(T_a - T_d) - c_v(T_b - T_c)}{c_v(T_a - T_d)} = 1 - \frac{T_b - T_c}{T_a - T_d}. \quad (241)$$

Since the expansion and compression are adiabatic, we have by equation (79)

$$T_b v_b^{\kappa - 1} = T_a v_a^{\kappa - 1}, \quad \text{and} \quad T_c v_c^{\kappa - 1} = T_d v_d^{\kappa - 1};$$

but the volumes at A and D are equal, as are also the volumes at B and C; consequently by division

$$\frac{T_b}{T_c} = \frac{T_a}{T_d};$$

consequently

$$\frac{T_b - T_c}{T_a - T_d} = \frac{T_b}{T_a} = \frac{T_c}{T_d} = \left(\frac{v_d}{v_c}\right)^{\kappa - 1},$$

and the expression for efficiency becomes

$$\eta = 1 - \left(\frac{v_d}{v_c}\right)^{\kappa - 1}, \quad \ldots\ldots \quad (242)$$

which shows that the efficiency depends only on the compression before explosion.

For example, if the volume of the clearance or compression space is one-third of the piston displacement, so that v_d is one-fourth of v_c, then the efficiency is

$$\eta = 1 - \left(\frac{1}{4}\right)^{0.405} = 0.43.$$

The pressure at the end of compression is

$$p_d = p_c\left(\frac{v_c}{v_d}\right)^{\kappa} = 14.7\left(\frac{4}{1}\right)^{1.405} = 103.1$$

pounds absolute, or 88.4 pounds by the gauge. The calculated efficiency is therefore not much less than the efficiencies found for other examples; it is notable that the efficiency is nearly the same as that calculated on page 205 for an engine with separate compression to 90 pounds by the gauge. For the case in hand, however, the pressure after explosion, which depends on the temperature, may exceed 300, as was shown in the preceding examples, for engines with compression in the cylinder.

The diagrams from engines of this type resemble Fig. 49, which was taken from an Otto engine tested under the direction of Professor Thurston. During the filling stroke the pressure is gradually reduced below that of the atmosphere; the explosion is nearly but not quite instantaneous, as is shown by the explosion line leaning toward the right and by the rounded curve joining it to the expansion line; there is a release before the end of the stroke and a little back-pressure at the beginning of exhaust. But the greatest difference between this diagram and the ideal diagram is found in the expansion curve, which is far from being an adiabatic, for the combustion does not cease when the maximum pressure is attained, but continues throughout the stroke; and at the same time heat is taken up energetically by the walls of the cylinder, which are cooled by a water-jacket to avoid overheating. These two effects, after-burning and loss of heat to the water-jacket, may nearly compensate, and then the expansion curve will resemble the adiabatic line in appearance.

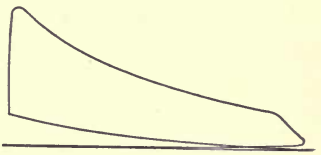

FIG. 49.

The engine from which this diagram was taken had a diameter of 8.5 inches and a stroke of 14 inches. At 158 revolutions per minute it indicated 9.6 horse-power and developed 8.1 horse-power on a brake. It consumed 24.5 cubic feet of illuminating-gas per horse-power per hour.

Professor Thurston estimates the distribution of heat as follows:

Work indicated in cylinder	17.0
Heat lost to cylinder walls	52.0
Heat carried away by exhaust gases	15.5
Heat lost by conduction and radiation	15.5
	100.0

so that the actual efficiency is 17 per cent.

The compression space was 38 per cent of the total cylinder volume, so that the efficiency of the ideal cycle was 42 per cent, and the ratio of the efficiencies was about 0.40. More modern and especially larger engines use less gas per horse-power per hour, and show a better efficiency both absolutely and relatively.

There are two questions that should now receive consideration if our calculations of ideal efficiency are to be taken as a guide either in the choice of a type of engine or the proportions of a type. They are the composition of the mixture in the cylinder both before and after explosion, and the probable temperature after explosion. As for the composition of the mixture, Clerk gives the following table showing the composition of Manchester illuminating-gas, the oxygen required for combustion, and the volume of the products after combustion, all reduced to normal pressure and temperature.

ANALYSIS OF MANCHESTER COAL-GAS. (Bunsen and Roscoe.)

		Amount required for Complete Combustion.	Products.
	vols.	vols. O.	vols.
Hydrogen, H	45.58	22.79	45.58, H_2O
Marsh-gas, CH_4	34.9	69.8	104.7, CO_2 & H_2O
Carbonic oxide, CO	6.64	3.32	6.64, CO_2
Ethylene, C_2H_4	4.08	12.24	16.32, CO_2 & H_2O
Tetrylene, C_4H_8	2.38	14.28	19.04, CO_2 & H_2O
Sulphuretted hydrogen, H_2S.	0.29	0.43	0.58, H_2O & SO_2
Nitrogen, N	2.46	.....	2.46
Carbonic acid, CO_2	3.67	.....	3.67
Total	100.00	122.86 O	198.99, CO_2, H_2O & SO_2

It appears in this case that the volume of the gas and oxygen is reduced by combustion from 223 to 199, both being at freezing-point, so that the error of neglecting such a change would appear to be about 10 per cent. But the gas is mixed with from 7 to 12 volumes of air when it is introduced into the cylinder of the engine so that the error is probably less than two per cent in any case; though of course it depends on the gas used, which may be illuminating-gas distilled from coal, so-called water-gas, or crude gas made in a special gas-producer.

To find the most economical mixture of gas and air and the probable temperature after combustion, Clerk made a large number of experiments on illuminating-gas distilled from coal. He found the pressure immediately after explosion to vary from 40 to 90 pounds per square inch above the atmosphere, depending on the kind of gas and the proportions of the mixture. Since this change of pressure takes place at constant volume (neglecting change of volume due to combustion) the absolute temperatures must be proportional to the absolute pressures, giving, for a temperature of 62° F. before explosion,

$$(460.7 + 62)\frac{14.7 + 40}{14.7} = 1940,$$

$$(460.7 + 62)\frac{14.7 + 90}{14.7} = 3720,$$

for the absolute temperatures, or from 1500° F. to 3000° F.

The temperature after explosion appears to be largely controlled by the phenomenon of dissociation, for a calculation of the temperature of combustion on the assumption that the total heat of combustion of the gas is developed during the explosion and is used in raising the temperature of the products of combustion gives temperatures which are about double those just stated. When the temperature of the gases after explosion has been reduced by expansion and by con-

duction and radiation to the cylinder of the engine, the combustion may be continued and may extend throughout the stroke; this phenomenon is called after-burning.

The performance of gas-engines has been improved by (1) raising the compression before explosion; (2) increasing the size of the engine; (3) clearing the cylinder of spent gases before introducing a new charge of gas.

The following table of tests on Crossley-Otto engines of about the same size illustrates the advantage of increasing the compression:

Diameter of cylinder	9.0	9.5	7.0
Stroke	18.0	18.0	15.0
Ratio of compression space to piston displacement	0.6	0.4	0.34
Pressures of compression above atmosphere.	38.0	61.6	87.5
Gas per horse-power per hour	24.0	20.5	14.8
Actual efficiency	0.17	0.21	0.25
Theoretical efficiency	0.33	0.40	0.428
Ratio of efficiencies	0.51	0.53	0.58

Since the greatest waste of the gas-engine is due to the heat carried away by the water-jacket, it is to be expected that a large engine will show a better performance than a small engine, since the area of the water-jacket for such an engine will have a smaller ratio to the volume of the cylinder. This is illustrated by the following results from engines of different sizes, but having about the same degree of compression:

Diameter of cylinder	7	12.5	9.5	14
Stroke	15	21	18	25
Relative capacity	1	3.77	1	2.97
Actual efficiency	0.25	0.275	0.21	0.277
Theoretical efficiency	0.428	0.428	0.40	0.41
Ratio of efficiencies	0.58	0.64	0.53	0.67

The amount of heat added to the gases during explosion depends to a large extent on the temperature before the explosion, since the temperature after explosion is largely controlled by dissociation. It is therefore important that the engine shall be charged with a cool mixture of air and gas.

But the compression, or *cartridge space*, as it is sometimes called, is commonly filled with hot spent gases which mingle with and raise the temperature of the fresh charge. Three methods of meeting this difficulty have been tried.

1. In the Clerk engine the charge of gas and air is drawn into an auxiliary cylinder by a piston driven by a crank which is ninety degrees ahead of the motor-crank. The charge is lightly compressed and is forced into the motor-cylinder just before the motor-piston reaches the end of the stroke, and drives out the spent gases through holes in the walls of the cylinder which are overrun and uncovered by the piston at the proper time. The return stroke of the motor-piston compresses the charge, which is exploded at the beginning of the next forward stroke. This engine therefore makes an explosion for every revolution. In practice it is found that the fresh charge tends to mingle with the spent gases and that unburned gas escapes in the exhaust, so that there is little if any gain in efficiency over the Otto engine.

2. The Atkinson engine has a double-beat linkage interposed between the piston and the crank, so that the piston makes four strokes of unequal length for each revolution. The diagram from the Atkinson engine has Fig. 47 for its ideal type. In the first place the piston has a very small clearance at the end of the exhaust stroke and the beginning of the filling stroke; and also the expansion reduces the pressure down towards that of the atmosphere before release. It appears that this engine showed marked improvement over the Otto engines with which it was compared, but the complicated linkage required to make the four unequal strokes gave much trouble, especially for large engines. More recent engines of the Otto type have, by increased size and compression, given better economy than the Atkinson engines which were made and tested. A comparison of the examples on pages 207 and 208 shows that the Otto cycle with sufficient compression may be expected to give nearly as good results as an engine with more expansion.

3. The third attempt to clear the cartridge space is found in the Crossley scavenging-engine, in which advantage is taken of the fact that the exhausts from the cylinder of an Otto engine set up a pulsation in the exhaust-pipe, and that between successive exhausts the pressure in that pipe is less than the pressure of the atmosphere. By making the exhaust-pipe of the right length (about 65 feet) the time of the reduced pressure between the pulsations may be made to coincide with the completion of the exhaust stroke. If the inlet air-valve is opened at this time fresh air sweeps through the cylinder and clears it of spent gases before the next charge of gas and air is drawn in. Clerk considers that a gain of five per cent may be attained by properly clearing the cylinder of spent gases; when larger gains have been reported they have been due in part to increased compression or increased size.

Gas-producers.—Illuminating gas must be refined so as to remove impurities which would give an offensive odor when burning, and is consequently an expensive fuel, and can seldom compete on even terms with coal burned to make steam. Large gas-engines have been supplied with a cheap gas made specially for them by distillation and partial combustion of coal and coke. The Dawson gas-producer, which has been successfully used, in practice has a closed furnace into which anthracite or coke is charged through a hopper and charging-valve. Air is forced into the furnace under a slight pressure by a steam-jet from a special small steam-boiler. The fire is kept about 18 inches thick to give incomplete combustion, and gives off a gas which in a certain instance had the composition:

	Dowson Gas.	Ideal Gas.
Nitrogen	48.98	45.0
Carbon monoxide	25.07	39.0
Hydrogen	18.73	16.0
Marsh-gas	0.31	
Olefiant gas	0.31	
Carbon dioxide	6.57	
Oxygen	0.03	
	100.00	

The ideal gas which would be produced if such a producer worked perfectly with pure carbon is given for the sake of comparison. A French analysis of Dowson gas gave over 11 per cent of carbon dioxide.

After the gas is formed it is passed through a scrubber of coke sprayed with water to cool it and remove the large proportion of sulphur, which would eventually injure the cylinder. From the scrubber the gas passes to a gas-holder, and from the holder through another scrubber to the engine. This gas-producer cannot be run with coal which produces a tarry distillate, and has consequently been restricted to anthracite and coke. Other producers have been devised which are intended, by mingling the distillate from the coal with steam and passing it to incandescent fuel, to break it up into lighter compounds like marsh-gas and olefiant gas. Such producers if successful could use bituminous and other cheaper coals.

The Dowson gas, from its composition, requires somewhat more than its own volume of air for complete combustion, and in general will give somewhat less mean effective pressure than illuminating-gas. Thus a scavenging-engine with Dowson gas gave 97.4 pounds mean effective pressure, while with coal-gas it gave 113.5 pounds. This was a large engine with diameter of 17 inches and a stroke of 24 inches; smaller engines may be expected to give about 65 pounds effective pressure with Dowson gas.

A recent test by Professor Spangler of a 100-H.P. Otto engine using gas made in a Taylor producer showed that the producer developed 69 per cent of the heat of the coal, and that the engine used 1.3 pounds of coal per horse-power per hour when developing 130 horse-power. The coal had the following composition:

Moisture	4.20
Volatile matter	6.88
Fixed carbon	80.41
Ash	8.51
Sulphur	0.74
	100.74

A test by Mr. Dowson of a Crossley engine showed one pound of coal per horse-power per hour; but as the coal in the producer was estimated at the beginning and end of the test, and as the test was only eight hours long, this result cannot be accepted without question.

Oil-engines.—A volatile oil like naphtha or gasoline, which can be readily vaporized at ordinary temperatures, can be used in any gas-engine. It is sufficient to draw a part of the air-supply through some simple vaporizer and to use this mixture of air and vapor in place of gas. But there is danger that such volatile oils may take fire accidentally or may leak out or escape from tanks and reservoirs and mingle with air in confined spaces and form an explosive mixture. Though engines of some size have been run with gasoline, the application of such volatile oils to explosive engines appears to be best adapted to motor-carriages or to small open boats where there is less danger from the formation of explosive mixtures with air in confined spaces. Insurance companies in America and the government in England interfere with the storing and use of gasoline and other volatile oils, and so have checked their use in gas-engines.

Much attention has been given in England to the development of engines which can use refined burning oils like kerosene. The difficulty in the use of such oils is that they cannot be entirely vaporized by the application of heat alone even though raised to 600° F. If heated under pressure they decompose, and yield, together with a light volatile product, another product which is heavier than the original oil. If they are brought to a very high temperature, by passing over a red-hot surface for example, they are likely to yield carbonaceous or tarry matter which clogs the vaporizer. The only exception appears to be the oil which is distilled from Scotch shales; it can be entirely vaporized at about 570° F., which may be due to the fact that it is made by a destructive distillation.

The usual way of treating kerosene and other safe oils

when they are used in engines is to spray or otherwise mingle them with hot air. Three ways have been devised for doing this. The first way was to spray the oil by an atomizer into an iron cylinder, which was heated by the exhaust gases while the engine was running. The air for the engine was drawn through this cylinder, and thus charged with oil in the form of vapor or perhaps partly in a spray or mist. In order to keep the vaporizing cylinder hot it was necessary to have an explosion for every complete cycle, as the omission of an explosion would cool the vaporizer too much. The engine was governed by reducing simultaneously the oil and air-supply, which was found to be a very wasteful way.

Another way of accomplishing the same thing is to have an extension to the cylinder which is not cooled by a water-jacket and which consequently becomes strongly heated. This extension, which has a constricted communication with the rest of the cylinder, is filled with hot spent gases after an explosion, into which the oil is sprayed and by which it is vaporized. The next charge of air is compressed into the extension to the cylinder, mingles with the hot gases in it, and an explosive mixture is formed which is ignited by the heat of the walls or the heat of compression when the proper proportions are attained. A variation of this device has the entire cylinder cooled with a water-jacket, but a closed receptacle smaller than the cylinder and open only at the forward end is attached to the cylinder-head. This receptacle is of course heated by the explosions, and can vaporize the oil which is injected into the hot gases remaining in it after the exhaust.

The third and perhaps the best way is to strongly heat a part of the air and draw it through oil in a hot vaporizer. This method brings the engine into much the condition of an engine using gasoline, and the usual way of governing by omitting explosions can be used. It is true that the preceding method may be governed by omitting explosions, but there is danger that the hot chamber may be too much cooled

if several explosions are omitted, and that the engine will stop.

All methods involving the use of hot air or hot gas for vaporizing the oil have the defect that the charge is at a high temperature before it is compressed. The compression must not be carried too far for fear of a premature explosion, and again, the rise of temperature is less than for a cool charge. Consequently the efficiency of the engine is less than that of a gas-engine.

The ideal way of treating safe oils for use in an explosive engine appears to be to decompose them in such a way as to get volatile products only. In refining petroleum and making kerosene it is customary to treat some of the heavier products by distillation and redistillation under pressure, and thus to gain a larger proportion of illuminating-oil. If a similar method could be used with kerosene, and thus a volatile product like gasoline could be made continuously as demanded by the engine, the main difficulty with the use of safe oils in explosive engines would be removed.

Ignition.—After the mixture of air and oil is compressed in the cylinder of an explosion-engine it must be ignited. There are four ways of accomplishing this: (1) by an electric spark, (2) by the flame method, (3) by a heated tube, and (4) by compression in a hot chamber.

The electric method requires a galvanic battery, an induction-coil, and a circuit-breaker—apparatus requiring some care and intelligence. It appears to be the favorite method in America, but is not liked in England. Older forms of this device had simply two points at the proper distance across which a spark leaped when the circuit was closed or broken by a circuit-breaker outside of the cylinder. The later form has the circuit-breaker inside the cylinder consisting of two points of platinum, one of which is pressed against or wiped over the other at the proper time by a gear driven by the engine. A spark is formed both when the circuit is closed and when it is broken; the charge is usually fired by the spark which is

formed when the circuit is opened as it is the more intense. This apparatus is made in a substantial way and does not appear to fail or give trouble in any way.

The flame ignition consists in conveying an ignited mass of gas and air from an igniting flame outside the cylinder of the engine to the explosive mixture inside. The space or cavity in which the burning gas is conveyed is usually in a slide which may or may not serve as the valve for charging and exhausting the cylinder. To convey the burning gas to the charge the cavity must be partially filled with gas; the gas is ignited at a small jet burning steadily near the cylinder; the cavity is then closed and a small supply of explosive gas from the cylinder is let in to raise the pressure in the cavity lest the sudden opening of the cavity to the cylinder should extinguish the flame; the cavity, where the pressure is still less than in the cylinder, is brought to the explosion-port and the charge is fired; the cavity must finally be ventilated to be ready for the next supply of gas. This method, using illuminating gas, works well up to 80 ignitions a minute. Clerk has a flame ignition in which the flame cavity is fed only from the compressed explosive gas in the cylinder. The cavity consequently does not require ventilation and is more quickly filled with gas. He has used it to make 300 ignitions a minute.

The hot tube requires only a small iron tube, which is kept red-hot by a Bunsen burner or other heating flame. The tube comes out horizontally from the cylinder and sometimes is turned upward for convenience in heating. At the proper time the explosive mixture in the cylinder is admitted to the tube by a valve which is worked by the engine. Sometimes the tube has an inlet-valve at the outer end to ventilate the tube with air drawn in during the filling stroke.

In the discussion of oil-engines reference has already been made to the fact that the charge may be exploded spontaneously by compressing it into a hot receptacle. In some engines this occurs when the hot gas with its charge of oil in

the receptacle is mingled with a proper amount of air to form an explosive mixture. In other engines the explosion takes place when the mixture of air and oil is compressed to a pressure which will cause an explosion in contact with the hot sides of the receptacle or with some other hot metallic surface. Considerable ingenuity has been shown in properly proportioning oil-engines so that the explosion shall take place spontaneously at the proper time. This method is better adapted to oil-engines than to gas-engines, as a mixture of oil and air is more readily exploded and at a lower temperature than a mixture of gas and air.

Gas-engine Governors.—The only effective way of governing a gas-engine is to omit an explosion from time to time, so that the engine shall run at less than full power. When the engine is running light two or more explosions in succession may be omitted. When the engine attains too high a speed the governor draws back the cam which opens the gas-valve and no gas is admitted; the engine consequently takes in air only, compresses and expands and then exhausts it, but no work is developed by the engine. The advantage of this method is that the engine is as efficient at small as at large loads; owing to the clearing of the cylinder of spent gases during the cycle when no explosion takes place the succeeding working cycle is more efficient. The disadvantage is that the engine runs irregularly, as the only source of energy when the engine fails to explode is the fly-wheel. This, and the fact that only one stroke in four is a working stroke, explains the use of heavy fly-wheels for explosive engines.

The reason why the engine must be governed by omitting an explosion is that the proportion of the mixture of gas and air can be varied only within narrow limits if it is to explode when ignited. Some engines for electric lighting have been made to diminish the gas-supply when the load is smaller than full power, but the variation of power by this means is very small and must be supplemented by omitting explosions if the load is much diminished. Large gas-engines with two cylinders

are of course more easily controlled than single-cylinder engines, since they have an explosion for every revolution.

The Diesel Motor.—A new form of internal-combustion engine was described by Rudolf Diesel in 1893, which does away with many of the difficulties of gas- and oil-engines and which at the same time gives a much higher efficiency. The essential feature of his engine consists in the adiabatic compression of atmospheric air to a sufficient temperature to ignite the fuel which is injected at a determined rate during part of the expansion or working stroke. The first experimental engines were built at Augsburg in 1894 and 1896, one of 12 and another of 20 horse-power; now the construction of these engines is undertaken at several places in Europe and in America.

The cycle of all of the engines thus far built under Diesel's patents is represented by Fig. 50, which represents four strokes of a single-acting piston or plunger. Atmospheric air is drawn in from *a* to *b* and is compressed from *b* to *c* to a pressure of 500 pounds to the square inch and a temperature of 1000° F. From *c* to *d* fuel is injected in a finely divided form, and as there is air in excess it burns completely at a rate that can be controlled by the injection mechanism. Thus far the only fuel used is petroleum or some other oil. At *d* the supply of fuel is interrupted and the remainder of the working stroke, *de*, is an adiabatic expansion. The cycle is completed by a release at *e* and a rejection of the products of combustion from *b* to *a*.

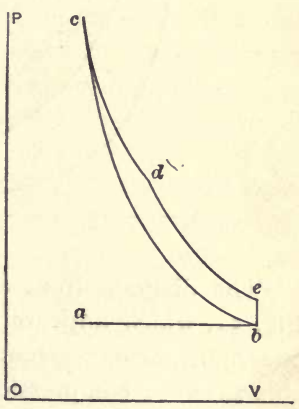

FIG 50.

The cycle has a resemblance to that of the Otto engine, but differs in that the air only is compressed in the cylinder and the combustion is accompanied by an expansion. Diesel, in his theoretic discussion of his engine, stipulates that the

rate of combustion shall be so regulated that the temperature shall not rise during the injection of fuel, and that the line *cd* shall therefore be very nearly an isothermal for a perfect gas. Since the fuel is added during the operation represented by the line *cd*, the weight of the material in the cylinder increases and its physical properties change, so that the line will not be a true isothermal. The fact that there is air in excess makes it probable that these changes of weight and properties will be insignificant. On the other hand, it is not probable that in practice the rate of injection of fuel will be regulated so as to give no rise of temperature, or that there is any great advantage in such a regulation if the temperature is not allowed to rise too high.

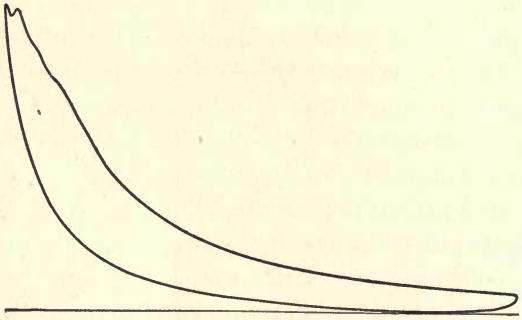

FIG. 51.

The diagram from an engine of this type is shown by Fig. 51, which appears to show an introduction of fuel for one-eighth or one-seventh of the working stroke. It is probable that the compression and the expansion after the cessation of the fuel supply are not really adiabatic, though as there is nothing but dry gas in the cylinder during those operations the deviation may not be large. The sides and heads of the cylinders of all the engines thus far constructed are water-jacketed, though the use of such a water-jacket and the consequent waste of heat was one of the difficulties in the use of internal-combustion engines that Diesel sought to avoid by controlling the rate of combustion.

The oil used as fuel is injected in form of a spray by air that is compressed separately in a small pump to 30 or 40 pounds pressure above that in the main cylinder; of course it is necessary to cool this portion of the air after compression to avoid premature ignition. The engines that have been used are described as giving a clear and nearly dry exhaust. In damp weather the exhaust shows a little moisture, probably from the combustion of hydrogen in the oil. The cylinder when opened shows a slight deposit of soot on the head. It appears therefore that Diesel has succeeded in constructing an engine for burning heavy oils with good economy and without the annoyances of an igniting device. The engines have the further advantage in that the work can be regulated by the amount of fuel supplied, which amount is not controlled, as in explosive engines, by the necessity to form an explosive mixture. The discussion of the theoretical efficiency of the cycle shows that the efficiency increases as the time of injection of fuel is shortened. In practice the engine shows a slight decrease in economy for light loads, due probably to the losses by radiation and to the water-jacket, which are nearly constant for all loads.

In the exposition of the theory of his motor, Diesel* claims that all kinds of fuel, solid, liquid, and gaseous, can be burned in his motor. As yet oil only has been used; the choice of petroleum or other heavy oil has probably been due to the low cost of such oils. It is evident that gas may be used in this type of engine; the gas can be compressed separately to a pressure somewhat higher than that in the main cylinder, much as the air is which is used for injecting oil. It does not appear necessary to cool the gas after compression, as it will burn only when supplied with air.

There appears to be no insurmountable difficulty in supplying powdered solid fuel to this engine. The presence of the ash from such fuel in the cylinder may, however, be expected to give trouble. Diesel claims that with a large

* *Rational Heat Motor*; Rudolf Diesel, trans. Bryan Donkin.

excess of air (for example, a hundred pounds of air for one pound of coal) the ash will be swept out of the cylinder with the spent gases and will not give trouble; but that claim is accompanied by an assumption that air may be compressed isothermally if water is injected into the cylinder of the compressor, and one idea appears to be as improbable as the other.

All the engines thus far built have single-acting pistons or plungers which perform all four operations for a cycle in one cylinder. Some of the engines have three cylinders with their plungers connected to a three-throw crank-shaft; but in that case each plunger has its own complete cycle performed in four strokes, just as for the single-cylinder engine. Diesel has, however, described a compound engine which has both compression and expansion divided into two stages. Such an engine may have a compression-pump, a combustion-cylinder, and an expansion-cylinder. The pump takes air from the atmosphere and compresses it to a moderate pressure and delivers it to a receiver. The combustion-cylinder has a plunger like that of the simple engine which performs a cycle in four strokes. It takes air at nearly constant pressure from the receiver on the first down stroke and compresses it to the proper pressure and temperature to ignite the fuel on the following up stroke; during a part of the second down stroke fuel is injected at such a rate as to produce isothermal expansion, and the remainder of that stroke gives adiabatic expansion with a reduction of temperature; the products of combustion cooled to a considerable extent by the partial adiabatic expansion in the combustion-cylinder are transferred to the expansion-cylinder during the fourth stroke of the plunger, which completes the cycle of the combustion-cylinder. The proportions of the three cylinders should be such that the fourth stroke of the plunger just mentioned should have for its terminal pressure the pressure in the receiver. The expansion-cylinder completes the expansion of the products of combustion and then discharges them into the atmosphere;

by its use the final volume of those gases when release occurs may be greater than the volume of the air drawn in by the compression-pump, and if desired, the expansion may be carried down to the pressure of the atmosphere. By assuming an isothermal compression in the compression-pump, Diesel works out a theoretic cycle which approaches very closely in appearance and efficiency to Carnot's cycle for a perfect gas; it appears that his attention was first directed to the motor he has invented by an attempt to describe an engine which could work on Carnot's cycle. But he assumes that an isothermal compression can be obtained by simply spraying water into the pump-cylinder during compression, which is contrary to experience in the use of air-compressors. Even if such an isothermal compression were practicable, it is doubtful whether it would be desirable; as it would require a much higher pressure to produce the proper ignition temperature in the combustion-cylinder than is now employed in single-cylinder engines; but the high pressure required to give ignition is one of the principal sources of difficulty in the single engine.

Since both compression-pump and expansion-cylinder, even with single-acting pistons, will have one working stroke for each revolution, while the combustion-cylinder can give a working stroke for two revolutions, it is clear that the compound engine must have at least two combustion-cylinders with their plungers working together, but having their operations half a cycle apart. The expansion-piston should have its crank opposite those of the combustion-plungers. The compression-pump may have its crank set at any convenient angle, since it delivers air to a reservoir.

A theoretical discussion of the efficiency of the cycle for the simple engine as represented by Fig. 50 may be obtained by considering that heat is added at constant temperature from c to d and that heat is rejected at constant volume from e to b, bearing in mind that bc and dc represent adiabatic changes.

From equation (75), page 65, the expression for the heat

supplied from c to d is, for one pound of working substance,

$$Q_1 = Ap_c v_c \log_e \frac{v_d}{v_c} = ART_c \log_e \frac{v_d}{v_e}.$$

The heat rejected at constant volume is

$$Q_2 = c_v(T_e - T_b) = \frac{c_p}{\kappa}(T_e - T_b).$$

Since the expansion de is adiabatic,

$$T_e = T_d\left(\frac{v_d}{v_e}\right)^{\kappa-1} = T_c\left(\frac{v_d}{v_e}\right)^{\kappa-1};$$

but since the compression bc is also adiabatic,

$$T_c = T_b\left(\frac{v_b}{v_c}\right)^{\kappa-1},$$

and consequently

$$T_e = T_b\left(\frac{v_b}{v_c}\right)^{\kappa-1}\left(\frac{v_d}{v_e}\right)^{\kappa-1} = T_b\left(\frac{v_d}{v_e}\right)^{\kappa-1}$$

for $v_e = b_b$. Replacing T_e by its value in the expression for Q_2, we have

$$Q_2 = \frac{c_p}{\kappa} T_b\left\{\left(\frac{v_d}{v_c}\right)^{\kappa-1} - 1\right\}.$$

Finally, the efficiency appears to be

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{\dfrac{c_p}{\kappa} T_b\left\{\left(\dfrac{v_d}{v_c}\right)^{\kappa-1} - 1\right\}}{ART_c \log_e \dfrac{v_d}{v_c}}.$$

Inspection of the equation shows that the efficiency may be increased by raising the temperature T_c or by reducing the temperature T_b. The latter is practically the temperature of the atmosphere, but T_c may be made any desired temperature by reducing the clearance of the cylinder and thus raising the pressure at the end of compression. Again, the efficiency

may be increased by reducing the time during which fuel is injected, that is, by reducing the ratio $v_d : v_c$, as may be proved by a series of calculations with different values for that ratio. This is a very important conclusion, as it shows that the engine will have in practice little if any falling off in efficiency at reduced loads.

It is reported that a clearance of something less than 7 per cent is associated with a compression to 500 pounds and a temperature of 1000° F., or more. Taking the pressure of the atmosphere at 14.7 pounds per square inch, adiabatic compression to 500 pounds above the atmosphere or to 514.7 pounds absolute requires a clearance of

$$v_a = v_b\left(\frac{p_b}{p_c}\right)^{\frac{1}{\kappa}} = v_b\left(\frac{14.7}{514.7}\right)^{\frac{1}{1.405}} = 0.0796v_b,$$

so that the clearance is

$$0.0796 \div \{1 - 0.0796\} = 0.0865$$

of the piston displacement.

If the temperature of the atmosphere be taken at 70° F. or 530.7 absolute, the temperature after adiabatic compression becomes

$$T_c = T_b\left(\frac{p_c}{p_b}\right)^{\frac{\kappa-1}{\kappa}} = 530.7\left(\frac{514.7}{14.7}\right)^{\frac{1.405-1}{1.405}} = 1480$$

absolute, or 1020° F.

If it be further assumed that fuel is supplied for one-tenth of the working stroke, then

$$v_d = 0.1(v_b - v_a) + v_a = 0.1(1 - 0.0796) + 0.0796v_b$$
$$= 0.1716v_b.$$

The equation for efficiency gives in this case

$$\eta = 1 - \frac{778 \times 0.2375 \times 530.7 \left\{ \left(\frac{0.1716}{0.0796} \right)^{0.405} - 1 \right\}}{1.405 \times 53.22 \times 1480 \log_e \frac{0.1716}{0.0796}} = 0.58.$$

An engine giving 26.6 indicated horse-power (cheval-à-vapeur) and exerting 19.2 horse-power at the brake is reported to consume 223 grams of petroleum per hour and to give a thermal efficiency of 28.6 per cent. Now if the heats of combustion of coal and petroleum are taken at ~~1400~~ B. T. U. and 20,000 B. T. U., respectively, one gram of petroleum is equivalent to $\frac{10}{7}$ of a gram of coal, and the performance of the Diesel engine is equal to the consumption of 333 grams of coal per horse-power per hour, or to about $\frac{3}{4}$ of a pound of coal per English horse-power per hour. To make the comparison complete the relative costs of coal and petroleum should be considered, together with the probability that a large demand for petroleum would be liable to affect its price.

CHAPTER XI.

THE STEAM-ENGINE.

THE steam-engine is, at the present time, the most important heat-engine. When of large size and properly designed and managed it has at least as good economy as any other heat-engine, though it may be excelled in this regard by explosive gas-engines when they are fully developed. It can be controlled, regulated, and reversed easily and positively—properties which are not possessed in like degree by other heat-engines. It is interesting to know that the theory of thermodynamics was developed mainly to account for the action and to provide methods of designing steam-engines; though neither object is entirely accomplished, on account of the fact that the engine-cylinder must be made of some metal to be hard and strong enough to endure service, for all metals are good conductors of heat and seriously affect the action of a condensable fluid like steam.

Carnot's Cycle for a steam-engine is represented by Fig. 52, in which *ab* and *cd* are isothermal lines representing the application and rejection of heat at constant temperature and at constant pressure. *bc* and *da* are adiabatic lines, representing change of temperature and pressure, without transmission of heat through the walls of the cylinder. The diagram representing Carnot's cycle has an external resemblance to the indicator-diagram from some actual engines, but it differs in essential particulars.

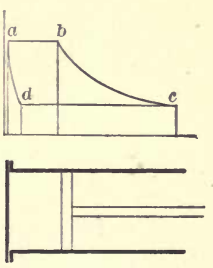

FIG. 52.

In the condition represented by the point a the cylinder contains a mixture of water and steam at the temperature t_1 and the pressure p_1. If connection is made with a source of heat at the temperature t_1, and heat is added, some of the water will be vaporized and the volume will increase at constant pressure as represented by ab. If thermal communication is now interrupted adiabatic expansion may take place as represented by bc till the temperature is reduced to t_2, the temperature of the refrigerator, with which thermal communication may now be established. If the piston is forced toward the closed end of the cylinder some of the steam in it will be condensed, and the volume will be reduced at constant pressure as represented by cd. The cycle is completed by an adiabatic compression represented by da.

If the absolute temperature of the source of heat is T_1, and if that of the refrigerator is T_2, then the efficiency is

$$\eta = \frac{T_1 - T_2}{T_1},$$

whatever may be the working fluid.

For example, if the pressure of the steam during isothermal expansion is 100 pounds above the atmosphere, and if the pressure during isothermal compression is equal to that of the atmosphere, then the temperatures of the source of heat and of the refrigerator are 337°.6 F. and 212° F., or 798.4 and 672.7 absolute, so that the efficiency is

$$\frac{798.4 - 672.7}{798.4} = 0.157.$$

The following table gives the efficiencies worked out in a similar way, for various steam-pressures,—both for t_2 equal to 212° F., corresponding to atmospheric pressure, and for t_2 equal to 116° F., corresponding to an absolute pressure of 1.5 pounds to the square inch:

EFFICIENCY OF CARNOT'S CYCLE FOR STEAM-ENGINES.

Initial Pressure by the Gauge, above the Atmosphere.	Atmospheric Pressure.	1.5 Pounds Absolute.
15	0.053	0.189
30	0.084	0.215
60	0.124	0.249
100	0.157	0.278
150	0.186	0.302
200	0.207	0.320
300	0.238	0.347

The column for atmospheric pressure may be used as a standard of comparison for non-condensing engines, and the column for 1.5 pounds absolute may be used for condensing engines.

It is interesting to consider the condition of the fluid in the cylinder at the different points of the diagram for Carnot's cycle. Thus if the fluid at the condition represented by b in Fig. 52 is made up of x_b part steam and $1 - x_b$ part water, then from equation (146) the condition at the point c is given by

$$x_c = \frac{T_2}{r_2}\left(\frac{r_1}{T_1}x_b + \theta_1 - \theta_2\right). \quad . \quad . \quad . \quad (244)$$

In like manner the condition of the mixture at the point d is

$$x_d = \frac{T_2}{r_2}\left(\frac{r_1}{T_1}x_a + \theta_1 - \theta_2\right). \quad . \quad . \quad . \quad (245)$$

It is interesting to note that if x_b is larger than one-half, that is, if there is more steam than water in the cylinder at b, then the adiabatic expansion is accompanied by condensation. Again, if x_a is less than one-half, then the adiabatic compression is also accompanied by condensation. Very commonly it is assumed that x_b is unity, so that there is dry saturated steam in the cylinder at b; and that x_a is zero so that there is water only in the cylinder at a; but there is no necessity for such assumptions, and they in no way affect the efficiency.

If the cylinder contains M pounds of steam and water, the heat absorbed by the working substance during isothermal expansion is

$$Q_1 = Mr_1(x_b - x_a), \quad . \quad . \quad . \quad . \quad . \quad (246)$$

and the heat rejected during isothermal compression is

$$Q_2 = Mr_2(x_c - x_d), \quad . \quad . \quad . \quad . \quad . \quad (247)$$

so that the heat changed into work during the cycle is

$$Q_1 - Q_2 = M\{r_1(x_a - x_b) - r_2(x_c - x_d)\}. \quad . \quad (248)$$

But from equations (244) and (245)

$$r_2(x_c - x_d) = \frac{T_2}{T_1} r_1(x_b - x_a), \quad . \quad . \quad . \quad (249)$$

and the expression for the heat changed into work becomes

$$Q_1 - Q_2 = Mr_1(x_b - x_a)\frac{T_1 - T_2}{T_1}. \quad . \quad . \quad (250)$$

This equation is deduced because it is convenient for making comparisons of various other volatile liquids and their vapors, with steam, for use in heat-engines. It is of course apparent that

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1},$$

from equations (250) and (246), a conclusion which is known independently, and indeed is necessary in the development of the theory of the adiabatic expansion of steam.

In the discussion thus far it has been assumed that the working fluid is steam, or a mixture of steam and water. But a mixture of any volatile liquid and its vapor will give similar results, and the equations deduced can be applied

directly. The principal difference will be due to the properties of the vapor considered, especially its specific pressures and specific volumes for the temperatures of the source of heat and the refrigerator.

For example, the efficiency of Carnot's cycle for a fluid working between the temperatures 160° C. and 40° C. is

$$\frac{160 - 40}{160 + 273.7} = 0.277.$$

The properties of steam and of chloroform at these temperatures are

	Steam.		Chloroform.	
	40° C.	160° C.	40° C.	160° C.
Pressure, mm. mercury	54.91	4651.4	369.26	8734.2
Volume, cubic metres	19.74	0.3035	0.4449	0.0243
Heat of vaporization, r	578.7	494.2	63.13	50.53
Entropy of liquid, θ	0.1364	0.4633	0.03196	0.11041

For simplicity, we may assume that one kilogram of the fluid is used in the cylinder for Carnot's cycle, and that x_b is unity while x_a is zero, so that from equation (250)

$$Q_1 - Q_2 = r_1 \frac{T_1 - T_2}{T_1},$$

and for steam

$$Q_1 - Q_2 = 494.2 \times 0.277 = 137 \text{ calories},$$

while for chloroform

$$Q_1 - Q_2 = 50.53 \times 0.277 = 14 \text{ calories}.$$

After adiabatic expansion the qualities of the fluid will be, from equation (244), for steam

$$x_c = \frac{40 + 273.7}{578.7}\left(\frac{494.2}{160 + 273.7} + 0.4633 - 0.1364\right) = 0.795,$$

and for chloroform

$$x_c = \frac{40+273.7}{63.13}\left(\frac{50.53}{160+273.7} + 0.11041 - 0.03196\right) = 0.969.$$

The specific volumes after adiabatic expansion are, consequently, for steam

$$v_c = x_c u_2 + \sigma = 0.795(19.74 - 0.001) + 0.001 = 15.7,$$

and for chloroform

$$v_c = x_c u_2 + \sigma = 0.969(0.4449 - 0.000655) + 0.000655 = 0.431.$$

These values for v_c just calculated are the volumes in the cylinder at the extreme displacement of the piston, on the assumption that one kilogram of the working fluid is vaporized during isothermal expansion. A better idea of the relative advantages of the two fluids will be obtained by finding the heat changed into work for each cubic metre of maximum piston-displacement, or for a cylinder having the volume of one cubic metre. This is obtained by dividing $Q_1 - Q_2$, the heat changed into work for each kilogram by v_c. For steam the result is

$$(Q_1 - Q_2) \div v_c = 137 \div 15.7 = 8.73,$$

and for chloroform it is

$$(Q_1 - Q_2) \div v_c = 14 \div 0.413 = 34;$$

from which it appears that for the same volume chloroform can produce more than three and a half times as much power. Even if we consider that the difference of pressure for chloroform,

$$8734.2 - 369.3 = 8364.9 \text{ mm.},$$

is nearly twice that for steam, which has only

$$4651.4 - 54.9 = 4596.5 \text{ mm.}$$

difference of pressure, the advantage appears to be in favor of chloroform. If, however, the difference of pressures given for chloroform is allowable also for steam, giving

$$8364.9 + 54.9 = 8419.8 \text{ mm.}$$

for the superior pressure, then the initial temperature for steam becomes 184°.9 C., and the efficiency becomes

$$\frac{184°.9 - 40}{184.9 + 273.7} = 0.318,$$

instead of 0.277. On the whole, steam is the more desirable fluid, even if we do not consider the inflammable and poisonous nature of chloroform. Similar calculations will show that on the whole steam is at least as well adapted for use in heat-engines as any other saturated fluid; in practice, the cheapness and incombustibility of steam indicate that it is the preferable fluid for such uses.

Non-conducting Engine.—The conditions required for alternate isothermal expansion and adiabatic expansion cannot be fulfilled for Carnot's cycle with steam any more than they could be for air. The diagram for the cycle with steam, however, is well adapted to production of power; the contrary is the case with air, which gives a much attenuated diagram.

In practice steam from a boiler is admitted to the cylinder of the steam-engine during that part of the cycle which corresponds to the isothermal expansion of Carnot's cycle, thus transferring the isothermal expansion to the boiler, where steam is formed under constant pressure. In like manner the isothermal compression is replaced by an exhaust at constant pressure, during which steam may be condensed in a separate condenser, cooled by cold water.

By proper valve-gear the expansion and compression of Carnot's cycle may be simulated, thus giving a diagram having an external resemblance to Carnot's cycle. The cylinder

is commonly made of cast iron, and is always some kind of metal; there is consequently considerable interference due to the conductivity of the walls of the cylinder, and the expansion and compression are never adiabatic. There is an advantage, however, in discussing first an engine with a cylinder made of some non-conducting material, although no such material proper for making cylinders is now known.

The diagram representing the operations in a non-conducting cylinder for a steam-engine can be represented by Fig. 53. *ab* represents the admission of dry saturated steam from the boiler; *bc* is an adiabatic expansion to the exhaust pressure; *cd* represents the exhaust; and *da* is an adiabatic compression to the initial pressure. It is assumed that the small volume, represented by *a*, between the piston and the head of the cylinder is filled with dry steam, and that the steam remains homogeneous during exhaust so that the quality is the same at *d* as at *c*. These conditions are consistent and necessary, since the change of condition due to adiabatic expansion (or compression) depends only on the initial condition and the initial and final pressures; so that an adiabatic expansion from *a* to *d* would give the same quality at *d* as that found at *c* after adiabatic expansion from *b*, and conversely adiabatic compression from *d* to *a* gives dry steam at *a* as required.

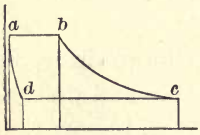

FIG. 53.

The cycle represented by Fig. 53 differs most notably from Carnot's cycle (Fig. 52) in that *ab* represents admission of steam and *cd* represents exhaust of steam, as has already been pointed out. It also differs in that the compression *da* gives dry steam instead of wet steam. The compression line *da* is therefore steeper than for Carnot's cycle, and the area of the figure is slightly larger on this account. This curious fact does not indicate that the cycle has a higher efficiency; on the contrary, the efficiency is less, and the cycle is incomplete. To complete the cycle for a non-conducting cylinder

the exhausted steam must be condensed, pumped back into the boiler, and reëvaporated.

If the pressure during admission (equal to the pressure in the boiler) is p_1, and if the pressure during exhaust is p_2, then the heat required to raise the water resulting from the condensation of the exhaust-steam is

$$q_1 - q_2,$$

where q_1 is the heat of the liquid at the pressure p_1 and q_2 is the heat of the liquid at the pressure p_2. The heat of vaporization at the pressure p_1 is r_1, so that the heat required to raise the feed-water from the temperature of the exhaust to the temperature in the boiler and evaporate it into dry steam is

$$Q_1 = r_1 + q_1 - q_2; \quad . \quad . \quad . \quad . \quad . \quad (251)$$

and this is the quantity of heat supplied to the cylinder per pound of steam.

The steam exhausted from the cylinder has the quality x_2 calculated by aid of the equation

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right),$$

and the heat that must be withdrawn when it is condensed is

$$Q_2 = x_2 r_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

this is the heat rejected from the engine. The heat changed into work per pound of steam is

$$Q_1 - Q_2 = r_1 + q_1 - q_2 - x_2 r_2. \quad . \quad . \quad . \quad (253)$$

But part of the work is used in pumping the condensed steam or feed-water into the boiler from the pressure p_2 to the pressure p_1. The heat equivalent of this work is

$$A(p_1 - p_2)\sigma, \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

where σ is the specific volume of water. This heat should be subtracted from the heat changed into work per pound of

steam to find the available energy developed by the engine. But the heat represented by the expression (254) is small compared with the heat changed into work as represented by equation (253), and may be neglected.

The efficiency of the cycle is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2}. \quad . \quad . \quad . \quad (255)$$

If values are assigned to p_1 and p_2 and the proper numerical calculations are made, it will appear that the efficiency for a non-conducting engine is always less than the efficiency for Carnot's cycle between the corresponding temperatures.

It should be remarked that the efficiency is not affected by the clearance or space between the piston and the head of the cylinder and the space in the steam-passages of the cylinder, provided that the clearance is filled with dry saturated steam as indicated in Fig. 52. This is evident from the fact that no term representing the clearance, or volume at a, Fig. 52, appears in equation (254). Or, again, we may consider that the steam in the cylinder at the beginning of the stroke, occupying the volume represented by a, expands during the adiabatic expansion and is compressed again during compression, so that one operation is equivalent to and counterbalances the other, and so does not affect the efficiency of the cycle.

Incomplete Cycle. — The cycle for a non-conducting engine may be incomplete because the expansion is not carried far enough to reduce the pressure to that of the back-pressure line, as is shown in Fig. 54. Such an incomplete cycle has less efficiency than a complete cycle, but in practice the advantage of using a smaller cylinder and of reducing friction is sufficient compensation for the small loss of efficiency due to a moderate drop at the end of the stroke, as shown in Fig. 54.

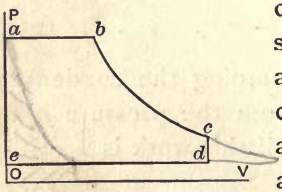

FIG. 54.

The discussion of the incomplete cycle is simplified by assuming that there is no clearance and no compression as is indicated by Fig. 54. It will be shown later that the efficiency will be the same with a clearance, provided the compression is complete.

The most ready way of finding the efficiency for this cycle is to determine the work of the cycle. Thus the work during admission is

$$p_1(u_1 + \sigma), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (256)$$

where u_1 is the increase of volume due to vaporization of a pound of steam and σ is the specific volume of water. The work during expansion is

$$E_b - E_c = \frac{1}{A}(\rho_1 + q_1 - x_c\rho_c - q_c), \quad . \quad . \quad . \quad (257)$$

where q_1 and ρ_1 are the heat of the liquid and the heat-equivalent of the external work during vaporization at the pressure p_1, while q_c and ρ_c are corresponding quantities for the pressure at c. x_c is to be calculated by the equation

$$x_c = \frac{T_c}{r_c}\left(\frac{r_1}{T_1} + \theta_1 - \theta_c\right).$$

The work done by the piston on the steam during exhaust is

$$p_2(x_c u_c + \sigma).$$

The total work of the cycle is obtained by adding the work during admission and expansion and subtracting the work during exhaust, giving

$$\frac{1}{A}(\rho_1 + Ap_1u_1 - x_c\rho_c - Ap_2x_cu_c + q_1 - q_c) + (p_1 - p_2)\sigma. \quad (258)$$

The last term is small, and may be neglected. Adding and subtracting $Ap_c x_c u_c$ and multiplying by A, we get for the heat-equivalent of the work of the cycle

$$Q_1 - Q_2 = r_1 - x_c r_c + A(p_c - p_2)u_c x_c + q_1 - q_c, \quad . \quad (259)$$

which is equal to the difference between the heat supplied and the heat rejected as indicated. The heat supplied is

$$Q_1 = r_1 + q_1 - q_2, \quad .$$

as was deduced for the complete cycle; the cost of making the steam remains the same, whether or not it is used efficiently. Finally, the efficiency of the cycle is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{r_1 + q_1 - x_c r_c - q_c + A(p_c - p_2)x_c u_c}{r_1 + q_1 - q_2};$$

$$\therefore \quad \eta = 1 - \frac{x_c r_c + q_c - q_2 - A(p_c - p_2)x_c u_c}{r_1 + q_1 - q_2}. \quad . \quad . \quad . \quad (260)$$

If p_c is made equal to p_2 in the preceding equation, it will be reduced to the same form as equation (254), because the cycle in such case becomes complete.

Steam-consumption of Non-conducting Engine. — A horse-power is 33000 foot-pounds per minute or 60 × 33000 foot-pounds per hour. But the heat changed into work per pound of steam by a non-conducting engine with complete expansion is, by equation (253),

$$r_1 + q_1 - q_2 x_2 r_2,$$

so that the steam required per horse-power per hour is

$$\frac{60 \times 33000}{778(r_1 + q_1 - q_2 - x_2 r_2)}.$$

Similarly, the steam per horse-power per hour for an engine with incomplete expansion, by aid of expression (258), is

$$\frac{60 \times 33000}{778(\rho_1 + Ap_1u_1 - x_c\rho_c - Ap_2x_cu_c + q_1 - q_c)}.$$

The value of x_2 or x_c is to be calculated by the general equation

$$x = \frac{T}{r}\left(\frac{r_1}{T_1} + \theta_1 - \theta\right).$$

The denominator in either of the above expressions for the steam per horse-power per hour is of course the work done per pound of steam and the parenthesis without the mechanical equivalent 778 is equal to $Q_1 - Q_2$. If then we multiply and divide by

$$Q_1 = r_1 + q_1 - q_2,$$

that is, by the heat brought from the boiler by one pound of steam, we shall have in either case

$$\frac{60 \times 33000 \times Q_1}{778(Q_1 - Q_2)Q_1} = \frac{60 \times 33000}{778\eta(r_1 + q_1 - q_2)}, \quad . \quad . \quad (261)$$

where

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

is the efficiency for the cycle.

Actual Steam-engine.—The indicator-diagram from an actual steam-engine differs from the cycle for a non-conducting engine in two ways: there are losses of pressure between the boiler and the cylinder and between the cylinder and the condenser, due to the resistance to the flow of steam through pipes, valves, and passages; and there is considerable interference of the metal of the cylinder with the action of the steam in the cylinder. The losses of pressure may be mini-

mized for a slow-moving engine by making the valves and passages direct and large. The interference of the walls of the cylinder cannot be prevented, but may be ameliorated by using superheated steam or by steam-jacketing.

When steam enters the cylinder of an engine, some of it is condensed on the walls which were cooled by contact with exhaust-steam, thereby heating them up nearly to the temperature of the steam. After cut-off the pressure of the steam is reduced by expansion and some of the water on the walls of the cylinder vaporizes. At release the pressure falls rapidly to the back-pressure, and the water remaining on the walls is nearly if not all vaporized. It is at once evident that so much of the heat as remains in the walls until release and is thrown out during exhaust is a direct loss; and again, the heat which is restored during expansion does work with less efficiency, because it is reëvaporated at less than the temperature in the boiler or in the cylinder during admission. A complete statement of the action of the walls of the cylinder of an engine, with quantitative results from tests on engines, was first given by Hirn. His analysis of engine tests, showing the interchanges of heat between the walls of the cylinder and the steam, will be given later. It is sufficient to know now that a failure to consider the action of the walls of the cylinder leads to gross errors, and that an attempt to base the design of an engine on the theory of a steam-engine with a non-conducting cylinder can lead only to confusion and disappointment.

The most apparent effect of the influence of the walls of the cylinder on the indicator-diagram is to change the expansion and the compression lines; the former exhibits this change most clearly. In the first place the fluid in the cylinder at cut-off consists of from twenty to fifty per cent hot water, which is found mainly adhering to the walls of the cylinder. Even if there were no action of the walls during expansion the curve would be much less steep than the adiabatic line for dry saturated steam. But the reëvaporation

during expansion still further changes the curve, so that it is usually less steep than the rectangular hyperbola.

It may be mentioned that the fluctuations of temperature in the walls of a steam-engine cylinder caused by the condensation and reëvaporation of water do not extend far from the surface, but that at a very moderate depth the temperature remains constant so long as the engine runs under constant conditions.

The performance of a steam-engine is commonly stated in pounds of steam per horse-power per hour. For example, a small Corliss engine, developing 16.35 horse-power when running at 61.5 revolutions per minute under 77.4 pounds boiler-pressure, used 548 pounds of steam in an hour. The steam consumption was

$$548 \div 16.35 = 33.5$$

pounds per horse-power per hour.

This method was considered sufficient in the earlier history of the steam-engine, and may now be used for comparing simple condensing or non-condensing engines which use saturated steam and do not have a steam-jacket, for the total heat of steam, and consequently the cost of making steam from water at a given temperature increases but slowly with the pressure.

The performance of steam-engines may be more exactly stated in British thermal units per horse-power per minute. This method, or some method equivalent to it, is essential in making comparisons to discover the advantages of superheating, steam-jacketing, and compounding. For example, the engine just referred to used steam containing two per cent of moisture, so that x, at the steam-pressure of 77.4 pounds was 0.98. The barometer showed the pressure of the atmosphere to be 14.7 pounds, and this was also the back-pressure during exhaust. If it be assumed that the feed-water was or could be heated to the corresponding temperature of 212° F., the

heat required to evaporate it against 77.4 pounds above the atmosphere or 92.1 pounds absolute was

$$x_1 r_1 + q_1 - q_2 = 0.98 \times 888.4 + 291.7 - 180.8 = 981.5 \text{ B. T. U.}$$

The thermal units per horse-power per minute were

$$\frac{981.5 \times 33.5}{60} = 548.$$

Efficiency of the Actual Engine.—When the thermal units per horse-power per minute are known or can be readily calculated, the efficiency of the actual steam-engine may be found by the following method: A horse-power corresponds to the development of 33000 foot-pounds per minute, which are equivalent to

$$33000 \div 778 = 42.42$$

thermal units. This quantity is proportional to $Q_1 - Q_2$, and the thermal units consumed per horse-power per minute are proportional to Q_1, so that the efficiency is

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{42.42}{\text{B. T. U. per H.P. per min.}}.$$

For example, the Corliss engine mentioned above had an efficiency of

$$42.42 \div 548 = 0.077.$$

This same method may evidently be applied to any heat-engine for which the consumption in thermal units per horse-power per hour can be applied.

From the tests reported on page 327 it appears that the engine in the laboratory of the Massachusetts Institute of Technology on one occasion used 13.73 pounds of steam per horse-power per hour, of which 10.86 pounds were supplied to the cylinders and 2.87 pounds were condensed in steam-

jackets on the cylinders. The steam in the supply-pipe had the pressure of 157.7 pounds absolute, and contained 1.2 per cent of moisture. The heat supplied to the cylinders per minute in the steam admitted was

$$10.86(x_1 r_1 + q_1 - q_2) \div 60$$
$$= 10.86(0.988 \times 858.3 + 334.2 - 126.4) \div 60 = 191.1 \text{ B. T. U.};$$

q_2 being the heat of the liquid at the temperature of the back-pressure of 4.5 pounds absolute.

The steam condensed in the steam-jackets was withdrawn at the temperature due to the pressure and could have been returned to the boiler at that temperature; consequently the heat required to vaporize it was r_1, and the heat furnished by the steam in the jackets was

$$2.87 \times 0.98 \times 858.3 \div 60 = 40.6 \text{ B. T. U.}$$

The heat consumed by the engine was

$$191.1 + 40.6 = 231.7 \text{ B. T. U.}$$

per horse-power per minute, and the efficiency was

$$\eta = 42.42 \div 231.7 = 0.183.$$

The efficiency of Carnot's cycle for the range of temperatures corresponding to 157.7 and 4.5 pounds absolute, namely, 822°.9 and 618°.4 absolute, is

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{822.9 - 618.4}{822.9} = 0.2485.$$

The efficiency for a non-conducting engine with complete expansion, calculated by equation (255), is for this case

$$\eta'' = 1 - \frac{x_2 r_2}{r_1 + q_1 - q_2} = 1 - \frac{0.8214 \times 1004.2}{858.3 + 334.2 - 126.1} = 0.227;$$

where x_2 is calculated by the equation

$$x_2 = \frac{T_2}{r_2}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right)$$

$$= \frac{618.4}{1004.2}\left(\frac{858.3}{822.9} + 0.5184 - 0.2278\right) = 0.8214.$$

During the test in question the terminal pressure at the end of the expansion in the low-pressure cylinder was 6 pounds absolute, which gives

$$x_c = \frac{T_c}{r_c}\left(\frac{r_1}{T_1} + \theta_1 - \theta_2\right)$$

$$= \frac{630.8}{995.2}\left(\frac{858.2}{822.9} + 0.5184 - 0.2480\right) = 0.8317,$$

and the efficiency by equation (260) was

$$\eta''' = 1 - \frac{x_c r_c - q_c + q_2 - A(p_c - p_2)x_c u_c}{r_1 + q_1 - q_2}$$

$$= 1 - \frac{0.8317 \times 995.2 - 138.6 + 126.1 + \frac{144}{778}(6 - 4.5)0.8317 + 61.65}{858.3 + 334.2 - 126.1}$$

$$= 0.222.$$

The real criterion of the perfection of the action of an engine is the ratio of its actual efficiency to that of a perfect engine. If for the perfect engine we choose Carnot's cycle the ratio is

$$\frac{\eta}{\eta'} = \frac{0.183}{0.2485} = 0.736.$$

But if we take for our standard an engine with a cylinder of non-conducting material the ratio for complete expansion is

$$\frac{\eta}{\eta''} = \frac{0.183}{0.227} = 0.807.$$

For incomplete expansion the ratio is

$$\frac{\eta}{\eta'''} = \frac{0.183}{0.222} = 0.824.$$

To complete the comparison it is interesting to calculate the steam-consumption for a non-conducting steam-engine by equation (261), both for complete and for incomplete expansion. For complete expansion we have

$$\frac{60 \times 33000}{778 \times 0.227(858.3 + 334.2 - 126.1} = 10.5 \text{ pounds},$$

and for incomplete expansion

$$\frac{60 \times 33000}{778 \times 0.222(858.3 + 334.2 - 126.1)} = 10.7 \text{ pounds}$$

per horse-power per hour.

But if these steam-consumptions are compared with the actual steam-consumption of 13.73 pounds per horse-power per hour, the ratios are

$$10.5 \div 13.73 = 0.766 \quad \text{and} \quad 10.7 \div 13.73 = 0.783,$$

which are very different from the ratios of the efficiencies. The discrepancy is due to the fact that more than a fourth of the steam used by the actual engine is condensed in the jackets and returned at full steam temperature to the boiler, while the non-conducting engine has no jacket, but is assumed to use all the steam in the cylinder.

From this discussion it appears that there is not a wide margin for improvement of a well-designed engine running under favorable conditions. Improved economy must be sought either by increasing the range of temperatures (raising the steam-pressure or improving the vacuum), or by choosing some other form of heat-motor, such as the gas-engine.

Attention should be called to the fact that the real

criterion of the economy of a heat-engine is the cost of producing power by that engine. The cost may be expressed in thermal units per horse-power per minute, in pounds of steam per horse-power per hour, in coal per horse-power per hour, or directly in money. The expression in thermal units is the most exact, and the best for comparing engines of the same class, such as steam-engines. If the same fuel can be used for different engines, such as steam- and gas-engines, then the cost in pounds of fuel per horse-power per hour may be most instructive. But in any case the money cost must be the final criterion. The reason why it is not more frequently stated in reports of tests is that it is in many cases somewhat difficult to determine, and because it is affected by market prices which are subject to change.

At the present time a pressure as high as 150 pounds above the atmosphere is used where good economy is expected. It appears from the table on page 233, showing the efficiency of Carnot's cycle for various pressures, that the gain in economy by increasing steam-pressure above 150 pounds is slow. The same thing is shown even more clearly by the following table:

EFFECT OF RAISING STEAM-PRESSURE.

Boiler-pressure by Gauge.	Efficiency, Carnot's Cycle.	Non-conducting Engine.		Probable Performance, Actual Engine.	
		Efficiency.	B. T. U. per H.P. per Minute.	B. T. U. per H.P. per Minute.	Steam per H.P. per Minute.
150	0.302	0.272	156	195	11.5
200	0.320	0.288	147	184	10.5
300	0.347	0.306	135	169	9.6

In the calculations for this table the steam is supposed to be dry as it enters the cylinder of the engine and the back-pressure is supposed to be 1.5 pounds absolute, while the expansion for the non-conducting engine is assumed to be complete. The heat-consumption of the non-conducting

engine is obtained by dividing 42.42 by the efficiency; thus for 150 pounds

$$42.42 \div 0.272 = 156.$$

The heat-consumption of the actual engine is assumed to be one-fourth greater than that of the non-conducting engine. The steam-consumption is calculated by the reversal of the method of calculating the thermal units per horse-power per minute from the steam per horse-power per hour, and for simplicity all of the steam is assumed to be supplied to the cylinder. But an engine which shall show such an economy for a given pressure as that set down in the table must be a triple or a quadruple engine and must be thoroughly steam-jacketed. The actual steam-consumption is certain to be a little larger than that given in the table, as steam condensed in a steam-jacket yields less heat than that passed through the cylinder.

It is very doubtful if the gain in fluid efficiency due to increasing steam-pressure above 150 or 200 pounds is not offset by the greater friction and the difficulty of maintaining the engine. Higher pressures than 200 pounds are used only where great power must be developed with small weight and space, as in torpedo-boats.

Condensers.—Two forms of condensers are used to condense the steam from a steam-engine, known as jet-condensers and surface-condensers. The former are commonly used for land engines; they consist of a receptacle having a volume equal to one-fourth or one-third of that of the cylinder or cylinders that exhaust into it, into which the steam passes from the exhaust-pipe and where it meets and is condensed by a spray of cold water.

If it be assumed that the steam in the exhaust-pipe is dry and saturated and that it is condensed from the pressure p and cooled to the temperature t^k, then the heat yielded per pound of steam is

$$\lambda - q_k,$$

where λ is the total heat of steam at the pressure p and q_k is the heat of the liquid at the temperature t_k. The heat acquired by each pound of condensing or injection water is

$$q_i - q_k,$$

where g_i is the heat of the liquid at the temperature t_i of the injection-water as it enters the condenser. Each pound of steam will require

$$G = \frac{\lambda - q_k}{q_k - q_i}$$

pounds of injection-water.

For example, steam at 4 pounds absolute has for the total heat 1128.6. If the injection-water enters with a temperature of 60° F. and leaves with a temperature of 120° F., then each pound of steam will require

$$\frac{\lambda - q_k}{q_k - q_i} = \frac{1128.6 - 88.1}{88.1 - 28.12} = 17.3$$

pounds of injection-water. This calculation is used only to aid in determining the size of the pipes and passages leading water to and from the condenser, and the dimensions of the air-pump. Anything like refinement is useless and impossible, as conditions are seldom well known and are liable to vary. From 20 to 30 times the weight of steam used by the engine is commonly taken for this purpose.

The jet-condensers cannot be used at sea when the boiler-pressure exceeds 40 pounds by the gauge; all modern steamers are consequently supplied with surface-condensers which consist of receptacles, which are commonly rectangular in shape, into which steam is exhausted, and where it is condensed on horizontal brass tubes through which cold sea-water is circulated. The condensed water is drained out through the air-pump and is returned to the boiler. Thus the feed-water is kept free from salt and other mineral matter that would be pumped into the boiler if a jet-condenser were

used, and if the feed-water were drawn from the mingled water and condensed steam from such a condenser. Much trouble is, however, experienced from oil used to lubricate the cylinder of the engine, as it is likely to be pumped into the boilers with the feed-water, even though attempts are made to strain or filter it from the water.

The water withdrawn from a surface-condenser is likely to have a different temperature from the cooling water when it leaves the condenser. If its temperature is t_1, then we have instead of equation (261)

$$G = \frac{\lambda - q_1}{q_k - q_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (262)$$

for the cooling water per pound of steam. The difference is really immaterial, as it makes little difference in the actual value of G for any case.

Cooling Surface.—Experiments on the quantity of cooling surface required by a surface-condenser are few and unsatisfactory, and a comparison of condensers of marine engines shows a wide diversity of practice. Seaton says that with an initial temperature of 60°, and with 120° for the feed-water, a condensation of 13 pounds of steam per square foot per hour is considered fair work. A new condenser in good condition may condense much more steam per square foot per hour than this, but allowance must be made for fouling and clogging, especially for vessels that make long voyages.

Seaton also gives the following table of square feet of cooling surface per indicated horse-power:

Absolute Terminal Pressure, Pounds per Square Inch.	Square Feet per I. H. P.
30	3
20	2.5
15	2.25
12½	2.
10	1.8
8	1.6
6	1.5

For ships stationed in the tropics, allow 20 per cent more; for ships which occasionally visit the tropics, allow 10 per cent more; for ships constantly in a cold climate, 10 per cent less may be allowed.

Designing Engines.—The only question that is properly discussed here is the probable form of the indicator-diagram, which gives immediately the method of finding the mean effective pressure and, consequently, the size of the cylinder of the engine.

The most reliable way of finding the expected mean effective pressure in the design of a new engine is to measure an indicator-diagram from an engine of the same or similar type and size, and working under the same conditions.

As it can hardly be expected that a diagram of exactly the required form will be at hand, a diagram like Fig. 55 may be drawn, using the proper cut-off, compression, and clearance. If an indicator-diagram taken from an engine under similar conditions is attainable, it may be used to determine exponential equations for the expansion and compression curves; usually the exponent will be different for the two curves, and must be determined separately. For ordinary work it is sufficient to use the hyperbola for both curves, and to assume the steam line *a* and the back-pressure line *c* to be parallel to the atmospheric line, while the lead of admission and exhaust may be neglected. It is also customary to assume a loss of pressure of two or more pounds between the boiler and the engine, and a back-pressure of a like amount above the pressure in the condenser or the pressure of the atmosphere, as the case may be.

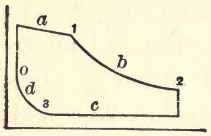

FIG. 55.

If the diagram is drawn to scale, the area and mean effective pressure may be found by measuring it; or, the form of the expansion and compression curves being assumed, the areas under the steam line, the expansion curve, the back-pressure line, and the compression curves may be calculated

separately, integrating between limits when necessary, and therefrom the resulting area of the diagram and the mean effective pressure may be determined. Ordinarily, the expansion and compression curves are assumed to be hyperbolæ.

Seaton* gives the following multipliers for finding the mean effective pressure from that calculated by the process described:

MULTIPLIERS FOR FINDING PROBABLE M. E. P., SIMPLE EXPANSIVE ENGINE.

(1) Special valve-gear, or with separate cut-off valve, engine jacketed	0.94
(2) Good ordinary valves, large ports, engine jacketed.	0.9–0.92
(3) Ordinary valves and gears as in general practice, unjacketed	0.80–0.85

To estimate the consumption of steam, we may calculate from the pressure and volume at release the weight of steam then present in the cylinder, and in a similar manner the weight of steam caught in the clearance space from the volume and pressure at compression, both under the assumption that the steam is dry and saturated. The difference is the steam exhausted per stroke under the assumption; but to get a fair estimate of the probable consumption, it is necessary to add a fraction of this amount, depending on the style and size of the engine and on the conditions under which it is to run. Sufficient data for this purpose seldom exist; so it is customary to add to the calculated amount one-fourth to one-third of itself, to get the probable consumption of non-condensing engines of medium size.

PROBLEM.—Required the dimensions of an engine to give 100 horse-power; revolutions, 120; gauge-pressure, 80 pounds; cut-off at $\frac{1}{3}$ stroke; release at end of stroke; compression at $\frac{1}{10}$ stroke, and clearance 5 per cent.

Assume the pressure during admission to be 78 pounds and during exhaust to be 1.3 pounds above the atmosphere, and assume hyperbolic expansion and compression.

* *Manual of Marine Engineering.*

The work during expansion is (Fig. 55)

$$p_1 v_1 \log_e \frac{v_2}{v_1} = 144(78+14)(0.333+0.05) \text{ pist. disp. } \log_e \frac{1+0.05}{0.333+0.05}$$

so that the mean pressure per square inch during expansion is

$$0.383 \times 92.7 \log_e \frac{1.05}{0.383}$$

and the mean effective pressure is

$$0.333 \times 92.7 + 0.383 \times 92.7 \log_e \frac{1.05}{0.383} - 16 \times 0.9$$

$$- 0.15 \times 16 \log_e \frac{0.15}{0.05} = 49.5 \text{ pounds.}$$

If, further, the stroke of the engine is twice the diameter, then

$$100 = \frac{\frac{\pi d^2}{4} \times \frac{2d}{12} \times 120 \times 2 \times 49.5}{33000};$$

$$\therefore \quad d = 12.85, \qquad s = 25.70.$$

The volume of the cylinder will be 1.93 cubic feet, and the terminal pressure will be 33.8 pounds absolute. At 33.8 pounds the density of steam is 0.08234, and at 16 pounds it is 0.04067. The consumption of steam per horse-power per hour, on the assumption of dry steam at release and compression, will be

$$\frac{(0.08234 \times 1.05 - 0.04067 \times 0.15) 1.93 \times 2 \times 120 \times 60}{100} = 22.3 \text{ pounds.}$$

If one-third of this quantity be added, then the estimated consumption of steam will be 30 pounds per horse-power per hour.

The calculated dimensions are stated in inches and hundredths, but in practice the engine would be made $12\frac{7}{8}$ inches in diameter by $25\frac{3}{4}$ inches stroke; or possibly the dimensions 13 by 25 would be chosen, since they give nearly the same volume.

CHAPTER XII.

COMPOUND ENGINES.

A COMPOUND engine has commonly two cylinders, one of which is three or four times as large as the other. Steam from a boiler is admited to the small cylinder, and after doing work in that cylinder it is transferred to the large cylinder, from which it is exhausted, after doing work again, into a condenser or against the pressure of the atmosphere. If we assume that the steam from the small cylinder is exhausted into a large receiver, the back-pressure in that cylinder and the pressure during the admission to the large cylinder will be uniform. If, further, we assume that there is no clearance in either cylinder, that the back-pressure in the small cylinder and the forward pressure in the large cylinder are the same, and that the expansion in the small cylinder reduces the pressure down to the back-pressure in that cylinder, the diagram for the small cylinder will be *ABCD*, Fig. 52, and

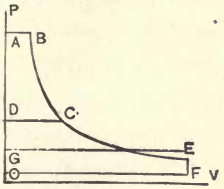

FIG. 56.

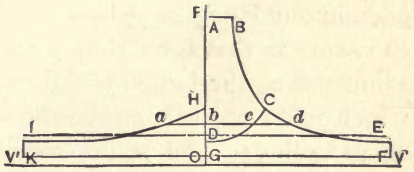

FIG. 57.

for the large cylinder *DCFG*. The volume in the large cylinder at cut-off is equal to the total volume of the small cylinder, since the large cylinder takes from the receiver the same weight of steam that is exhausted by the small cylinder, and, in this case, at the same pressure.

The case just discussed is one extreme. The other

extreme occurs when the small cylinder exhausts directly into the large cylinder without an intermediate receiver. In such engines the pistons must begin and end their strokes together. They may both act on the beam of a beam engine, or they may act on one crank or on two cranks opposite each other.

For such an engine, *ABCD*, Fig. 57, is the diagram for the small cylinder. The steam line and expansion line *AB* and *BC* are like those of a simple engine. When the piston of the small cylinder begins the return stroke, communication is opened with the large cylinder, and the steam passes from one to the other, and expands to the amount of the difference of the volume, it being assumed that the communication remains open to the end of the stroke. The back-pressure line *CD* for the small cylinder, and the admission line *HI* for the large cylinder, gradually fall on account of this expansion. The diagram for the large cylinder is *HIKG*, which is turned toward the left for convenience.

To combine the two diagrams, draw the line *abcd*, parallel to *V'OV*, and from *b* lay off *bd* equal to *ca*; then *d* is one point of the expansion curve of the combined diagram. The point *C* corresponds with *H*, and *E*, corresponding with *I*, is as far to the right as *I* is to the left.

For a non-conducting cylinder, the combined diagram for a compound engine, whether with or without a receiver, is the same as that for a simple engine which has a cylinder the same size as the large cylinder of the compound engine, and which takes at each stroke the same volume of steam as the small cylinder, and at the same pressure. The only advantage gained by the addition of the small cylinder to such an engine is a more even distribution of work during the stroke, and a smaller initial stress on the crank-pin.

Compound engines may be divided into two classes—those with a receiver and those without a receiver; the latter are called "Woolf engines" on the continent of Europe. Engines without a receiver must have the pistons begin and end their strokes at the same time; they may act on the same

crank or on cranks 180° apart. The pistons of a receiver compound engine may make strokes in any order. A form of receiver compound engine with two cylinders, commonly used in marine work, has the cranks at 90° to give handiness and certainty of action. Large marine engines have been made with one small cylinder and two large or low-pressure cylinders, both of which draw steam from the receiver and exhaust to the condenser. Such engines usually have the cranks at 120°, though other arrangements have been made.

Nearly all compound engines have a receiver, or a space between the cylinders corresponding to one, and in no case is the receiver of sufficient size to entirely prevent fluctuations of pressure. In the later marine work the receiver has been made small, and frequently the steam-chests and connecting pipes have been allowed to fulfil that function. This contraction of size involves greater fluctuations of pressure, but for other reasons it appears to be favorable to economy.

Under proper conditions there is a gain from using a compound engine instead of a simple engine, which may amount to ten per cent or more. This gain is to be attributed to the division of the change of temperature from that of the steam at admission to that of exhaust into two stages, so that there is less fluctuation of temperature in a cylinder and consequently less interchange of heat between the steam and the walls of the cylinder.

Compound Engine without Receiver.—The indicator-diagrams from a compound engine without a receiver are represented by Fig. 58. The steam line and expansion line of the small cylinder, *AB* and *BC*, do not differ from those of a simple engine. At *C* the exhaust opens, and the steam suddenly expands into the space between the cylinders and the clearance of the large cylinder, and the pressure falls from *C* to *D*. During the return stroke the volume in the large cylinder increases more rapidly than that of the small cylinder

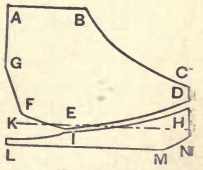

FIG. 58.

decreases, so that the back-pressure line *DE* gradually falls, as does also the admission line *HI* of the large cylinder, the difference between these two lines being due to the resistance to the flow of steam from one to the other. At *E* the communication between the two cylinders is closed by the cut-off of the large cylinder; the steam is then compressed in the small cylinder and the space between the two cylinders to *F*, at which the exhaust of the small cylinder closes; and the remainder of the diagram *FGA* is like that of a simple engine. From *I*, the point of cut-off of the large cylinder, the remainder of the diagram *IKLMNH* is like the same part of the diagram of a simple engine.

The difference between the lines *ED* and *HI* and the "drop" *CD* at the end of the stroke of the small piston indicate waste or losses of efficiency. The compression *EFG* and the accompanying independent expansion *IK* in the large cylinder show a loss of power when compared with a diagram like Fig. 56 for an engine which has no clearance or intermediate space; but compression is required to fill waste spaces with steam. The compression *EF* is required to reduce the drop *CD*, and the compression *FG* fills the clearance in anticipation of the next supply from the boiler. Neither compression is complete in Fig. 58.

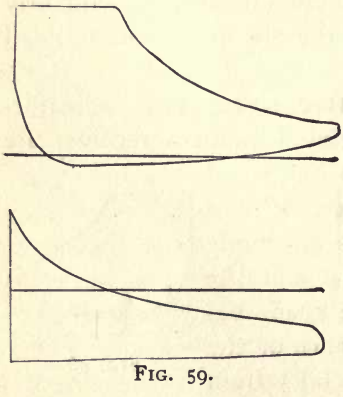

FIG. 59.

Diagrams from a pumping engine at Lawrence, Massachusetts, are shown by Fig. 59. The rounding of corners due to the indicator make it difficult to determine the location of points like *D*, *E*, *F*, and *I* on Fig. 58. The low-pressure diagram is taken with a weak spring, and so has an exaggerated height.

Compound Engine with Receiver.—It has already been pointed out that some receiver space is required if the cranks

of a compound engine are to be placed at right angles. When the receiver space is small, as on most marine engines, the fluctuations of pressure in the receiver are very notable. This is exhibited by the diagrams in Fig. 60, which were taken from a yacht engine. An intelligent conception of the causes and meaning of such fluctuations can be best obtained by constructing ideal diagrams for special cases, as explained on page 269.

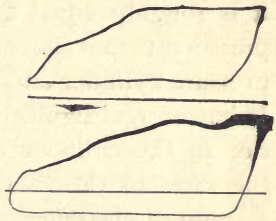

FIG. 60.

Triple and Quadruple Compound Engines.—The same influences which introduced the compound engines, when the common steam-pressure changed from forty to eighty pounds to the square inch, have brought in the successive expansion through three cylinders (the high-pressure, intermediate, and low-pressure cylinders) now that 125 to 170 pounds pressure are employed. Just as three or more cylinders are combined in various ways for compound engines, so four, five, or six cylinders have been arranged in various manners for triple-expansion engines; for example, a compound engine with two cylinders may be conveniently changed into a triple-expansion engine by the addition of a small high-pressure cylinder over each of the existing cylinders.

Quadruple engines with four successive expansions have been employed with high-pressure steam, but with the advisable pressures for present use the extra complication and friction make it a doubtful expedient.

Total Expansion.—In Figs. 56 and 57, representing the diagrams for compound engines without clearance and without drop between the cylinders, the total expansion is the ratio of the volumes at E and at A; that is, of the low-pressure piston displacement to the displacement developed by the high-pressure piston at cut-off. The same ratio is called the total or equivalent expansion for any compound engine, though it may have both clearance and drop. Such a con-

ventional total expansion is commonly given for compound and multiple-expansion engines, and is a convenience because it is roughly equal to the ratio of the initial and terminal pressures; that is, of the pressure at cut-off in the high-pressure cylinder and at release in the low-pressure cylinder. It has no real significance, and is not equivalent of the expansion in the cylinder of a simple engine, by which we mean the ratio of the volume at release to that at cut-off, taking account of clearance. And further, since the clearance of the high- and the low-pressure cylinders are different there can be no real equivalent expansion.

If the ratio of the cylinders is R and the cut-off of the high-pressure cylinder is at $\frac{1}{e}$ of the stroke, then the total expansion is represented by

$$E = eR \quad . \quad . \quad . \quad . \quad . \quad . \quad (263)$$

and

$$\frac{1}{e} = R \div E. \quad . \quad . \quad . \quad . \quad . \quad . \quad (264)$$

This last equation is useful in determining approximately the cut-off of the high-pressure cylinder.

For example, if the initial pressure is 100 pounds absolute and the terminal pressure is to be 10 pounds absolute, then the total expansions will be about 10. If the ratio of the cylinders is $3\frac{1}{2}$, then the high-pressure cut-off will be about

$$\frac{1}{e} = 3\tfrac{1}{2} \div 10 = 0.35$$

of the stroke.

Low-pressure Cut-off.—The cut-off of the low-pressure cylinders in Figs. 56 and 57 is controlled by the ratio of the cylinders, since the volumes in the low-pressure cylinder at cut-off is in each case made equal to the high-pressure piston displacement; this is done to avoid a drop. If the cut-off

were lengthened there would be a loss of pressure or drop at the end of the stroke of the high-pressure piston, as is shown by Fig. 61, for an engine with a large receiver and no clearance. Such a drop will have some effect on the character of the expansion line $C''E$ of the low-pressure cylinder, both for a non-conducting and for the actual engine with or without a clearance. Its principal effect will, however, be on the distribution of work between the cylinders; for it is evident that if the cut-off of the low-pressure cylinder is shortened the pressure at C'' will be increased because the same weight of steam is taken in a smaller volume. The back-pressure DC' of the high-pressure cylinder will be raised and the work will be diminished; while the forward pressure DC'' of the low-pressure cylinder will be raised, increasing the work in that cylinder.

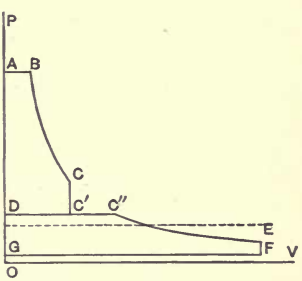

FIG. 61.

Ratio of Cylinders.—In designing compound engines, more especially for marine work, it is deemed important for the smooth action of the engine that the total work shall be evenly distributed upon the several cranks of the engines, and that the maximum pressure on each of the cranks shall be the same, and shall not be excessive. In case two or more pistons act on one crank, the total work and the resultant pressure on those pistons are to be considered; but more commonly each piston acts on a separate crank, and then the work and pressure on the several pistons are to be considered.

In practice both the ratio of the cylinders and the total expansions are assumed, and then the distribution of work and the maximum loads on the crank-pins are calculated, allowing for clearance and compression. Designers of engines usually have a sufficient number of good examples at hand to enable them to assume these data. In default of such data it may be necessary to assume proportions, to make prelimi-

nary calculations, and to revise the proportions till satisfactory results are obtained. For compound engines using 80 pounds steam-pressure the ratio is 1 : 3 or 1 : 4. For triple-expansion engines the cylinders may be made to increase in the ratio 1 : 2 or 1 : $2\frac{1}{2}$.

Approximate Indicator-diagrams.—The indicator-diagrams from some compound and multiple-expansion engines are irregular and apparently erratic, depending on the sequence of the cranks, the action of the valves, and the relative volumes of the cylinders and the receiver spaces. A good idea of the effects and relations of these several influences can be obtained by making approximate calculations of pressures at the proper parts of the diagrams by a method which will now be illustrated.

For such a calculation it will be assumed that all expansion lines are rectangular hyperbolæ, and in general that any change of volume will cause the pressure to change inversely as the volume. Further, it will be assumed that when communication is opened between two volumes where the pressures are different, the resultant pressure may be calculated by adding together the products of each volume by its pressure, and dividing by the sum of the volumes. Thus if the pressure in a cylinder having the volume v_c is p_c, and if the pressure is p_r in a receiver where the volume is v_v, then after the valve opens communication from the cylinder to the receiver the pressure will be

$$p = \frac{p_c v_c + p_r v_r}{v_c + v_r}.$$

The same method will be used when three volumes are put into communication.

It will be assumed that there are no losses of pressure due to throttling or wire-drawing; thus the steam line for the high-pressure cylinder will be drawn at the full boiler-pressure, and the back-pressure line in the low-pressure cylinder will be drawn to correspond with the vacuum in the condenser.

Again, cylinders and receiver spaces in communication will be assumed to have the same pressure.

For sake of simplicity the motions of pistons will be assumed to be harmonic.

Diagrams constructed in this way will never be realized in any engine; the degree of discrepancy will depend on the type of engine and the speed of rotation. For slow-speed pumping-engines the discrepancy is small and all irregularities are easily accounted for. On the other hand the discrepancies are large for high-speed marine-engines, and for compound locomotives they almost prevent the recognition of the events of the stroke.

Direct-expansion Engine.—If the two pistons of a compound engine move together or in opposite directions the diagrams are like those shown by Fig. 62. Steam is admitted to the high-pressure cylinder from *a* to *b*; cut-off occurs at *b*, and *bc* represents expansion to the end of the stroke; *bc* being a rectangular hyperbola referred to the axis *OV* and *OP*, from which *a*, *b*, and *c* are laid off to represent absolute pressures and volumes, including clearance.

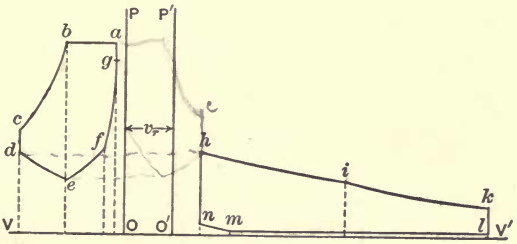

FIG. 62.

At the end of the stroke release from the high-pressure cylinder and admission to the low-pressure cylinder are assumed to take place, so that communication is opened from the high-pressure cylinder through the receiver space into the low-pressure cylinder. As a consequence the pressure falls from *c* to *d*, and rises from *n* to *h* in the low-pressure cylinder. The line *O'P'* is drawn at a distance from *OP*,

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which corresponds to the volume of the receiver space, and the low-pressure diagram is referred to $O'P'$ and $O'V'$ as axes.

The communication between the cylinders is maintained until cut-off occurs at i for the low-pressure cylinder. The lines de and hi represent the transfer of steam from the high-pressure to the low-pressure cylinder, together with the expansion due to the increased size of the large cylinder. After the cut-off at i, the large cylinder is shut off from the receiver and the steam in it expands to the end of the stroke. The back-pressure and compression lines for that cylinder are not affected by compounding, and are like those of a simple engine. Meanwhile the small piston compresses steam into the receiver, as represented by ef, till compression occurs, after which compression into the clearance space is represented by fg. The expansion and compression lines ik and mn are drawn as hyperbolæ referred to the axis $O'P'$ and $O'V'$. The compression line ef is drawn as an hyperbola referred to $O'V$ and $O'P'$, while fg is referred to OV and OP.

In Fig. 62 the two diagrams are drawn with the same scale for volume and pressure, and are placed so as to show to the eye the relations of the diagrams to each other. Diagrams taken from such an engine resemble Figs. 59, which have the same length, and different vertical scales depending on the springs used in the indicators.

Some engines have only one valve to give release and compression for the high-pressure cylinder and admission and cut-off for the low-pressure cylinder. In such case there is no receiver space, and the points e and f coincide.

When the receiver is closed by the compression of the high-pressure cylinder it is filled with steam with the pressure represented by f. It is assumed that the pressure in the receiver remains unchanged till the receiver is opened at the end of the stroke. It is evident that the drop cd may be reduced by shortening the cut-off of the low-pressure cylinder so as to give more compression from e to f and consequently a higher pressure at f when the receiver is closed.

Representing the pressure and volume at the several points by p and v with appropriate subscript letters, and representing the volume of the receiver by v_r, we have the following equations:

$$p_a = p_b = \text{initial pressure};$$
$$p_l = p_m = \text{back-pressure};$$
$$p_c = p_b v_b \div v_c;$$
$$p_n = p_m v_m \div v_n;$$
$$p_d = p_h = (p_c v_c + p_n v_n + p_f v_r) \div (v_c + v_n + v_r);$$
$$p_e = p_i = p_d(v_c + v_n + v_r) \div (v_e + v_i + v_r);$$
$$p_f = p_e(v_e + v_r) \div (v_f + v_r);$$
$$p_g = p_f v_f \div v_g;$$
$$p_k = p_i v_i \div v_k.$$

The pressures p_c and p_n can be calculated directly. Then the equations for p_d, p_e, and p form a set of three simultaneous equations with three unknown quantities, which can be easily solved. Finally, p_g and p_k may be calculated directly.

For example, let us find the approximate diagram for a direct-expansion engine which has the low-pressure piston displacement equal to three times the high-pressure piston displacement. Assume that the receiver space is half the smaller piston displacement, and that the clearance for each cylinder is one-tenth of the corresponding piston displacement. Let the cut-off for each cylinder be at half-stroke, and the compression at nine-tenths of the stroke; let the admission and release be at the end of the stroke. Let the initial pressure be 65.3 pounds by the gauge or 80 pounds absolute, and let the back-pressure be two pounds absolute.

If the volume of the high-pressure piston displacement be taken as unity, then the several required volumes are:

$$v_b = 0.5 + 0.1 = 0.6 \qquad v_h = v_n = 3 \times 0.1 = 0.3$$
$$v_c = v_d = 1.0 + 0.1 = 1.1 \qquad v_i = 3(0.5 + 0.1) = 1.8$$
$$v_e = 0.5 + 0.1 = 0.6 \qquad v_k = v_l = 3(1.0 + 0.1) = 3.3$$
$$v_f = 0.1 + 0.1 = 0.2 \qquad v_m = 3(0.1 + 0.1) = 0.6$$
$$v_g = 0.1 \qquad v_r = 0.5$$

The pressures may be calculated as follows:

$$p_a = p_b = 80; \qquad p_l = p_m = 2;$$

$$p_c = p_b v_b \div v_c = 80 \times 0.6 \div 1.1 = 43.6;$$

$$p_m = p_m v_m \div v_n = 2 \times 0.6 \div 0.3 = 4;$$

$$p_e = p_d(v_c + v_n + v_r) \div (v_e + v_i + v_r) = p_d(1.1 + 0.3 + 0.5) \div (0.6 + 1.8 + 0.5) = 0.655 p_d;$$

$$p_f = p_e(v_e + v_r) \div (v_f + v_r) = p_e(0.6 + 0.5) \div (0.2 + 0.5) = 1.57 p_e = 1.57 \times 0.655 p_d = 1.03 p_d;$$

$$p_d = (p_c v_c + p_n v_n + p_f v_r) \div (v_c + v_n + v_r)$$
$$= (80 \times 0.6 + 4 \times 0.3 + 0.5 p_f) \div 0.6 + 0.3 + 0.5$$
$$= 25.89 + 0.26 p_f;$$

$$p_d = 25.89 + 0.26 \times 1.03 p_d; \qquad p_d = 35.36;$$

$$p_e = p_i = 0.655 p_d = 0.655 \times 35.36 = 23.2;$$

$$p_f = 1.03 p_d = 1.03 \times 35.36 = 36.5;$$

$$p_g = p_f v_f \div v_g = 36.5 \times 0.2 \div 0.1 = 73;$$

$$p_k = p_i v_i \div v_k = 23.2 \times 1.8 \div 3.3 = 12.6.$$

As the pressures and volumes are now known the diagrams of Fig. 62 may be drawn to scale. Or, if preferred, diagrams like Fig. 59 may be drawn, making them of the same length and using convenient vertical scales of pressure. If the engine runs slowly and has abundant valves and passages the diagrams thus obtained will be very nearly like those taken from the engine by indicators. If losses of pressure in valves and passages are allowed for, a closer approximation can be made.

The mean effective pressures of the diagrams may be readily obtained by the aid of planimeters, and may be used for estimating the power of the engine. For this purpose we should either use the modified diagrams allowing for losses of pressure, or we should affect the mean effective pressures by a multiplier obtained by comparison of the approximate with the actual diagrams from engines of the same type. For a slow-speed pumping-engine the multiplier may be as large as 0.9 or even more; for high-speed engines it may be as small as 0.6.

The mean effective pressure of the diagrams may be calculated from the volumes and pressures if desired, assuming, of course, that the several expansion and compression curves are hyperbolæ. The process can be best explained by applying it to the example already considered. Begin by finding the mean pressure during the transfer of steam from the high-pressure cylinder to the low-pressure cylinder as represented by *de* and *hi*. The net effective work during the transfer is

$$\int pdv = p_1 v_1 \log_e \frac{v_2}{v_1} = 144 p_d (v_d + v_h + v_r) \log_e \frac{v_e + v_i + v_r}{v_d + v_h + v_r}$$

$$= 144 \times 35.4 (1.1 + 0.3 + 0.5) \log_e \frac{0.6 + 1.8 + 0.5}{1.1 + 0.3 + 0.5}$$

$$= 4120 \text{ foot-pounds}$$

for each cubic foot of displacement of the high-pressure piston. This corresponds with our previous assumption of unity for the displacement of that piston. The increase of volume is

$$v_e + v_i + v_r - (v_d + v_h + v_r) = 0.6 + 1.8 + 0.5 - (1.1 + 0.3 + 0.5) = 1 ;$$

so that the mean pressure during the transfer is

$$4120 \div 1 \times 144 = 28.6 = p_x$$

pounds per square inch, which acts on both the high- and the low-pressure pistons.

The effective work for the small cylinder is obtained by adding the works under *ab* and *bc* and subtracting the works under *de*, *ef*, and *fg*. So that

$$W_H = 144 \left\{ p_a (v - v_a) + p_b v_b \log_e \frac{v_c}{v_b} - p_x (v_d - v_c) \right.$$

$$\left. - p_d (v_e + v_r) \log_e \frac{v_e + v_r}{v_f + v_r} - p\ v_f \log_e \frac{v_f}{v_g} \right\}$$

$$= 144 \left\{ 80 (0.6 - 0.1) + 80 \times 0.6 \log_e \frac{1.1}{0.6} - 28.6 (1.1 - 0.6) \right.$$

$$\left. - 23.2 (0.6 + 0.5) \log_e \frac{0.6 + 0.5}{0.2 + 0.5} - 36.5 \times 0.2 \log_e \frac{0.2}{0.1} \right\}$$

$$= 144 \times 33.26 = 4789 \text{ foot-pounds.}$$

This is the work for each cubic foot of the high-pressure piston displacement, and the mean effective pressure on the small piston is evidently 33.26 pounds per square inch.

In a like manner the work of the large piston is

$$W_L = 144\left\{ p_x(v_i - v_k) + p_i v_i \log_e \frac{v_k}{v_i} - p_l(v_l - v_m) - p_m v_m \log_e \frac{v_m}{v_n} \right\}$$

$$= 144\left\{ 28.6(1.8 - 0.3) + 23.2 \times 1.8 \log_e \frac{3.3}{1.8} \right.$$

$$\left. - 2(3.3 - 0.6) - 2 \times 0.6 \log_e \frac{0.6}{0.3} \right\} = 144 \times 61.92 = 8916 \text{ foot-pounds.}$$

Since the ratio of the piston displacements is 3, the work for each cubic foot of the low-pressure piston displacement is one-third of the work just calculated; and the mean effective pressure on the large piston is

$$61.92 \div 3 = 20.64$$

pounds per square inch.

The proportions given in the example lead to a very uneven distribution of work; that of the large cylinder being nearly twice as much as is developed in the small cylinder. The distribution can be improved by shortening the cut-off of the small cylinder, or lengthening that of the large cylinder, or by increasing the size of the large cylinder.

As has already been pointed out, the works just calculated and the corresponding mean effective pressures are in excess of those that will be actually developed, and must be affected by multipliers which may vary from 0.6 to 0.9, depending on the type and speed of the engine.

Cross-compound Engine.—A two-cylinder compound engine with pistons connected to cranks at right angles with each other is frequently called a cross-compound engine. Unless a large receiver is placed between the cylinders the pressure in the space between the cylinders will vary widely.

Two cases arise in the discussion of this engine according as the cut-off of the large cylinder is earlier or later than half-

stroke; in the latter case there is a species of double admission to the low-pressure cylinder, as is shown in Fig. 63. For sake of simplicity the release, and also the admission for each cylinder, is assumed to be at the end of the stroke. If the release is early the double admission occurs before half-stroke.

The admission and expansion of steam for the high-pressure cylinder are represented by *ab* and *bc*. At *c* release occurs, putting the small cylinder in communication with the intermediate receiver, which is then open to the large cylinder.

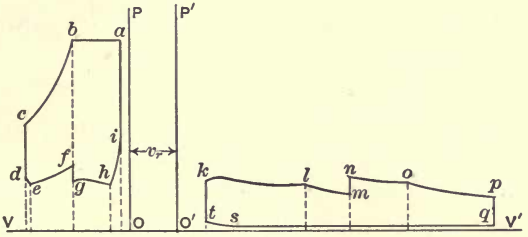

FIG. 63.

There is a drop at *cd* and a corresponding rise of pressure *mn* on the large piston, which is then at half-stroke; it will be assumed that the pressures at *d* and at *n* are identical. From *d* to *e* the steam is transferred from the small to the large cylinder, and the pressure falls because the volume increases; *no* is the corresponding line on the low-pressure diagram. The cut-off at *o* for the large cylinder interrupts this transfer, and steam is then compressed by the small piston into the intermediate receiver with a rise of pressure as represented by *ef*. The admission to the large cylinder, *tk*, occurs when the small piston is at the middle of its stroke, and causes a drop, *fg*, in the small cylinder. From *g* to *h* steam is transferred through the receiver from the small to the large cylinder. The pressure rises at first because the small piston moves rapidly while the large one moves slowly until its crank gets away from the dead-point; afterwards the pressure falls. The line *kl* represents this action on the low-pressure diagram. At *h* compression occurs for the small cylinder, and *hi* shows

the rise of pressure due to compression. For the large cylinder the line *lm* represents the supply of steam from the receiver, with falling pressure which lasts till the double admission at *mn* occurs.

The expansion, release, exhaust, and compression in the large cylinder are not affected by compounding.

Strictly, the two parts of the diagram for the low-pressure cylinder, *mnopq* and *stklm* belong to opposite ends of the cylinder, one belonging to the head end and one to the crank end. With harmonic motion the diagrams from the two ends are identical, and no confusion need arise from our neglect to determine which end of the large cylinder we are dealing with at any time. Such an analysis for the two ends of the cylinder, taking account of the irregularity due to the action of the connecting-rod, would lead to a complexity that would be unprofitable.

A ready way of finding corresponding positions of two pistons connected to cranks at right angles with each other is by aid of the diagram of Fig. 60. Let O be the centre of the crank-shaft and pR_yR_xq be the path of the crank-pin. When one piston has the displacement py and its crank is at OR_y, the other crank may be 90° ahead at OR_x and the corresponding piston displacement will be px.

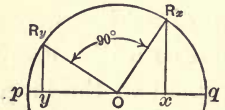

FIG. 64.

The same construction may be used if the crank is 90° behind or if the angle R_yOR_x is other than a right angle. The actual piston position and crank angles when affected by the irregularity due to the connecting-rod will differ from those found by this method, but the position found by such a diagram will represent the average positions very nearly.

The several pressures may be found as follows:

$p_b = p_a =$ initial pressure;
$p_s = p_g =$ back-pressure;
$p_c = p_b v_b \div v_c$;

$$p_t = p_s v_s \div v_t;$$
$$p_d = p_n = \{p_c v_c + p_m(v_m + v_r)\} \div (v_c + v_m + v_r);$$
$$p_e = p_o = p_d(v_c + v_m + v_r) \div (v_e + v_o + v_r);$$
$$p_f = p_e(v_e + v_r) \div (v_f + v_r);$$
$$p_g = p_k = \{p_f(v_f + v_r) + p_t v_t\} \div (v_f + v_t + v_r);$$
$$p_h = p_l = p_g(v_f + v_t + v_r) \div v_h + v_l + v_r);$$
$$p_m = p_l(v_l + v_r) \div (v_m + v_r);$$
$$p_i = p_k v_k \div v_i;$$
$$p_p = p_o v_o \div v_p.$$

The pressures p_c and p_n can be found directly from the initial pressure and the back-pressure, and finally the last two equations give direct calculations for p_i and p_p so soon as p_k and p_o are found. There remain six equations containing six unknown quantities, which can be readily solved after numerical values are assigned to the known pressures and to all the volumes.

The expansion and compression lines, *bc* and *hi*, for the high-pressure diagrams are hyperbolæ referred to the axis *OV* and *OP*; and in like manner the expansion and compression lines *op* and *st*, for the low-pressure diagram are hyperbolæ referred to *O'V'* and *O'P'*. The curve *ef* is an hyperbola referred to *O'V* and *O'P'*, and the curve *lm* is an hyperbola referred to *OV'* and *OP*. The transfer lines *de* and *no*, *gh* and *kl*, are not hyperbolæ. They may be plotted point by point by finding corresponding intermediate piston positions, p_x and p_y, by aid of Fig. 64, and then calculating the pressure for these positions by the equation

$$p_x = p_y = p_d(v_d + v_m + v_r) \div (v_x + v_y + v_r).$$

The work and mean effective pressure may be calculated in a manner similar to that given for the direct-expansion engine; but the calculation is tedious, and involves two transfers, *de* and *no*, and *gh* and *kl*. The first involves only an expansion, and presents no special difficulty; the second consists of a compression and an expansion, which can be dealt

with most conveniently by a graphical construction. All things considered, it is better to plot the diagrams to scale and determine the areas and mean effective pressures by aid of a planimeter.

If the cut-off of the low-pressure is earlier than half-stroke so as to precede the release of the high-pressure cylinder the transfer represented by *de* and *no*, Fig. 63, does not occur. Instead there is a compression from *d* to *f* and an expansion from *l* to *m*. The number of unknown quantities and the number of equations are reduced. On the other hand a release before the end of the stroke of the high-pressure piston requires an additional unknown quantity and one more equation.

Triple Engines.—The diagrams from triple and other multiple-expansion engines are likely to show much irregularity, the form depending on the number and arrangement of the cylinders and the sequence of the cranks. A common arrangement for a triple engine is to have three pistons acting on cranks set equidistant around the circle, as shown by Fig. 65. Two cases arise depending on the sequence of the cranks, which may be in the order, from one end of the engine, of high-pressure, low-pressure, and intermediate, as shown by Fig. 65; or in the order of high-pressure, intermediate, and low-pressure.

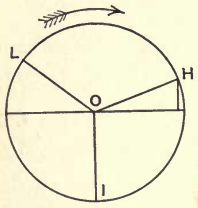

FIG. 65.

With the cranks in the order high-pressure, low-pressure, and intermediate, as shown by Fig. 65, the diagrams are like those given by Fig. 66. The admission and expansion for the high-pressure cylinder are represented by *abc*. When the high-pressure piston is at release, its crank is at *H*, Fig. 65, and the intermediate crank is at *I*, so that the intermediate piston is near half-stroke. If the cut-off for that cylinder is later than half-stroke, it is in communication with the first receiver when its crank is at *I*, and steam may pass through the first receiver from the high-pressure to the intermediate

cylinder, and there is a drop *cd*, and a corresponding rise of pressure *no* in the intermediate cylinder. The transfer continues till cut-off for the intermediate cylinder occurs at *p*, corresponding to the piston position *e* for the high-pressure

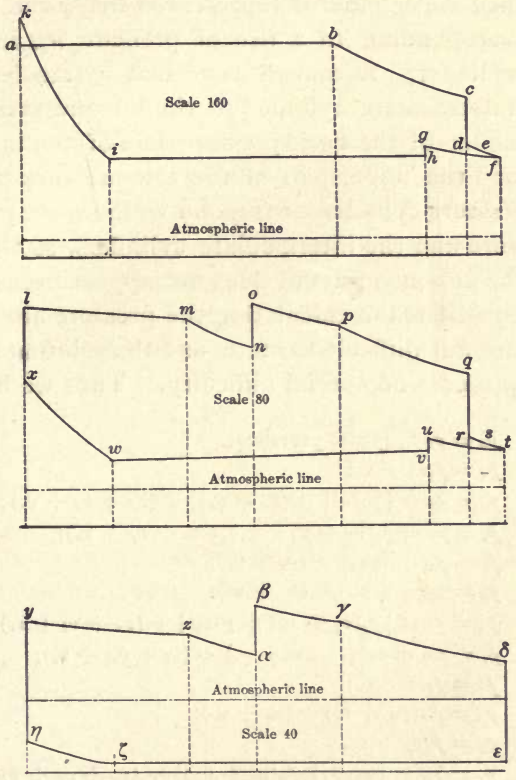

FIG. 66.

cylinder. From the position *e* the high-pressure piston moves to the end of the stroke at *f*, causing an expansion, and then starts to return, causing the compression *fg*. When the high-pressure piston is at *g* the intermediate cylinder takes steam at the other end, causing the drop *gh* and the rise of pressure *xl*. Then follows a transfer of steam from the high-pressure to the intermediate cylinder represented by *hi* and *lm*. At *i* the high-pressure compression *ik*

begins, and is carried so far as to produce a loop at k. After compression for the high-pressure cylinder shuts it from the first receiver, the steam in that receiver and in the intermediate cylinder expand as shown by mn. The expansion in the intermediate cylinder is represented by pq and the release by qr, corresponding to a rise of pressure $\alpha\beta$ in the low-pressure cylinder. rs and $\beta\gamma$ represent a transfer of steam from the intermediate cylinder to the low-pressure cylinder. The remainder of the back-pressure line of the intermediate cylinder and the upper part of the low-pressure diagram for the low-pressure cylinder correspond to the same parts of the high-pressure and the intermediate cylinders, so that a statement of the actions in detail does not appear necessary.

The equations for calculating the pressure are numerous, but they are not difficult to state, and the solution for a given example presents no special difficulty. Thus we have

$$p_a = p_b = \text{initial pressure};$$
$$p_c = p_b v_b \div v_c;$$
$$p_d = p_o = \{ p_c v_c + p_n(v_o + v_r) \} \div (v_d + v_o + v_r);$$
$$p_e = p_p = p_d(v_d + v_o + v_r) \div v_e + v_p + v_r);$$
$$p_f = p_e(v_e + v_r) \div (v_f + v_r);$$
$$p_g = p_f(v_f + v_r) \div (v_g + v_r);$$
$$p_h = p_l = \{ p_g(v_g + v_r) + p_x v_x \} \div (v_h + v_l + v_r);$$
$$p_i = p_m = p_h(v_h + v_l + v_r) \div (v_i + v_m + v_r);$$
$$p_k = p_i v_i \div v_k;$$
$$p_n = p_m(v_m + v_r) \div (v_n + v_r);$$
$$p_g = p_p v_p \div v_q;$$
$$p_r = p_\beta = \{ p_q v_q + p_a(v_a + v_R) \} \div (v_r + v_a + v_R);$$
$$p_s = p_\gamma = p_r(v_r + v_a + v_R) \div (v_s + v_\gamma + v_R);$$
$$p_t = p_s(v_s + v_R) \div (v_t + v_R);$$
$$p_u = p_t(v_t + v_R) \div (v_u + v_R);$$
$$p_v = \{ p_u(v_u + v_R) + p_\eta v_\eta \} \div (v_v + v_\eta + v_R);$$
$$p_w = p_v(v_v + v_\eta + v_R) \div (v_w + v_z + v_R);$$
$$p_x = p_w v_w \div v_x;$$
$$p_a = (v_z + v_R) \div (v_a + v_R);$$
$$p_\delta = p_a v_a \div v_\delta;$$
$$p_\epsilon = p_\zeta = \text{back-pressure};$$
$$p_\eta = p_\zeta v_\zeta \div v_\eta.$$

The pressures at c and at η can be calculated immediately from the initial pressure and from the back-pressure. Then it will be recognized that there are four individual equations for finding p_f, p_k, p_t, and p_δ. The fourteen remaining equations, solved as simultaneous equations, give the corresponding fourteen required pressures, some of which are used in calculating the four pressures which are determined by the four individual equations.

If the cut-off for the intermediate cylinder occurs before the release of the high-pressure cylinder, then the transfer represented by *de* and *op* does not occur. In like manner, if the cut-off for the low-pressure cylinder occurs before the release for the intermediate cylinder, the transfer represented by *rs* and $\beta\gamma$ does not occur. The omission of a transfer of course simplifies the work of deducing and of solving equations.

In much the same way, equations may be deduced for calculating pressures when the cranks have the sequence high-pressure, intermediate, and low-pressure. The diagrams take forms which are distinctly unlike those for the other sequence of cranks. In general, the case solved, i.e., high-pressure, low-pressure, and intermediate, gives a smoother action for the engine.

For example, the engines of the U. S. S. *Machias* have the following dimensions and proportions:

	High-pressure.	Intermediate.	Low-pressure.
Diameter of piston, inches	15¾	22½	35
Piston displacement, cubic feet	2.71	5.53	13.39
Clearance, per cent	13	14	7
Cut-off, per cent stroke	66	66	66
Release, " "	93	93	93
Compression, per cent stroke	18	18	18
Ratio of piston displacements	1	2.04	4.94
Volume first receiver, cubic feet		2.22	
Volume second " " "		6.26	
Ratio of receiver volumes to high-pressure piston displacement	0.82		2.31
Stroke, inches		24	
Boiler-pressure, absolute, pounds per sq. in.		180	
Pressure in condenser, " " " "		2	

If the volume of the high-pressure piston displacement is taken to be unity, then the volumes required in the equations

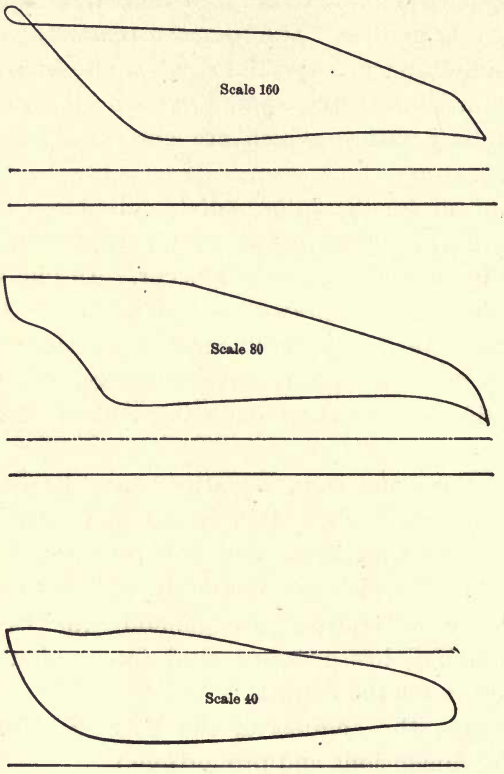

FIG. 67.

for calculating pressures, when the cranks have the order high-pressure, low-pressure, and intermediate, are as follows:

$v_b = 0.79$ $v_c = v_d = 1.06$ $v_e = 1.10$ $v_f = 1.13$ $v_g = v_h = 0.88$ $v_i = 0.31$ $v_k = v_a = 0.13$

$v_l = v_x = 0.29$ $v_m = 0.98$ $v_n = v_o = 1.26$ $v_p = 1.63$ $v_q = v_r = 2.18$ $v_s = 2.28$ $v_t = 2.33$ $v_u = v_v = 1.85$ $v_w = 0.63$

$v_y = v_\eta = 0.35$ $v_z = 2.02$ $v_\alpha = v_\beta = 2.72$ $v_\gamma = 3.60$ $v_\delta = v_\epsilon = 4.94$ $v_\zeta = 1.23$

The required pressures are:

$p_a = p_b = 150$	$p_k = 165$	$p_w = p_z = 25.6$
$p_c = 112$	$p_n = 60.0$	$p_x = 52.3$
$p_d = p_o = 76.5$	$p_q = 50.5$	$p_\alpha = 22.1$
$p_e = p_p = 67.5$	$p_r = p_\beta = 28.3$	$p_\delta = 18.5$
$p_f = 67.5$	$p_s = p_\gamma = 25.3$	$p_\epsilon = p_\zeta = 5$
$p_g = 76.9$	$p_t = 25.1$	$p_\eta = 17.6$
$p_h = p_l = 73.5$	$p_u = 29.0$	
$p_i = p_m = 69.3$	$p_v = p_y = 28.2$	

Diagrams with the volumes and pressures corresponding to this example are plotted in Fig. 66 with convenient vertical scales. Actual indicator-diagrams taken from the engine are given by Fig. 67. The events of the stroke come at slightly different piston positions on account of the irregularity due to the connecting-rod, and the drops and other fluctuations of pressure are gradual instead of sudden; moreover, there is considerable loss of pressure from the boiler to the engine, from one cylinder to another, and from the low-pressure cylinder to the condenser. Nevertheless the general forms of the diagrams are easily recognized and all apparent erratic variations are accounted for.

Designing Compound Engines.—The designer of compound and multiple-expansion engines should have at hand a well-systematized fund of information concerning the sizes, proportions, and powers of such engines, together with actual indicator-diagrams. He may then, by a more or less direct method of interpolation or exterpolation, assign the dimensions and proportions to a new design, and can, if deemed advisable, draw or determine a set of probable indicator-diagrams for the new engines. If the new design differs much from the engines for which he has information he may determine approximate diagrams both for an actual engine from which indicator-diagrams are at hand, and for the new design. He may then sketch diagrams for the new engine, using the approximate diagrams as a basis and taking a comparison of

the approximate and actual diagrams from the engine already built, as a guide. It is convenient to prepare and use a table showing the ratios of actual mean effective pressures and approximate mean effective pressures. Such a table enables the designer to assign mean effective pressures to a cylinder of the new engine and to infer very closely what its horse-power will be. It is also very useful as a check in sketching probable diagrams for a new engine, which diagrams are not only useful in estimating the power of the new engine, but in calculating stresses on the members of that engine.

A rough approximation of the power of an engine may be made by calculating the power as though the total or equivalent expansion took place in the low-pressure cylinder, neglecting clearance and compression. The power thus found is to be affected by a factor which according to the size and type of the engine may vary from 0.6 to 0.9 for compound engines and from 0.5 to 0.8 for triple engines. Seaton and Rounthwaite * give the following table of multipliers for compound marine engines:

MULTIPLIERS FOR FINDING PROBABLE M.E.P. COMPOUND AND TRIPLE MARINE ENGINES.

Description of Engine.	Jacketed.	Unjacketed.
Receiver-compound, screw engines	0.67 to 0.73	0.58 to 0.68
do paddle engines		0.55 to 0.65
Direct expansion		0.71 to 0.75
Three cylinder triple, merchant ships	0.64 to 0.68	0.60 to 0.66
do naval vessels	0.55 to 0.65	
do gunboats and torpedo boats		0.60 to 0.67

For example, let the boiler-pressure be 80 pounds by the gauge, or 94.7 pounds absolute; let the back-pressure be 4 pounds absolute; and let the total number of expansions be six, so that the volume of steam exhausted to the condenser is six times the volume admitted from the boiler. Neglect-

* *Pocket Book of Marine Engineering.*

ing the effect of clearance and compression, the mean effective pressure is

$$94.7 \times \tfrac{1}{6} + 94.7 \times \tfrac{1}{6} \log_e \tfrac{6}{1} - 4 \times 1 = 40.06 = \text{M.E.P.}$$

If the large cylinder is 30 inches in diameter, and the stroke is 4 feet, the horse-power at 60 revolutions per minute is

$$\frac{\pi 30^2}{4} \times 40.06 \times 2 \times 4 \times 60 \div 33000 = 412 \text{ H.P.}$$

If the factor to be used in this case is 0.75, then the actual horse-power will be about

$$0.75 \times 400 = 300 \text{ H.P.}$$

CHAPTER XIII.

TESTING STEAM-ENGINES.

THE principal objects of tests of steam-engines is to determine the cost of power. Series of engine tests are made to determine the effect of certain conditions, such as superheating and steam-jackets, on the economy of the engine. Again, tests may be made to investigate the interchanges of heat between the steam and the walls of the cylinder by the aid of Hirn's analysis, and thus find how certain conditions produce effects that are favorable or unfavorable to economy.

The two main elements of an engine test are, then, the measurement of the power developed and the determination of the cost of the power in terms of thermal units, pounds of steam, or pounds of coal. Power is most commonly measured by aid of the steam-engine indicator, but the power delivered by the engine is sometimes determined by a dynamometer or a friction break; sometimes, when an indicator cannot be used conveniently, the dynamic or break power only is determined. When the engine drives a dynamo-electric generator the power applied to the generator may be determined electrically, and thus the power delivered by the engine may be known.

When the cost of power is given in terms of coal per horse-power per hour, it is sufficient to weigh the coal for a definite period of time. In such case a combined boiler and engine test is made, and all the methods and precautions for a careful boiler test must be observed. The time required for such a test depends on the depth of the fire on the grate and

the rate of combustion. For factory boilers the test should be twenty-four hours long if exact results are desired.

When the cost of power is stated in terms of steam per horse-power per hour, one of two methods may be followed. When the engine has a surface condenser the steam exhausted from the engine is condensed, collected, and weighed. One hour is usually sufficient for tests under favorable conditions; shorter intervals, ten or fifteen minutes, give fairly uniform results. The chief objection to this method is that the ar pumped from the condenser is saturated with moisture which is not accounted for. The error from this source is probably not important, for the results of tests by this method and by determining the feed-water supplied to the boiler are substantially the same. In tests on non-condensing and on jet-condensing engines the steam consumption is determined by weighing or measuring the feed-water supplied to the boiler or boilers that furnish steam to the engine. Steam used for any other purpose than running the engine, for example, for pumping, heating, or making determinations of the quality of the steam, must be determined and allowed for. The most satisfactory way is to condense and weigh such steam, but small quantities, as for determining quality by a steam calorimeter, may be gauged by allowing it to flow through an orifice. Tests which depend on measuring the feed-water should be long enough to minimize the effect of the uncertainty of the amount of water in a boiler corresponding to an apparent height of water in a water-gauge; for the apparent height of the water-level depends largely on the rate of vaporization and the activity of convection currents.

When the cost of power is expressed in thermal units it is necessary to measure the steam pressure, the amount of moisture in the steam supplied to the cylinder, and the temperature of the condensed steam when it leaves the condenser. If steam is used in jackets or reheaters it must be accounted for separately. But it is customary in all engine tests to take pressures and temperatures, so that the cost may usually be calculated in

thermal units, even when the experimenter is content to state it in pounds of steam.

For a Hirn's analysis it is necessary to weigh or measure the condensing water, and to determine the temperatures at admission to and exit from the condenser.

Important engines, with their boilers and other appurtenances, are commonly built under contract to give a certain economy, and the fulfilment of the terms of a contract is determined by a test of the engine or of the whole plant. The test of the entire plant has the advantage that it gives, as one result, the cost of power directly in coal; but as the engine is often watched with more care during a test than in regular service, and as auxiliary duties, such as heating and banking fires, are frequently omitted from such an economy test, the actual cost of power can be more justly obtained from a record of the engine in regular service, extending for weeks or months. The tests of engine and boilers, though made at the same time, need not start and stop at the same time, though there is a satisfaction in making them strictly simultaneous. This requires that the tests shall be long enough to avoid drawing the fires at beginning and end of the boiler test.

In anticipation of a test on an engine it must be run for some time under the conditions of the test, to eliminate the effects of starting or of changes. Thus engines with steam-jackets are commonly started with steam in the jackets, even if they are to run with steam excluded from the jackets. An hour or more must then be allowed before the effect of using steam in the jackets will entirely pass away. An engine without steam-jackets, or with steam in the jackets, may come to constant conditions in a fraction of that time, but it is usually well to allow at least an hour before starting the test.

It is of the first importance that all the conditions of a test shall remain constant throughout the tests. Changes of steam-pressure are particularly bad, for when the steam-pressure rises the temperature of the engine and all parts affected

by the steam must be increased, and the heat required for this purpose is charged against the performance of the engine ; if the steam-pressure falls a contrary effect will be felt. In a series of tests one element at a time should be changed, so that the effect of that element may not be confused by other changes, even though such changes have a relatively small effect. It is, however, of more importance that steam-pressure should remain constant than that all tests at a given pressure should have identically the same steam-pressure, because the total heat of steam varies more slowly than the temperature.

All the instruments and apparatus used for an engine test should be tested and standardized either just before or just after the test ; preferably before, to avoid annoyance when any instrument fails during the test and is replaced by another.

Thermometers.—Temperatures are commonly measured by aid of mercurial thermometers, of which three grades may be distinguished. For work resembling that done by the physicist the highest grade should be used, and these must ordinarily be calibrated, and have their boiling- and freezing-points determined by the experimenter or some qualified person; since the freezing-point is liable to change, it should be redetermined when necessary. For important data good thermometers must be used, such as are sold by reliable dealers, but it is preferable that they should be calibrated or else compared with a thermometer that is known to be reliable. For secondary data or for those requiring little accuracy common thermometers with the graduation on the stem may be used, but these also should have their errors determined and allowed for. Thermometers with detachable scales should be used only for crude work.

Gauges.—Pressures are commonly measured by Bourdon gauges, and if recently compared with a correct mercury column these are sufficient for engineering work. The columns used by gauge-makers are commonly subject to minor errors, and are not usually corrected for temperature. It is important that such gauges should be frequently retested. From their

convenience vacuum-gauges of the same form are used, even where a mercurial gauge could easily be applied.

The pressure of the atmosphere may be taken with either a mercurial or an aneroid barometer, but if the latter is used its errors must be known. It should be easy to make the barometric errors only a fraction of the unavoidable gauge errors.

Dynamometers.—The standard for measurement of power is the friction-brake. For smooth continuous running it is essential that the brake and its band shall be cooled by a stream of water that does not come in contact with the rubbing surfaces. Sometimes the wheel is cooled by a stream of water circulating through it, sometimes the band is so cooled, or both may be. A rubbing surface which is not cooled should be of non-conducting material. If both rubbing surfaces are metallic they must be freely lubricated with oil. An iron wheel running in a band furnished with blocks of wood requires little or no lubrication.

To avoid the increase of friction on the brake-bearings due to the load applied at a single brake-arm two equal arms may be used with two equal and opposite forces applied at the ends to form a statical couple.

With care and good workmanship a friction-brake may be made an instrument of precision sufficient for physical investigations, but with ordinary care and workmanship it will give results of sufficient accuracy for engineering work.

All forms of transmission dynamometers should be standardized, and should have their errors determined by comparison with a friction-brake.

Indicators.—The most important and at the same time the least satisfactory instrument used in engine-testing is the indicator. Even when well made and in good condition it is liable to have an error of two per cent or more when used at moderate speeds. At high speeds, three hundred revolutions per minute and over, it is likely to have two or three times as much error. As a rule, an indicator cannot be used at more than four hundred revolutions per minute.

The mechanism for reducing the motion of the crosshead of the engine and transferring it to the paper drum of an indicator should be correct in design and free from undue looseness. It should require only a very short cord leading to the paper drum, because all the errors due to the paper drum are proportional to the length of the cord and may be practically eliminated by making the cord short.

The weighing and recording of the steam-pressure by the indicator-piston, pencil-motion, and pencil are affected by errors which may be classified as follows :

1. Scale of the spring.
2. Design of the pencil-motion.
3. Inertia of moving parts.
4. Friction and backlash.

Good indicator-springs, when tested by direct loads out of the indicator, usually have correct and uniform scales ; that is, they collapse under pressure the proper amount for each load applied. The springs are perceptibly weaker at high temperatures than at ordinary temperatures, but when used on a steam-engine they are exposed to steam but little, if any, above the pressure of the atmosphere, and are but little affected thereby. Some recent indicators have the spring above the cylinder, so that it is not exposed to steam. If the scale of a spring is uniform, but is either more or less than the rated scale, a correction can easily be applied. Thus a spring marked for 50 pounds to the inch may be really a 48-pound spring, if the indicator-pencil rises an inch for 48-pounds increase of pressure. But a spring having an irregular scale must be rejected, as there is no convenient way of applying corrections for irregular errors, especially when the area of the diagram is measured by a planimeter.

The motion of the piston of the indicator is multiplied five or six times by the pencil-motion, which is usually an approximate parallel motion. Within the proper range of motion (about two inches) the pencil draws a line which is nearly

straight when the paper drum is at rest, and it gives a nearly uniform scale provided that the spring is uniform. The errors due to the geometric design of this part of the indicator are always small.

When steam is suddenly let into the indicator, as at admission to the engine-cylinder, the indicator-piston and attached parts forming the pencil-motion are set into vibration, with a natural time of vibration depending on the stiffness of the spring. A weak spring used for indicating a high-speed engine may throw the diagram into confusion, because the pencil will give a few deep undulations which make it impossible to recognize the events of the stroke of the engine, such as cut-off and release. A stiffer spring will give more rapid and less extensive undulations, which will be much less troublesome. As a rule, when the undulations do not confuse the diagram the area of the diagram is but little affected by the undulations due to the inertia of the piston and pencil-motion.

The largest and most troublesome errors of the indicator are due to friction and backlash. The various joints at the piston and in the pencil-motion are made as tight as can be without undue friction, but there is always some looseness and some friction at those joints. There is also some friction of the piston in the cylinder and of the pencil on the paper. Errors from this source are commonly one or two per cent, and may be excessive unless the instrument is used with care and skill. A blunt pencil pressed up hard on the paper will reduce the area of the diagram five per cent or more; on the other hand a medium pencil drawing a faint but legible line will affect the area very little. Any considerable friction of the piston of the indicator will destroy the value of the diagram.

Errors of the scale of the spring can be readily determined and investigated by loading the spring with known weights, when properly supported, out of the indicator. Other tests of the indicator are difficult and are likely to be misleading, as in general such tests consist in comparing the indicator with either a mercury column or a gauge when subjected to the same

pressure. Now a gauge, and still more a mercury column, can be used only for measuring steady pressure, while an indicator-piston is sure to show excessive friction if it does not stick fast when exposed to a steady pressure. The consequence is that a test made by changing pressure progressively on both an indicator and a gauge or mercury column is likely to lead to errors in the indications of both instruments. The only satisfactory way of testing an indicator is to subject it to changing pressure, so as to simulate the action of steam in the cylinder of an engine, and to measure the pressure with a mercury column or a gauge. The two pressures may differ by five or ten pounds and may be varied progressively at a rate adapted to the use of a mercury column or gauge. Very few instruments for thus testing indicators have been made, and they have not yet been adapted to commercial work. Such investigations show that the error of an indicator need not be more than one or two per cent.

Scales.—Weighing should be done on scales adapted to the load; overloading leads to excessive friction at the knife-edges and to lack of delicacy. Good commercial platform scales, when tested with standard weights, are sufficient for engineering work.

Coal and ashes are readily weighed in barrows, for which the tare is determined by weighing empty. Water is weighed in barrels or tanks. The water can usually be pumped in or allowed to run in under a head, so that the barrel or tank can be filled promptly. Large quick-opening valves must be used to allow the water to run out quickly. As the receptacle will seldom drain properly, it is not well to wait for it to drain, but to close the exit-valve and weigh empty with whatever small amount of water may be caught in it. Neither is it well to try to fill the receptacle to a given weight, as the jet of water running in may affect the weighing. With large enough scales and tanks the largest amounts of water used for engine tests may be readily handled.

Measuring Water.—When it is not convenient to weigh

water directly it may be measured in tanks or other receptacles of known volume. Commonly two are used, so that one may fill while the other is emptied. The volume of a receptacle may be calculated from its dimensions or may be determined by weighing in water enough to fill it. When desired a receptacle may be provided with a scale showing the depth of the water, and so partial volumes can be determined. A closed receptacle may be used to measure hot water or other fluids.

Water metres are of two kinds: some measure the water by the displacement of a piston or of pistons; in others the water is recorded by a rotating piece which is commonly made of a material having nearly the density of water. The latter may serve to check the use of water drawn from a city water-supply, but they are seldom reliable enough for use in engine-testing. Piston water-metres can be made to give almost any desired degree of accuracy; good commercial piston-metres seldom have more than one or two per cent of error. They should always be tested under the conditions of the test, taking account of both the amount and the temperature of the water measured. Tests of marine engines can hardly be made without the aid of a metre. For such tests the metre may be placed on a by-pass through which the feed-water from the surface-condenser can be made to pass by closing a valve on the direct line of feed-pipe. If necessary the metre can be quickly shut off and the direct line can be opened. The chief uncertainty in the use of a metre is due to air in the water; to avoid error from this source the metre should be frequently vented to allow air to escape without being recorded by the metre.

Weirs and Orifices.—So far as possible the use of weirs and orifices for water during engine tests should be avoided, for, in addition to the uncertainties unavoidably connected with such hydraulic measurements, difficulties are liable to arise from the temperature of the water and from the oil in it. A very little oil is enough to sensibly affect the coefficient for a weir or orifice. The water flowing from the hot-well of a

jet-condensing engine is so large in amount that it is deemed advisable to measure it on a weir; the effect of temperature and oil is less than when the same method is used for measuring condensed steam from a surface-condenser.

Calorimeters.—When superheated steam is supplied to an engine it is sufficient to take the temperature of the steam in the steam-pipe near the engine. When moist steam is used the condition of the steam must be determined by a calorimetric experiment. Four kinds of calorimeters will be described out of a large number that have been used by different experimenters and at different times. They are the barrel calorimeter, the Barrus continuous water-calorimeter, the throttling-calorimeter, and the separating calorimeter.

The Barrel Calorimeter.—A wooden barrel set on scales is provided with a large valve for emptying it, and provision is made for filling it with cold water, usually from a hydrant-pipe, and for bringing the steam to be tested. Some form of stirrer must be used, a good form being a wooden propeller-wheel on a wooden shaft with a hand-crank.

The method of making a test is as follows: The barrel is weighed empty, and a suitable quantity of cold water is run in and weighed. The temperature of the cold water should be taken as it enters. The steam-pipe usually terminates in a piece of rubber hose which may be swung into or out of the barrel. When the barrel is nearly filled with cold water the steam-valve may be opened until all condensed water is blown from the pipe and the hose is warmed up; then the hose may be swung into the barrel and steam may be run into the water till a proper amount is condensed. A preliminary calculation will determine the proper weights of water and steam to give a good range of temperatures in the calorimeter. After the steam is run in the water in the barrel may be well stirred and the highest temperature taken as the final temperature.

To eliminate the action of the wood of the barrel one or more tests are made and rejected, and the times of running in

water and steam are made equal, so that the barrel, which is already warmed by the preceding test, may give up as much heat during one part of the process as it receives during the other part.

If the pressure of the steam is p, and the part of each pound of the mixture which is steam is represented by x, while the initial and final temperatures of the water are t_1 and t_2, and the weights of the water and steam are W and w, then

$$w(xr + q - q_2) = W(q_2 - q_1);$$

$$\therefore \quad x = \frac{W(q_2 - q_1) - w(q - q_2)}{wr}, \quad . \quad . \quad . \quad (265)$$

r and q being the latent heat and heat of the liquid for the pressure p, and q_1 and q_2 being the heats of the liquid for the temperatures t_1 and t_2.

For example, suppose that 180 pounds of water at the temperature of 60°.2 F. are run into a barrel calorimeter, and that the final temperature of the water in the calorimeter is 103°.6 F., after $7\frac{1}{4}$ pounds of steam at 73.8 pounds by the gauge are run in and condensed. At an absolute pressure of 88.5 pounds $r = 890.4$, $q = 288.8$; the heats of the liquid at 60°.2 and 103°.6 are 28.32 and 71.6.

$$x = \frac{180(71.6 - 28.32) - 7.25(288.8 - 71.6)}{7.25 \times 890.4} = 0.963;$$

consequently the per cent of priming is 3.7.

It is to be remarked of this kind of calorimeter that satisfactory results are difficult to attain even when every care and precaution are used, and that a small error in determining the weight of steam, which is obtained by subtraction, makes a large difference in the result.

Continuous Water-calorimeter.—The difficulty of obtaining the weight of steam with sufficient accuracy which occurs in the use of the barrel calorimeter is avoided in the use of the continuous water-calorimeter, represented by Fig. 68. This calorimeter is essentially a small surface-condenser of

special form, so arranged that the condensed steam is weighed separately from the cooling water.

Steam is brought to the calorimeter by the pipe *j*, with the gauge *i* for giving the pressure. The pipe *a*, which forms the

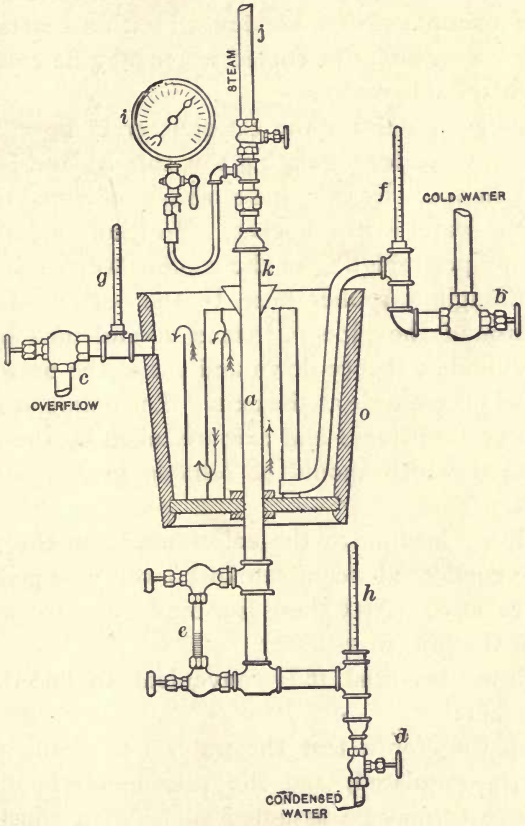

FIG. 68.

condensing surface, and which may conveniently be made of brass pipe one inch in diameter, should have the joints, above and below, clear of the bucket containing the cooling water. Steam is let into the pipe *a* at full boiler-pressure, and the condensed water gathers in the pipe below, where the water-level is shown at *e*. The height of the water at *e* is kept constant by aid of the valve at *d*, which may have a long wooden handle

attached for convenient regulation. At *h* there is a thermometer to determine the temperature of the condensed steam. Since this temperature is only a little less than that due to the boiler-pressure, the condensed water should be led through a cooler like a simple surface-condenser, with a separate stream of cooling water, and the cooled water may be collected and weighed on suitable scales.

The cooling water for the calorimeter is brought by the pipe *b* with a valve for regulating the supply, and is led away to a barrel on scales by the pipe *c*, with a valve to regulate the height of the water in the bucket. To insure a good circulation and a proper mingling of the cooling water the current is directed through a rubber hose to the bottom of the inner cylinder around the pipe *a*, thence up and into the top of the outer cylinder, thence down and out at the bottom of this cylinder and over a weir at the exit. The temperatures of the cooling water at entrance and exit are taken by the thermometers *f* and *g*, which should be reliable to $\frac{1}{20}$ of one degree Fahrenheit.

The pipe *j* leading to the calorimeter and the pipe containing the condensed steam should be well wrapped as far as to the valve at *d*. At *k* there is a brass cone to protect the covering of the pipe from water.

Though not essential, it is convenient to line the bucket with sheet metal.

In preparing for a test the water and steam are let on and properly regulated, and the calorimeter is allowed to run till all parts may be assumed to be at a constant temperature; the cooling water from *c* and the condensed steam are then directed into the receptacles for weighing, and the time is noted as the beginning of the test. The steam-pressure and the several temperatures are taken at intervals and recorded. At the end of half an hour or an hour the cooling water and condensed water are diverted from the weighing receptacles, and the time is noted as the end of the test. The quantities of the cooling and condensed water can

be weighed at the end of the test, or the test may be made continuous for any desired length of time by having two weighing receptacles for each, and filling and emptying them alternately.

The radiation in thermal units per hour must be determined by running the calorimeter without cooling water and with the bucket filled with hair-felt.

In this or any form of calorimeter that is capable of giving accurate results it is essential that the steam-pressure should not change during a test, since a considerable change of pressure will vitiate the results on account of the heat absorbed or yielded by the pipes leading to the condenser.

Let W and w be the weights of the cooling water for the test, and let p be the steam-pressure and t_2 the final temperature of the condensed steam taken by the thermometer at h, while t_1 and t_2 are the initial and final temperatures of the cooling water; finally, let the radiation during the test be e thermal units.

Then

$$w(xr + q - q_2) = W(q_2 - q_1) + e;$$

$$\therefore x = \frac{W(q_2 - q_1) + e - w(q - q_2)}{wr}. \quad . \quad . \quad (266)$$

Example.—The following are the data of a test made in the laboratory of the Institute of Technology:

Initial temperature of cooling water, .	37°.49 F.
Final " " " " .	83°.84 F.
Temperature of condensed steam, . .	304°.88 F.
Pressure of the atmosphere, ...	14.8 lbs. per sq. in.
Pressure of steam by gauge, ...	72.4 " " " "
Duration of test, ...	40 minutes.
Radiation per hour, ...	180 B. T. U.
Weight of cooling water, ...	573.5 pounds.
" " condensed water, ...	29.89 "

$$x = \frac{573.5(51.91 - 5.53) + 120 - 29.89(287.6 - 274.4)}{29.89 \times 891.2};$$

$$x = 0.988.$$

Per cent of priming, 1.2.

It is apparent that any surface-condenser may be used in the same manner as a calorimeter, except that it is not usually convenient to fill such a condenser with steam at boiler-pressure. Since the wire-drawing of steam in a well-wrapped valve is accompanied with little loss of heat, this need not interfere with such a use of a condenser. In an engine test the quality of exhaust-steam flowing to a jet or a surface condenser can be determined by equation (265) or (266), except that the external radiation cannot always be satisfactorily determined.

Throttling-calorimeter.—A simple form of calorimeter, devised by the author, is shown by Fig. 69, where *A* is a reservoir about four inches in diameter and about 10 inches long to which steam is admitted through a half-inch pipe *b*, with a throttle valve near the reservoir. Steam flows away through an inch pipe *d*. At *f* is a gauge for measuring the pressure, and at *e* there is a deep cup for a thermometer to measure the temperature. The boiler-pressure may be taken from a gauge on the main steam-pipe near the calorimeter. It should not be taken from a pipe in which there is a rapid flow of steam as in the pipe *b*, since the velocity of the steam will affect the gauge-reading, making it less than the real pressure. The reservoir is wrapped with hair-felt and lagged with wood to reduce radiation of heat.

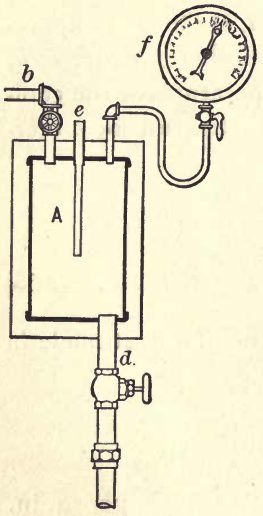

FIG. 69.

When a test is to be made, the valve on the pipe *d* is opened wide (this valve is frequently omitted) and the valve at *b* is opened wide enough to give a pressure of five to fifteen pounds in the reservoir. Readings are then taken of the boiler-gauge, of the gauge at *f*, and of the thermometer at *e*.

It is well to wait about ten minutes after the instrument is started before taking readings so that it may be well heated. Let the boiler-pressure be p, and let r and q be the latent heat and heat of the liquid corresponding. Let p_1 be the pressure in the calorimeter, and λ_1 and t_1 the total heat and the temperature of saturated steam at that pressure, while t_s is the temperature of the superheated steam in the calorimeter. Then

$$xr + q = \lambda_1 + c_p(t_s - t_1);$$

$$\therefore \quad x = \frac{\lambda_1 + c_p(t_s - t_1) - q}{r} \quad . \quad . \quad . \quad . \quad (267)$$

Example.—The following are the data of a test made with this calorimeter:

Pressure of the atmosphere.....	14.8	pounds;
Steam-pressure by gauge.....	69.8	"
Pressure in the calorimeter, gauge...	12.0	"
Temperature in the calorimeter.....	268°.2	F.

$$x = \frac{1156.4 + 0.48(268.2 - 243.9) - 285.3}{892.7} = 0.988;$$

Per cent of priming, 1.2.

A little consideration shows that this type of calorimeter can be used only when the priming is not excessive; otherwise the throttling will fail to superheat the steam, and in such case nothing can be told about the condition of the steam either before or after throttling. To find this limit for any pressure t_s may be made equal to t_1 in equation (267); that is, we may assume that the steam is just dry and saturated at that limit in the calorimeter. Ordinarily the lowest convenient pressure in the calorimeter is the pressure of the atmosphere, or 14.7 pounds to the square inch. The table following has been calculated for several pressures in the manner indicated. It shows that the limit is higher for higher pressures, but that the calorimeter can be applied only where the priming is moderate.

When this calorimeter is used to test steam supplied to a condensing-engine the limit may be extended by connecting

the exhaust to the condenser. For example, the limit at 100 pounds absolute, with 3 pounds absolute in the calorimeter, is 0.064, instead of 0.046 with atmospheric pressure in the calorimeter.

LIMITS OF THE THROTTLING CALORIMETER.

Pressure.		Priming.
Absolute.	Gauge.	
300	285.3	0.077
250	235.3	0.070
200	185.3	0.061
175	160.3	0.058
150	135.3	0.052
125	110.3	0.046
100	85.3	0.040
75	60.3	0.032
50	35.3	0.023

In case the calorimeter is used near its limit—that is, when the superheating is a few degrees only—it is essential that the thermometer should be entirely reliable; otherwise it might happen that the thermometer should show superheating when the steam in the calorimeter was saturated or moist. In any other case a considerable error in the temperature will produce an inconsiderable effect on the result. Thus at 100 pounds absolute with atmospheric pressure in the calorimeter 10° F. of superheating indicates 0.035 priming, and 15° F. indicates 0.032 priming. So also a slight error in the gauge-reading has little effect. Suppose the reading to be apparently 100.5 pounds absolute instead of 100, then with 10° of superheating the priming appears to be 0.033 instead of 0.032.

It has been found by experiment that no allowance need be made for radiation from this calorimeter if made as described, provided that 200 pounds of steam are run through it per hour. Now this quantity will flow through an orifice one-fourth of an inch in diameter under the pressure of 70 pounds by the gauge, so that if the throttle-valve be replaced by such an orifice the question of radiation need not be considered. In such case a stop-valve will be placed on the pipe to shut

off the calorimeter when not in use; it is opened wide when a test is made. If an orifice is not provided the throttle-valve may be opened at first a small amount, and the temperature in the calorimeter noted; after a few minutes the valve may be opened a trifle more, whereupon the temperature may rise, if too little steam was used at first. If the valve is opened little by little till the temperature stops rising it will then be certain that enough steam is used to reduce the error from radiation to a very small amount.

Various modifications of the throttling-calorimeter have been proposed, mainly with a view to reducing its size and weight. Almost any of them will prove satisfactory in practice, but some will be found to be liable to error from radiation or from the fact that there is not sufficient opportunity for the steam to come to rest and properly develop the superheating due to throttling.

Separating Calorimeter.—If steam contains more than three per cent of moisture the priming may be determined by a good separator which will remove nearly all the moisture. It remains to measure the steam and water separately. The water may be best measured in a calibrated vessel or receiver, while the steam may be condensed and weighed, or may be gauged by allowing it to flow through an orifice of known size. A form of separating calorimeter devised by Prof. Carpenter* is shown by Fig. 70.

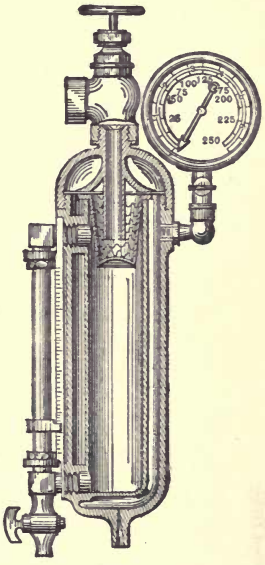

FIG. 70.

Steam enters a space at the top which has sides of wire gauze and a convex cup at the bottom. The water is thrown against the cup and finds its way through the gauze into an inside chamber or receiver and rises in a

* *Trans. Am. Soc. Mech. Engs.*, vol. xvii, p. 608.

water-glass outside. The receiver is calibrated by trial, so that the amount of water may be read directly from a graduated scale. The steam meanwhile passes into the outer chamber which surrounds the inner receiver and escapes from an orifice at the bottom. The steam may be determined by condensing, collecting, and weighing it; or it may be calculated from the pressure and the size of the orifice. When the steam is weighed there is no radiation error, since the inner chamber is protected by the steam in the outer chamber. This instrument may be guarded against radiation by wrapping and lagging, and then if steam enough is used the radiation will be insignificant, just as was found to be the case for the throttling-calorimeter.

There is a question whether this instrument or the throttling-calorimeter should be called a calorimeter; perhaps they may be better considered to be priming-gauges.

Method of Sampling Steam—It is customary to take a sample of steam for a calorimeter or priming-gauge through a small pipe leading from the main steam-pipe. The best method of securing a sample is an open question; indeed, it is a question whether we ever get a fair sample. There is no question but that the composition of the sample is correctly shown by any of the calorimeters described, when the observer makes tests with proper care and skill. It is probable that the best way is to take steam through a pipe which reaches at least halfway across the main steam-pipe, and which is closed at the end and drilled full of small holes. It is better to have the sampling-pipe at the side or top of the main, or otherwise arranged so that water trickling along the bottom of the main shall not enter the calorimeter. Again, it is better to take a sample from a pipe through which steam flows vertically upward. The sampling-pipe should be short and well wrapped to avoid radiation.

Indirect Engine Test.—It is often difficult, if not impossible, to determine the steam used by an engine directly if it has not a surface-condenser, and if at the same time the boiler

or boilers supplying it with steam do other work also. If the engine has a jet-condenser then the overflow from the hot-well may be determined by allowing it to flow over a weir. At the same time the engine may be indicated and the temperatures of the cold injection-water and of the overflow from the hot-well may be determined. If possible the priming in the steam-pipe should be determined by aid of a throttling calorimeter, and if the engine has a steam-jacket the radiation may be found from the condensation of steam in the jackets while the engine is at rest. For approximate work the steam in the supply-pipe may be assumed to be dry, and the radiation may be inferred from comparison with an engine of the same size and type; or, since the radiation is nearly constant, it may be treated as a constant unknown error.

Suppose that the engine uses M pounds of steam per horse-power per hour, and that G pounds of condensing water are used per pound of exhaust-steam; then

$$M(1+G) = A \quad . \quad . \quad . \quad . \quad . \quad (268)$$

is equal to the overflow of the hot-well in pounds per hour as determined by the aid of a weir.

If the injection-water comes in at the temperature t_i and flows from the hot-well with the condensed steam at the temperature t_k, then the heat taken up by the condensing water is

$$MG(q_k - q_i),$$

where q_k and q_i are the heats of the liquid corresponding to the temperatures t_k and t_i. The heat radiated per hour may be represented by R. The heat changed into work will be

$$\frac{60 \text{ H.P.}}{33000},$$

where H.P. represents the horse-power of the engine determined from the indicator-diagrams.

The heat supplied to the engine per hour will be

$$M(xr + q - q_k),$$

where r and q are the heat of vaporization and heat of the liquid at the pressure p in the steam-pipe, and x is the quality of the steam; usually, x is between 0.98 and unity. This equation assumes that the feed-water is drawn from the hot-well at the temperature t_k.

But the heat supplied to the engine is equal to the heat accounted for as work, by radiation, and as carried away by the cooling water, so that we have

$$M(xr + q - q_k) = \frac{60 \text{ H.P.}}{33000} + R + MG(q_k - q_i). \quad (269)$$

From equations (268) and (269) we have

$$M = \frac{\frac{60 \text{ H.P.}}{33000} + R + A(q_k - q_i)}{xr + q - q_i}, \quad . \quad . \quad . \quad (270)$$

by which the steam per horse-power can be readily calculated, and the thermal units per horse-power per minute can be found from the expression

$$M(xr + q - q_k) \div 60. \quad . \quad . \quad . \quad . \quad (271)$$

CHAPTER XIV.

INFLUENCE OF THE CYLINDER WALLS.

THE difference between the action of steam in the cylinder of a steam-engine and the calculated action of steam in a non-conducting cylinder is due mainly to the influence of the walls of the cylinder, on which steam condenses during admission and from which water is vaporized during expansion and exhaust. This influence is so intense, and at the same time so complicated, that any attempt to base the design of a steam-engine on the theoretical discussion of the cycle for a non-conducting engine leads only to confusion and disappointment. This became evident as soon as an attempt was made to use the results of the thermodynamic investigations of Rankine and Clausius for that purpose, and in consequence the whole thermodynamic treatment of the properties of steam and of its application to the production of power was looked upon with disfavor by engine-builders. Tests on engines showed an actual steam-consumption of quarter or half more than was given by theoretical calculations; and, again, the conditions which the theory indicated as favorable to economy often gave the worst results. This discrepancy between theory and practice was most notable in the case of cut-off and expansion in the cylinder. Thus on page 238 it is shown that complete expansion to the back-pressure gives a higher efficiency than incomplete expansion with a drop at release. But many experiments, for example those by Isherwood on the U. S. S. *Michigan* in 1861, showed conclusively that the best economy was obtained by use of a moderately short cut-off.

From the accompanying table of conditions and results it appears that for the engines of the *Michigan* the best economy in steam-consumption is given for a cut-off at $\frac{4}{9}$ of the stroke. The same result is shown even more clearly by a diagram in which each cut-off is laid off as an abscissa and the corresponding steam-consumption is laid off as an ordinate, as shown by Fig. 71. To make the diagram clear and compact, the axis of abscissæ is taken at 30 pounds of steam per horse-power per hour. An inspection of this diagram and of the figures in the table show a regularity in the results which can be attained only when tests are made with care and skill. The only condition purposely varied is the cut-off; the only condition showing important accidental variation is the vacuum, and consequently the back-pressure in the cylinder. To allow for the small variations in the back-pressure Isherwood changed the mean effective pressure for each test by adding or subtracting, as the case might require, the difference between the actual back-presure and the mean back-pressure of 2.7 pounds per square inch, as deduced from all the tests.

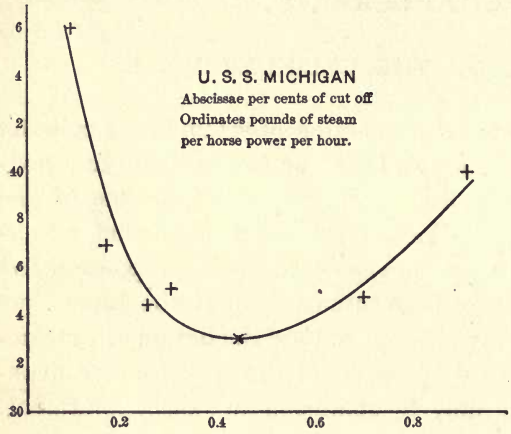

FIG. 71.

An inspection of any such a series of tests having a wide range of expansions will show that the steam-consumption decreases as the cut-off is shortened till a minimum is reached, usually at $\frac{1}{3}$ to $\frac{1}{6}$ stroke; any further shortening of the cut-off will be accompanied by an increased steam-consumption, which may become excessive if the cut-off is made very short. Some insight into the reason for this may be had from the

TABLE II.

TESTS ON THE ENGINE OF THE U. S. S. MICHIGAN.

CYLINDER DIAMETER, 36 INCHES ; STROKE, 8 FEET.

By Chief-Engineer ISHERWOOD, *Researches in Experimental Steam Engineering.*

	I.	II.	III.	IV.	V.	VI.	VII.
Duration, hours	72	72	72	72	72	72	72
Cut-off	11/12	7/10	4/9	3/10	1/4	1/6	4/45
Revolutions per minute	20.6	15.6	17.3	13.7	13.9	11.2	14.1
Boiler-pressure, pounds per sq.in. above atmosphere	21.0	19 5	21.0	21.0	21.0	21.0	22.0
Barometer, inches of mercury	30.1	29.8	29.7	30.1	29.9	29.9	29.9
Vacuum, inches of mercury	26.5	26.1	26.3	25.8	25.8	25.6	24.1
Steam per horse-power per hour, pounds	38.0	33.8	32.7	34.7	34.5	36.8	41.4
Per cent of water in cylinder at release.	10.7	15.3	27.2	41.7	39.6	42.1	45.1

per cent of water in the cylinder, calculated from the dimensions of the cylinder and the pressures in the cylinder taken from the indicator-diagram. The method of the calculation will be given in detail a little later in connection with Hirn's analysis. It will be sufficient now to notice that the amount of water in the cylinder of the engine of the *Michigan* at release increased from 10.7 per cent for a cut-off at $\frac{11}{12}$ of the stroke to 45.1 per cent for a cut-off at $\frac{4}{45}$ of the stroke. Now all the water in the cylinder at release is vaporized during the exhaust, the heat for this purpose being abstracted from the cylinder walls, and the heat thus abstracted is wasted, without any compensation. The walls may be warmed to some extent in consequence of the rise of pressure and temperature during compression, but by far the greater part of the heat abstracted during exhaust must be supplied by the incoming steam at admission. There is therefore a large condensation of steam during admission and up to cut-off, and the greater part of the steam thus condensed remains in the form of water and does little if anything toward producing work. This may be seen by inspection of the table of results of Dixwell's tests on page 371. With saturated steam and with cut-off at 0.217 of the stroke, 52.2 per cent of the working substance in the cylinder

was water. Of this 19.8 per cent was reëvaporated during expansion and 32.4 per cent remained at release to be re-evaporated during exhaust. When the cut-off was lengthened to 0.689 of the stroke, there was 27.9 per cent of water at cut-off and 23.9 per cent at release. The statement in percentages gives a correct idea of the preponderating influence of the cylinder walls when the cut-off is unduly shortened; it is, however, not true that there is more condensation with a short than with a long cut-off. On the contrary there is more water condensed in the cylinder when the cut-off is long, only the condensation does not increase as fast as do the weight of steam supplied to the cylinder and the work done, and consequently the condensation has a less effect.

Hirn's Analysis.—Though the method just illustrated gives a correct idea of the influence of the walls of the cylinder of a steam-engine, our first clear insight into the action of the walls is due to Hirn,* who accompanied his exposition by quantitative results from certain engine tests. The statement of his method which will be given here is derived from a memoir by Dwelshauvers-Dery.†

Let Fig. 72 represent the cylinder of a steam-engine and the diagram of the actual cycle. For sake of simplicity the diagram is represented without lead of admission or release, but the equations to be deduced apply to engines having either or both. The points 1, 2, 3, and 0 are the points of cut-off, release, compression, and admission. The part of the cycle from 0 to 1, that is, from admission to cut-off, is represented by a; in like manner, b, c, and d represent the parts of the cycle during expansion, exhaust, and compression. The numbers will be used as subscripts to designate the properties

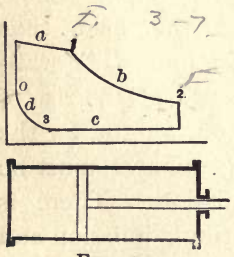

FIG. 72.

* *Bulletin de la Soc. Ind. de Mulhouse*, 1873; *Théorie Méchanique de la Chaleur*, vol. ii., 1876.

† *Revue universelle des Mines*, vol. viii. p. 362, 1880.

of the working fluid under the conditions represented by the points indicated, and the letters will be used in connection with the operations taking place during the several parts of the cycle. Thus at cut-off the pressure is p_1, and the temperature, heat of the liquid, heat of vaporization, condition, etc., are represented by t_1, q_1, r_1, x_1, etc. The external work from cut-off to release is W_b, and the heat yielded by the walls of the cylinder due to reëvaporation is Q_b.

Suppose that M pounds of steam are admitted to the cylinder per stroke, having in the supply-pipe the pressure p and the condition x; that is, each pound is x part steam mingled with $1 - x$ of water. The heat brought into the cylinder per stroke, reckoned from freezing-point, is

$$Q = M(q + xr). \quad . \quad . \quad . \quad . \quad . \quad (272)$$

Should the steam be superheated in the supply-pipe to the temperature t_s, then

$$Q = M[\lambda + c_p(t_s - t)], \quad . \quad . \quad . \quad . \quad (273)$$

in which $c_p = 0.4805$ is the specific heat of superheated steam at constant pressure.

Let the heat-equivalent of the intrinsic energy of the entire weight of water and steam in the cylinder at any point of the cycle be represented by I; then at admission, cut-off, release, and compression we have

$$I_0 = M_0(q_0 + x_0\rho_0); \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

$$I_1 = (M + M_0)(q_1 + x_1\rho_1); \quad . \quad . \quad . \quad (275)$$

$$I_2 = (M + M_0)(q_2 + x_2\rho_2); \quad . \quad . \quad . \quad . \quad (276)$$

$$I_3 = M_0(q_3 + x_3\rho_3); \quad . \quad . \quad . \quad . \quad . \quad . \quad (277)$$

in which ρ is the heat-equivalent of the internal work due to vaporization of one pound of steam, and M_0 is the weight of water and steam caught in the cylinder at compression, calculated in a manner to be described hereafter.

If the steam is superheated at any point of the cycle the corresponding intrinsic energy may be calculated by aid of

equation (179), page 133. For example the heat-equivalent of the intrinsic energy at release may be

$$I_2 = (M + M_0)\left\{\frac{A}{k-1}p_2(v_2 - s_2) + q_2 + \rho_2\right\}, \quad (278)$$

where v_2 is the specific volume of the superheated steam and s_2 is the specific volume of saturated steam at the pressure p_2. k is, of course, the constant $\frac{4}{3}$, as given on page 129.

At admission the heat-equivalent of the fluid in the cylinder is I_0, and the heat supplied by the entering steam up to the point of cut-off is Q. Of the sum of these quantities a part, AW_a, is used in doing external work, and a part remains as intrinsic energy at cut-off. The remainder must have been absorbed by the walls of the cylinder, and will be represented by Q_a. Hence

$$Q_a = Q + I_0 - I_1 - AW_a.$$

From cut-off to release the external work W_b is done, and at release the heat-equivalent of the intrinsic energy is I_2. Usually the walls of the cylinder, during expansion, supply heat to the steam and water in the cylinder. To be more explicit, some of the water condensed on the cylinder walls during admission and up to cut-off is evaporated during expansion. This action is so energetic that I_2 is commonly larger than I_1. Since heat absorbed by the walls is given a positive sign, the contrary sign should be given to heat yielded by them; it is, however, convenient to give a positive sign to all the interchanges of heat in the equations, and then in numerical problems a negative sign will indicate that heat is yielded during the operation under consideration. For expansion, then,

$$Q_b = I_1 - I_2 - AW_b.$$

During the exhaust the external work W_c is done by the engine on the steam, the water resulting from the condensation

of the steam in the condenser carries away the heat Mq_4, the cooling water carries away the heat $G(q_k - q_i)$, and there remains at compression the heat-equivalent of intrinsic energy I_2. So that

$$Q_c = I_2 - I_3 - Mq_4 - G(q_k - q_i) + AW_c,$$

in which q_4 is the heat of the liquid of the condensed steam, and G is the weight of cooling water per stroke which has on entering the heat of the liquid q_i, and on leaving the heat of the liquid q_k.

During compression the external work W_d is done by the engine on the fluid in the cylinder, and at the end of compression, i.e., at admission, the heat-equivalent of the intrinsic energy is I_0. Hence

$$Q_d = I_3 - I_0 + AW_d.$$

It should be noted (Fig 72) that the work W_a is represented by the area which is bounded by the steam line, the ordinates through 0 and 1 and by the axis OV. And in like manner the works W_b, W_c, and W_d are represented by areas which extend to the axis OV. In working up the analysis from a test the line of absolute zero of pressure may be drawn under the atmospheric line as in Fig. 73, or proper allowance may be made after the calculation has been made with reference to the atmospheric line.

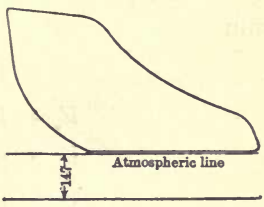

FIG. 73.

For convenience these four equations will be assembled as follows:

$$Q_a = Q + I_0 - I_1 - AW_a; \quad . \quad . \quad . \quad . \quad . \quad . \quad (279)$$

$$Q_b = I_1 - I_2 - AW_b; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (280)$$

$$Q_c = I_2 - I_3 - M_{q_4} - G(q_k - q_i) + AW_c; \quad . \quad (281)$$

$$Q_d = I_3 - I_0 + AW_d. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (282)$$

A consideration of these equations shows that all the quantities of the right-hand members can be obtained directly from the proper observations of an engine test except the several values of I, the heat-equivalents of the intrinsic energies in the cylinder. These quantities are represented by equations (274) to (277), in which there are five unknown quantities, namely x_0, x_1, x_2, x_3, and M_0. Should the steam be superheated at any point, as at release, the proper modification must be made as indicated by equation (278), in which case we have the unknown v_2 instead of x_2.

Let the volume of the clearance-space between the valve and the piston when it is at the end of its stroke be V_0; and let the volumes developed by the piston up to cut-off and release be V_1 and V_2; finally, let V_3 represent the corresponding volume at compression. The specific volume of one pound of mixed water and steam is

$$v = xu + \sigma,$$

and the volume of M pounds is

$$V = Mv = M(xu + \sigma).$$

At the points of admission, cut-off, release, and compression

$$V_0 = M_0(x_0u_0 + \sigma); \quad . \quad . \quad . \quad . \quad . \quad (283)$$

$$V_0 + V_1 = (M + M_0)(x_1u_1 + \sigma); \quad . \quad . \quad (284)$$

$$V_0 + V_2 = (M + M_0)(x_2u_2 + \sigma); \quad . \quad . \quad (285)$$

$$V_0 + V_3 = M_0(x_3u_3 + \sigma). \quad . \quad . \quad . \quad . \quad (286)$$

There is sufficient evidence that the steam in the cylinder at compression is nearly if not quite dry, and as there is comparatively little steam present at that time, there cannot be much error in assuming

$$x_3 = 1.$$

This assumption gives, by equation (286),

$$M_0 = \frac{V_0 + V_3}{u_3 + \sigma} = \frac{V_0 + V_3}{s_3} = (V_0 + V_3)\gamma_3, \quad . \quad (287)$$

in which γ_3 is the density or weight of one cubic foot of dry steam at compression.

Applying this result to equations (283) to (286) gives

$$x_0 = \frac{V_0}{M_0 u_0} - \frac{\sigma}{u_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (288)$$

$$x_1 = \frac{V_0 + V_1}{(M + M_0)u_1} - \frac{\sigma}{u_1}; \quad . \quad . \quad . \quad . \quad (289)$$

$$x_2 = \frac{V_0 + V_2}{(M + M_0)u_2} - \frac{\sigma}{u_2}. \quad . \quad . \quad . \quad . \quad (290)$$

We are now in condition to find the values of I_0, I_1, I_2, and I_3, and consequently can calculate all the interchanges of heat by equations (279) to (282).

If the steam is superheated at some point, as at release, proper allowance must of course be made. Thus we may have

$$V_0 + V_2 = (M + M_0)V_2$$

or

$$V_2 = \frac{V_0 + V_2}{M + M_0}, \quad . \quad . \quad . \quad . \quad (291)$$

which gives the means of calculating I_2 by equation (278). The fact that steam is superheated will be indicated by an apparent value of x greater than unity when calculated by one of the equations (288), (289), or (290). It is probable that any test of an engine with much compression will give superheated steam at admission when x_0 is calculated by equation (288). Recent tests by Callandar and Nicolson confirm the conclusions from such calculations. There will, however, be little error in most cases from assuming x_0 to be unity as well

as x_3, and the work will be a little simplified by such an assumption. It may be remarked in passing that when the specific volume and pressure of superheated steam are known the temperature may be calculated by equation (177), page 130. There is, however, no reason except general interest for making such a calculation.

In the diagram, Fig. 72, the external work during exhaust is all work done by the piston on the fluid, since the release is assumed to be at the end of the stroke. If the release occurs before the end of the stroke, some of the work, namely, from release to the end of the stroke, will be done by the steam on the piston, and the remainder, from the end of the stroke back to compression, will be done by the piston on the fluid. In such case W_c will be the difference between the second and the first quantities. If an engine has lead of admission, a similar method may be employed; but at that part of the diagram the curves of compression and admission can be distinguished with difficulty, if at all, and little error can arise from neglecting the lead.

The several pressures at admission, cut-off, release, and compression are determined by the aid of the indicator-diagram, and the pressures in the steam-pipe and exhaust-pipe or condenser are determined by gauges. The weight M of steam supplied to the cylinder per stroke is best determined by condensing the exhaust-steam in a surface-condenser and collecting and weighing it in a tank. If the engine is non-condensing, or if it has a jet-condenser, or if for any reason this method cannot be used, then the feed-water delivered to the boiler may be determined instead. The cooling or condensing water, either on the way to the condenser or when flowing from it, may be weighed, or for engines of large size may be measured by a metre or gauged by causing it to flow over a weir or through an orifice. The several temperatures t_4, t_i, and t_k must be taken by proper thermometers. When a jet-condenser is used, and the condensing water mingles with the steam, t_4 is identical with t_k. The quality x of the steam in

the supply-pipe must be determined by a steam-calorimeter. A boiler with sufficient steam-space will usually deliver nearly dry steam; that is, x will be nearly unity. If the steam is superheated, its temperature t_s may be taken by a thermometer.

Let the heat lost by radiation, conduction, etc., be Q_e; this is commonly called the radiation. Let the heat supplied by the jacket be Q_j. Of the heat supplied to the cylinder per stroke, a portion is changed into work, a part is carried away by the condensed steam and the cooling or condensing water, and the remainder is lost by radiation; therefore

$$Q_e = Q + Q_j - Mq_4 - G(q_k - q_i) - A(W_a + W_b - W_c - W_d). \quad (292)$$

The heat Q_j supplied by a steam-jacket may be calculated by the equation

$$Q_j = m(x'r' + q' - q''), \quad . \quad . \quad . \quad . \quad (293)$$

in which m is the weight of water collected per stroke from the jacket; x', r', and q' are the quality, the heat of vaporization, and the heat of the liquid of the steam supplied; and q'' is the heat of the liquid when the water is withdrawn. When the jacket is supplied from the main steam-pipe, x' is the same as the quality in that pipe. When supplied direct from the boiler, x' may be assumed to be unity. If the jacket is supplied through a reducing-valve, the pressure and quality may be determined either before or after passing the valve, since throttling does not change the amount of heat in the steam. Should the steam applied to the jacket be superheated from any cause, we may use the equation

$$Q_j = m[\lambda' + c_p(t_x' - t') - q''], \quad . \quad . \quad . \quad (294)$$

in which λ' is the total heat of saturated steam at the temperature t', and t_x' is the temperature of the superheated steam.

Equation (292) furnishes a method of calculating the heat lost by radiation and conduction; but since Q_e is obtained by

subtraction and is small compared with the quantities on the right-hand side of the equation, the error of this determination may be large compared with Q_e itself. The usual way of determining Q_e for an engine with a jacket is to collect the water condensed in the jacket for a known time, an hour for example, when the engine is at rest, and then the radiation of heat per hour may be calculated. If it be assumed that the rate of radiation at rest is the same as when the engine is running, the radiation for any test may be inferred from the time of the test and the determined rate. But the engine always loses heat more rapidly when running than when at rest, so that this method of determining radiation always gives a result which is too small.

If a steam-engine has no jacket it is difficult or impossible to determine the rate of radiation. The only available way appears to infer the rate from that of some similar engine with a jacket. Probably the best way is to get an average value of Q_e from the application of equation (292) to a series of carefully made tests.

It is well to apply equation (292) to any test before beginning the calculation for Hirn's analysis, as any serious error is likely to be revealed, and so time may be saved.

When the radiation Q_e is known from a direct determination of the rate of radiation, we may apply Hirn's analysis to a test on an engine even though the quantities depending on the condenser have not been obtained. For from equation (292)

$$-Mq_4 - G(q_k - q_i) = Q_e - Q - Q_j + A(W_a + W_b - W_c - W_d),$$

and consequently

$$Q_c = I_2 - I_3 - Q - Q_j + Q_e + A(W_a + W_b - 2W_c - W_d). \quad (294)$$

Thus it is possible to apply the analysis to a non-condensing engine or to the high-pressure cylinder of a compound engine.

It is apparent that the heat Q_c, thrown out from the walls

of the cylinder during exhaust, passes without compensation to the condenser, and is a direct loss. Frequently it is the largest source of loss, and for this reason Hirn proposed to make it a test of the performance and perfection of the engine; but such a use of this quantity is not justifiable, and is likely to lead to confusion.

The heat Q_b that is restored during expansion is supplied at a varying and lower temperature than that of the source of heat, namely, the boiler, and though not absolutely wasted, is used at a disadvantage. It has been suggested that an early compression, as found in engines with high rotative speed, warms up the cylinder and so checks initial condensation, thereby reducing Q_a and finally Q_c also. Such a storing of heat during compression and restoring during expansion is considered to act like the regenerator of a hot-air engine, and to make the efficiency of the actual cycle approach the efficiency of the ideal cycle more nearly than would be the case without compression. It does not, however, appear that engines of that type have exceeded, if they have equalled, the performance of slow-speed engines with small clearance and little compression.

Application.—In order to show the method of applying Hirn's analysis the complete calculation for a test made on a small Corliss engine in the laboratory of the ~~Massachusetts~~ California Institute of Technology will be given:

Diameter of the cylinder	8 inches.
Stroke of the piston	2 feet.
Piston displacement: crank end	0.6791 cu. ft.
head end	0.7016 "
Clearance, per cent of piston displacement:	
crank end	3.75
head end	5.42
Boiler-pressure by gauge	77.4 pounds.
Barometer	14.8 "
Condition of steam, two per cent of moisture.	

Events of the stroke:

Cut-off: crank end ..... 0.306 of stroke.
head end ..... 0.320 "

Release at end of stroke.

Compression: crank end ..... 0.013 of stroke.
head end ..... 0.0391 "

Duration of the test, one hour.

Total number of revolutions 3692

Weight of steam used ..... 548 pounds.

Weight of condensing water used ... 14,568 "

Temperatures:

Condensed steam ..... $t_4 = 141°.1$ F.

Condensing water: cold ..... $t_i = 52°.9$ F.
warm ..... $t_k = 88°.3$ F.

ABSOLUTE PRESSURES, FROM INDICATOR-DIAGRAMS, AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

	Crank End.				Head End.			
	p	q	ρ	u	p	q	ρ	u
Cut-off	83.6	284.6	813.0	5.190	83.3	284.4	813.2	5.207
Release	29.2	217.8	864.8	13.924	31.9	222.9	860.8	12.804
Compression ..	14.8	181.1	893.2	26.464	14.8	181.1	893.2	26.464
Admission	21.8	201.5	877.4	18.344	29.8	219.0	863.9	13.664

MEAN PRESSURES, AND HEAT-EQUIVALENTS OF EXTERNAL WORKS.

	Crank End.		Head End.	
	Mean Pressures.	Equivalents of Work.	Mean Pressures.	Equivalents of Work.
Admission	87.7	3.369	89.3	3.711
Expansion	44.5	3.877	47.1	4.159
Exhaust	14.8	1.836	14.8	1.847
Compression	18.3	0.0299	21.8	0.1104

VOLUMES, CUBIC FEET.

	Crank End.	Head End.
At cut-off, $V_0 + V_1$	0.2333	0.2626
At release, $V_0 + V_2$	0.7046	0.7396
At compression, $V_0 + V_3$	0.0343	0.0655
At admission, V_0	0.02550	0.03806

At the boiler-pressure, 92.1 pounds absolute, we have

$$r = 888.4, \qquad q = 291.7.$$

The steam used per stroke is

$$M = \frac{548}{2 \times 3692} = 0.0742 \text{ pound.}$$

The steam caught in the clearance space at compression, on the assumption that the steam is then dry and saturated, is obtained by multiplying the mean volume at that point by the weight of one cubic foot of steam at the pressure at compression, which is 0.03781 of a pound.

$$\therefore \quad M_0 = \frac{0.0343 + 0.0655}{2} \times 0.03781 = 0.0019 \text{ of a pound;}$$

$$M + M_0 = 0.0742 + 0.0019 = 0.0761 \text{ pound.}$$

The condensing water used per stroke is

$$G = \frac{14568}{2 \times 3692} = 1.973.$$

$$Q = M(xr + q) = 0.0742(0.98 \times 888.3 + 291.8) = 86.243;$$

$$x_0 = \frac{V_0}{M_0 u_0} - \frac{\sigma}{u_0};$$

$$\therefore \quad x_0 = \frac{\frac{1}{2}(0.02550 + 0.03806)}{0.0019 \times \frac{1}{2}(18.344 + 13.664)} - \frac{1}{62.4 \times \frac{1}{2}(18.344 + 13.664)}$$
$$= 1.043.$$

This indicates that the steam is superheated at admission. Such may be the case, or the appearance may be due to an error in the assumption of dry steam at compression, or to errors of observation. It is convenient to assume $x_0 = 1$.

$$x_1 = \frac{V_0 + V_1}{(M + M_0)u_1} - \frac{\sigma}{u_1};$$

$$\therefore \quad x_1 = \frac{\frac{1}{2}(0.2333 + 0.2626)}{0.0761 \times \frac{1}{2}(5.190 + 5.207)} - \frac{1}{62.4 \times \frac{1}{2}(5.190 + 5.207)}$$
$$= 0.6236.$$

$$x_2 = \frac{V_0 + V_2}{(M + M_0)u_2} - \frac{\sigma}{u_2};$$

$$\therefore \quad x_2 = \frac{\frac{1}{2}(0.7046 + 0.7396)}{0.0761 \times \frac{1}{2}(13.924 + 12.804)} - \frac{1}{62.4 \times \frac{1}{2}(13.924 + 12.804)}$$
$$= 0.7088.$$

$$I_0 = M_0(q_0 + x_0\rho_0);$$

$$\therefore \quad I_0 = \tfrac{1}{2} \times 0.0019[201.5 + 219.0 + 1.00(877.4 + 863.9)]$$
$$= 2.054.$$

$$I_1 = (M + M_0)(q_1 + x_1\rho_1);$$

$$\therefore \quad I_1 = \tfrac{1}{2} \times 0.0761[284.6 + 284.4 + 0.6236(813.0 + 813.2)]$$
$$= 60.238.$$

$$I_2 = (M + M_0)(q_2 + x_2\rho_2);$$

$$\therefore \quad I_2 = \tfrac{1}{2} \times 0.0761[217.8 + 222.9 + 0.7088(864.8 + 861.8)]$$
$$= 63.311.$$

$$I_3 = M_0(q_3 + x_3\rho_3);$$

$$\therefore \quad I_3 = 0.0019(181.1 + 893.2) = 2.041.$$

$$Q_a = Q + I_0 - I_1 - AW_a;$$

$$\therefore \quad Q_a = 86.243 + 2.054 - 60.238 - \tfrac{1}{2}(3.369 + 3.711) = 24.519.$$

$$Q_b = I_1 - I_2 - AW_b;$$

$$\therefore \quad Q_b = 60.238 - 63.311 - \tfrac{1}{2}(3.877 + 4.159) = -7.091.$$

$$Q_c = I_2 - I_3 - Mq_4 - G(q_k - q_i) + AW_c;$$

$$\therefore\ Q_c = 63.311 - 2.041 - 0.0742 \times 109.3 - 1.973(56.35 - 21.01) + \tfrac{1}{2}(1.836 + 1.847)$$
$$= -14.721.$$

$$Q_d = I_2 - I_0 + AW_d;$$

$$\therefore\ Q_d = 2.041 - 2.054 + \tfrac{1}{2}(0.0299 + 0.1104) = 0.157.$$

$$Q_e = Q_a + Q_b + Q_c + Q_d = 2.764.$$

Also, equation (292) for this case gives

$$Q_e = Q - Mq_1 - G(q_k - q_i) - AW$$
$$= 86.243 - 8.110 - 69.723 - (3.540 + 4.018 - 1.841 - 0.070)$$
$$= 86.243 - 8.110 - 69.723 - 5.647 = 2.764.$$

It is to be remembered that the heat lost by radiation and conduction per stroke, when estimated in this manner, is affected by the accumulated errors of observation and computation, which may be a large part of the total value of Q_e.

Dropping superfluous significant figures, we have in B. T. U.

$$Q = 86.2, \qquad Q_a = 24.5, \qquad Q_b = -7.1,$$
$$Q_c = -14.7, \qquad Q_d = .06, \qquad Q_e = 2.8.$$

The horse-power of the engine is

$$\frac{778 \times 5.647 \times 3692 \times 2}{60 \times 33000} = 16.35 \text{ H.P.};$$

and the steam per horse-power per hour is

$$\frac{548}{16.35} = 33.5 \text{ pounds.}$$

The data and results for this test and for four others made at the same time are given in Table II.

TABLE II.

APPLICATION OF HIRN'S ANALYSIS TO A SMALL CORLISS ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

DIAMETER, 8 INCHES; STROKE, 24 INCHES.

Number.	Duration, minutes.	Total number of revolutions.	Weight of steam used, pounds.	Weight of condensing-water, pounds.	Per cent of priming in steam. $1-x$	Temperatures, degrees Fahrenheit. Condensed steam. t_4	Condensing-water, cold. t_i	Condensing-water, warm. t_k	Barometer, pounds on the square inch.	Boiler-pressure by gauge, pounds. p	Absolute pressures, pounds. Cut-off. p_1 C. E.	Cut-off. p_1 H. E.	Release. p_2 C. E.	Release. p_2 H. E.	Compression. p_3 C. E.	Compression. p_3 H. E.	Admission. p_0 C. E.	Admission. p_0 H. E.
1	30	1839	143	3853	2.	118.6	39.2	74.3	14.8	68.7	77.1	76.3	16.3	16.8	14.8	14.8	24.8	30.8
2	60	3609	327	10077	2.	130.5	54.8	86.0	14.7	73.0	83.7	82.7	17.7	19.7	14.7	14.7	21.7	25.7
3	30	1839	170	4294	2.	118.0	39.2	77.0	14.9	68.5	74.2	77.5	18.6	20.3	14.9	14.9	20.9	24.9
4	45	2672	295	8881	2.	132.8	53.6	132.8	14.8	72.9	77.1	80.5	21.8	23.8	14.8	14.8	23 8	34.8
5	60	3692	548	14568	2.	141.1	52.9	88.3	14.8	77.4	83.6	83.3	29.2	31.9	14.8	14.8	21.8	29 8

Number.	Events of the stroke—per cent of stroke from beginning. Cut-off. C. E.	Cut-off. H. E	Release. C. E	Release. I. E.	Compression. C. E.	Compression. H. E.	Heat-equivalents of work. E. T. U. Admission. AW_a C. E.	Admission. AW_a H. E.	Expansion. AW_b C. E.	Expansion. AW_b H. E.	Exhaust. AW_c C. E.	Exhaust. AW_c H. E.	Compression. AW_d C E	Compression. AW_d H. E.	Interchanges of heat. B. T. U. Heat in M pounds of moist steam. Q	Absorbed by walls during admission. Q_a	Yielded by walls during expansion. Q_b	Yielded by walls during exhaust. Q_c	Absorbed by walls during compression. Q_d	Radiation and error. Q_e	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	10 0	9.0	100	100	2.2	7.7	1.027	0.979	3.110	3.284	1.817	1.773	0.052	0.248	45.10	20.91	12.27	6.43	0.43	2.65	6.53	44.3
2	12 2	12.2	100	100	1.3	4.6	1.305	1.386	3.285	3.783	1.822	1.820	0.035	0.114	52.61	22.26	11.60	9.28	0.15	1.53	8.08	38.6
3	15.0	15.0	100	100	0.9	3.4	1.471	1.633	3.202	3.564	1.854	1.868	0.022	0.099	53.60	21.28	11.15	7.77	0.05	2.42	8.72	39.0
4	19.5	19.5	100	100	1.5	4.0	2.108	2.217	3.527	3.929	1.830	1 844	0 039	0.116	64.10	23.39	10.64	9.43	0.06	3.38	11.13	35.3
5	30.6	32.0	100	100	1.3	3.9	3.369	3.711	3.877	4.159	1.836	1.847	0.0299	0.1104	86.24	24.52	7.09	14.72	0.06	2.76	16.35	33.5

OF CALIFORNIA

Effect of Varying Cut-off.—An inspection of the interchanges of heat show that the values of Q_a, the heat absorbed by the walls during admission, increases regularly as the cut-off is lengthened, and that the heat returned during expansion decreases at the same time so that there is a considerable increase in the value of the heat Q_c which is rejected during exhaust. Nevertheless there is a large gain in economy from restricting the cut-off so that it shall not come earlier than one-third stroke. Unfortunately tests on this engine with longer cut-off than one-third stroke have not been made, and consequently the poorer economy for long cut-off cannot be shown for this engine as for the engine of the *Michigan*.

Hallauer's Tests.—In Table III are given the results of a number of tests made by Hallauer on two engines, one built by Hirn having four flat gridiron valves, and the other a Corliss engine having a steam-jacket. Two tests were made on the former with saturated steam and six with superheated steam. Three tests were made on the latter with saturated steam and with steam supplied to the jackets. These tests have a historic interest, for though not the first to which Hirn's analysis was applied, they are the most widely known, and brought about the acceptance of his method. They have also a great intrinsic value, as they exhibit the action of two different methods of ameliorating the effect of the action of the cylinder walls, namely, by the use of superheated steam and of the steam-jacket. In all these tests there was little compression, and Q_d, the interchange of heat during compression, is ignored.

Superheated Steam.—Steam from a boiler is usually slightly moist, x, the quality, being commonly 0.98 or 0.99. Some boilers, such as vertical boilers with tubes through the steam space, give steam which is somewhat superheated, that is, the steam has a temperature higher than that of saturated steam at the boiler-pressure. Strongly superheated steam is commonly obtained by passing moist steam from a boiler

TABLE III.

TESTS ON A HIRN AND A CORLISS ENGINE.

By HALLAUER. *Bulletin de la Soc. Ind. de Mulhouse*, vol. XLVII, 1877.

	Condition.	Temperature of superheated steam, Fahr.	Revolutions per minute.	Expansions.	Boiler-pressure absolute, lbs. per square inch.	Back-pressure absolute, lbs. per square inch.	Horse-power.	Steam per H.P. per hour, pounds.	B. T. U. per H.P. per minute.	Exchanges of heat in per cent of total heat furnished per stroke.					B. T. U. rejected per stroke.
										Q_a	Q_b	Q_c	Q_e	Q_f	Q_c
1		....	30.4	6.1	70.7	3.0	106.4	19.9	353	28.3	5.0	21.6	1.5	....	147
2		....	30.6	3.9	66.0	5.2	134.6	22.5	390	23.9	7.3	15.4	1.0	....	149
3	Superheated	383	30.0	6.1	71.1	2.7	111.6	16.0	296	22.4	8.4	12.5	1.6	....	75
4		419	30.0	4.7	71.0	2.7	134.0	15.7	295	....	....	9.7	....	....	70
5		448	30.2	3.9	69.6	5.2	142.4	17.2	320	11.0	2.0	7.8	1.2	....	66
6		433	30.3	2.2	68.4	2.7	124.5	18.2	341	....	....	10.5	1.3	....	81
7		428	30.1	2.2	71.6	2.5	89.6	20.2	380	....	....	14.2	1.6	....	87
8*		428	30.0	3.5	62.2	....	77.3	27.9	....	....	....	†—1.2	....	....	—7
9	Jacketed		50.4	12		2.1	103.6	17.8	....	26.4	17.5	12.3	1.65	5.1	44
10			51.1	8		2.4	129.1	17.7	....	21.1	15.0	9.8	1.3	4.9	44
11			49.3	6		2.6	155.9	17.8	....	15.4	10.4	8.0	1.1	4.1	44

* Non-condensing. † The true value is probably zero.

The throttle-valve was partly closed for 6, 7, and 8.

through a coil of pipe, or a system of piping, which is exposed to hot gases beyond the boiler.

Superheated steam may yield a considerable amount of heat before it begins to condense; consequently where superheated steam is used in an engine a portion of the heat absorbed by the walls during admission is supplied by the superheat of the steam and less condensation of steam occurs. This is very evident in Dixwell's tests given by Table XXVII, on page 371, where the water in the cylinder at cut-off is reduced from 52.2 per cent to 27.4 per cent, when the cut-off is two-tenths of the stroke, by the use of superheated steam; with longer cut-off the effect is even greater. This reduction of condensation is accompanied by a very marked gain in economy.

The way in which superheated steam diminishes the action of the cylinder-walls and improves the economy of the engine is made clear by Hallauer's tests in Table III. A comparison of tests 1 and 3, having six expansions, shows that the heat Q_a absorbed during admission is reduced from 28.3 to 22.4 per cent of the total heat supplied, and that the exhaust waste is correspondingly reduced from 21.6 to 12.5 per cent. A similar comparison of tests 2 and 5, having nearly four expansions, shows even more reduction of the action of the cylinder-walls. The effect on the restoration of heat Q_b during expansion appears to be contradictory: in one case there is more and in the other case less. It does not appear profitable to speculate on the meaning of this discrepancy, as it may be in part due to errors and is certainly affected by the unequal degree of superheating in tests 3 and 5. It may be noted that the actual value of Q_c in calories is nearly the same for tests 1 and 2, there being a small apparent increase with the increase of cut-off, which is, however, less than the probable error of the tests. The exhaust waste Q_e is much more irregular for tests 3 to 7 for superheated steam. The increase from 81 to 87 B. T. U. from test 6 to test 7 may properly be attributed to a less degree of superheating; the

increase from 66 to 81 B. T. U. for tests 5 and 6 is due to longer cut-off and less superheating; finally, the steady reduction from 75 to 66 B. T. U. for the three tests 3, 4, and 5 is probably due to the rise of temperature of the superheated steam, which more than compensates for the effect of lengthening the cut-off. Finally, in test 8 the exhaust waste is practically reduced to zero by the use of strongly superheated steam in a non-condensing engine; this shows clearly that the exhaust waste Q_e by itself is no criterion of the value of a certain method of using steam.

Steam-jackets.—If the walls of the cylinder of a steam-engine are made double, and if the space between the walls is filled with steam, the cylinder is said to be steam-jacketed. Both barrel and heads may be jacketed, or the barrel only may have a jacket; less frequently the heads only are jacketed. The principal effect of a steam-jacket is to supply heat during the vaporization of any water which may be condensed on the cylinder-walls. The consequence is that more heat is returned to the steam during expansion and the walls are hotter at the end of exhaust than would be the case for an unjacketed engine. This is evident from a comparison of tests 1 and 11 in Table III. In test 1 only a small part of the heat absorbed during admission is returned during expansion, and by far the larger part is wasted during exhaust. In test 11 the heat returned during expansion is equal to two-thirds that absorbed during admission, though a part of this heat of course comes from the jacket. About half as much is wasted during exhaust as is absorbed during admission. The condensation of steam is thus reduced indirectly; that is, the chilling of the cylinder during expansion, and especially during exhaust, is in part prevented by the jacket, and consequently there is less initial condensation and less exhaust waste, and in general a gain in economy. But though there are many tests of engines both with and without steam in the jackets, there is so much diversity of results that engineers are not agreed as to the conditions under which it is profitable to use

a steam-jacket, nor as the amount of gain to be expected under a given set of conditions.

The jacket on the cylinder of a simple engine is usually supplied with steam from the main supply-pipe, so that the pressure in the jacket is little if any more than that in the cylinder during admission. As the steam in the cylinder comes in direct contact with the inner surface of the walls, while the jacket steam is in contact with the outer surface of the walls, the heat absorbed by the walls during admission comes mainly if not entirely from the steam in the cylinder. It is probable that the action is much the same when the pressure in the jackets is higher than that of the steam which is supplied to the cylinder, as is the case when full boiler-pressure is let into the jacket of the low-pressure cylinder of a compound engine. Now the heat supplied during expansion, though it does some work, is first subjected to a loss of temperature in passing from the steam in the jacket to the cooler water on the walls of the cylinder, and such a non-reversible process is necessarily accompanied by a loss of efficiency. On the other hand, the heat supplied by a jacket during exhaust passes with the steam directly into the exhaust-pipe. It appears, then, that the direct effect of a steam-jacket is to waste heat; the indirect effect (drying and warming the cylinder) reduces the initial condensation and the exhaust waste and often gives a notable gain in economy.

Application to Multiple-expansion Engines.—The application of Hirn's analysis to the high-pressure cylinder of a compound or multiple-expansion engine may be made by using equations (279), (280), and (282) for calculating Q_a, Q_b, and Q_d, while equation (294) may be used to find Q_c.

A similar set of equations may be written for the next cylinder, whether it be the low-pressure cylinder of a compound engine or the intermediate cylinder of a triple engine, provided we can determine the value of Q', the heat supplied to that cylinder. But of the heat supplied to the high-pres-

TABLE IV.

APPLICATION OF HIRN'S ANALYSIS TO THE EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

COMPOUND; CYLINDER DIAMETERS, 9 AND 24 INCHES; STROKE, 30 INCHES.

Technology Quarterly, vol. XI, p. 43.

Test numbers.	I.	II.	III.	IV.
Duration of test, minutes	60	60	60	60
Total number of revolutions	4976	4974	4903	4882
Revolutions per minute	82.93	82.90	81.72	81.37
Steam-consumption during test, pounds:				
Passing through cylinders	844.5	820.0	1059.0	1086.0
Condensation in h. p. jacket	42.0	39.0	42.5	44.0
" in receiver jacket	37.5	51.5	73.0	80.0
" in l. p. jacket	82.8	74.5	88.5	88.0
Total	1006.8	985.0	1263.0	1298.0
Condensing water for test, pounds	16066	16287	21800	21960
Priming, by calorimeter	1.0%	0.9%	1.13%	0.9%
Temperatures, Fahrenheit:				
Condensed steam	93.4	91.4	86.0	89.2
Condensing water, cold	52.2	52.2	52.5	52.7
Condensing water, hot	105.8	104.2	102.6	104.0
Pressure of the atmosphere, by the barometer, pounds per square inch	14.5	14.5	14.5	14.5
Boiler-pressure, lbs. per sq. in., absolute	114.7	115.5	115.4	115.9
Vacuum in condenser, inches of mercury	26.6	26.6	26.3	26.3
Events of the stroke, per cent:				
High-pressure cylinder—				
Cut-off, crank end	16.8	15.8	23.8	25.2
head end	15.7	15.4	24.2	25.6
Release, both ends	100.0	100.0	100.0	100.0
Compression, crank end	7.5	7.5	7.5	7.5
head end	11.5	11.5	10.5	10.5
Low-pressure cylinder—				
Cut-off, crank end	18.0	18.0	24.5	24.5
head end	18.0	18.0	25.0	25.0
Release, both ends	100.0	100.0	100.0	100.0
Compression, crank end	3.5	3.5	3.5	3.5
head end	5.5	5.5	4.0	4.0
Absolute pressures in the cylinder, pounds per square inch:				
High-pressure cylinder—				
Cut-off, crank end	106.5	109.5	107.5	108.3
head end	107.1	107.7	107.5	108.1
Release, crank end	29.9	29.5	37.0	38.2
head end	24.7	24.8	33.2	33.7
Compression, crank end	17.2	17.7	18.0	18.7
head end	17.8	19.5	20.1	21.3
Admission, crank end	32.7	35.5	36.5	38.2
head end	42.2	42.5	45.2	46.9
Low-pressure cylinder—				
Cut-off, crank end	14.4	14.5	14.7	15.1
head end	13.6	13.9	14.2	14.9
Release, crank end	3.4	3.3	4.2	4.2
head end	3.3	3.3	4.4	4.5
Compression, crank end	1.9	2.0	2.2	2.2
head end	2.4	2.5	2.8	2.9
Admission, crank end	3.4	3.5	4.1	3.7
head end	4.4	4.7	5.8	4.9
Heat-equivalents of external work, B. T. U. from areas on indicator-diagram to line of absolute vacuum:				
High-pressure cylinder—				
During admission, AWa, crank end	3.59	3.38	5.13	5.45
head end	3.49	3.47	5.43	5.83

TABLE IV—*Continued.*

Test numbers.	I.	II.	IV.	IV.
High-pressure cylinder—				
During expansion, AWb, crank end....	7.94	8.05	8.38	8.43
head end....	7.81	7.97	8.48	8.57
During exhaust, AWc, crank end....	2.86	2.96	3.11	3.28
head end.....	2.91	3.03	3.18	3.54
During compression, AWd, crank end.	0.35	0.37	0.38	0.40
head end..	0.67	0.72	0.72	0.72
Low-pressure cylinder—				
During admission, AWa, crank end....	4.13	4.15	5.93	6.11
head end....	4.07	4.13	5.09	6.25
During expansion, AWb, crank end....	7.19	7.25	7.78	7.99
head end.....	7.05	7.12	7.88	8.16
During exhaust, AWc, crank end.....	2.64	2.55	2.87	2.98
head end.....	3.25	3.10	3.98	4.00
During compression, AWd, crank end..	0.12	0.13	0.17	0.17
head end ..	0.33	0.32	0.28	0.29
Quality of the steam in the cylinder. At admission and at compression the steam was assumed to be dry and saturated:				
High-pressure cylinder—				
At cut-off ..... x_1	70.18	71.11	73.60	76.24
At release .. .... x_2	83.98	85.59	84.97	83.73
Low-pressure cylinder—				
At cut-off..... x_1.....	91.94	95.20	92.13	92.58
At release..... x_2.....	87.62	88.19	88.57	87.86
Interchanges of heat between the steam and the walls of the cylinders, in B. T. U. Quantities affected by the positive sign are absorbed by the cylinder-walls; quantities affected by the negative sign are yielded by the walls:				
High-pressure cylinder—				
Brought in by steam.... Q	99.82	97.10	126.90	131.30
During admission ..... Qa	25.22	23.85	27.14	25.83
During expansion ..... Qb	−15.01	−15.84	−16.13	−12.60
During exhaust .. .. Qc	−11.13	−10.04	−12.99	−15.47
During compression..... Qd.....	0.98	0.45	0.46	0.47
Supplied by jacket..... Qf	3.70	3.45	3.81	3.97
Lost by radiation..... Qe..... ...	1.87	1.87	2.20	2.20
Second intermediate receiver—				
Supplied by jacket..... Qr	3.30	4.53	6.50	7.19
Lost by radiation..... Qre.....	0.69	0.69	1.60	1.60
Low-pressure cylinder—				
Brought in by steam.... Q'	96.24	94.26	123.40	128.53
During admission ..... $Q'a$.....	7.63	5.44	9.63	10.57
During expansion ..... $Q'b$.....	− 1.75	− 1.12	− 2.03	−0.97
During exhaust..... $Q'c$	−11.11	− 7.53	− 9.92	−16.53
During compression ..... $Q'd$.....	0.33	2.21	0.11	0.64
Supplied by jacket .. .. $Q'f$.....	7.30	6.56	7.91	7.85
Lost by radiation ..... $Q'e$.....	4.44	4.44	5.70	5.70
Total loss by radiation:				
By preliminary test..... ΣQe	7.39	7.39	9.52	9.52
By equation	7.00	5.78	7.83	4.42
Power and economy:				
Heat-equivalents of works per stroke:				
High-pressure cylinder.. AW.....	8.02	7.89	10.01	10.17
Low-pressure cylinder... AW'.....	8.10	8.27	10.04	10.53
Totals	16.12	16.16	20.05	20.70
Total heat furnished by jackets.....	14.30	14.54	18.22	19.01
Distribution of work:				
High-pressure cylinder.....	1.00	1.00	1.00	1.00
Low-pressure cylinder.....	1.01	1.05	1.00	1.04
Total horse-power.....	63.07	63.19	78.13	80.47
Steam per horse-power per hour	15.96	15.59	16.16	16.14
B. T. U. per horse-power per minute....	282.1	275.1	284.4	283.8

sure cylinder a part is changed into work, a part is radiated, and a part is rejected in the exhaust waste. The heat rejected is represented by

$$Q + Q_j - AW - Q_e, \quad . \quad . \quad . \quad . \quad . \quad (295)$$

where Q is the heat supplied by the steam entering the cylinder, Q_j is the heat supplied by the jacket, AW is the heat-equivalent of the work done in the cylinder, and Q_e is the heat radiated. Suppose the steam from the high-pressure cylinder passes to an intermediate receiver, which by means of a tubular reheater or by other means supplies the heat Q_r, while there is an external radiation Q_{re}. The heat supplied to the next cylinder is consequently

$$Q' = Q + Q_j - AW - Q_e + Q_r - Q_{re}. \quad . \quad . \quad (296)$$

In a like manner we may find the heat Q'' supplied to the next cylinder; for example, to the low-pressure cylinder of a triple engine.

It is clear that such an application of Hirn's analysis can be made only when the several steam-jackets on the high- and the low-pressure cylinders, and the reheater of the receiver, etc., can be drained separately, so that the heat supplied to each may be determined individually.

Table IV gives applications of Hirn's analysis to four tests on the experimental engine in the laboratory of the Massachusetts Institute of Technology, using the high- and low-pressure cylinders only as a compound engine.

Table V gives application to four tests on the same engine running as a triple engine.

It will be noted that in both sets of tests the steam in the cylinders becomes drier in its course through the engine, under the influence of thorough steam-jacketing with steam at boiler-pressure. In the triple-expansion tests the steam is practically dry at release in the low-pressure cylinder. All of the tests show superheating in the low-pressure cylinder, which is of course possible, for the steam in the jackets is at

TABLE V.

APPLICATION OF HIRN'S ANALYSIS TO THE EXPERIMENTAL ENGINE IN THE LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

TRIPLE-EXPANSION; CYLINDER DIAMETERS, 9, 16, AND 24 INCHES; STROKE, 30 INCHES.

Trans. Am. Soc. Mech. Engrs., vol. XII, p. 740.

	I.	II.	III.	IV.
Duration of test, minutes	60	60	60	60
Total number of revolutions	5299	5228	5173	5148
Revolutions per minute	88.3	87.1	86.2	85.8
Steam-consump'n during test, lbs.:				
Passing through cylinders	1193	1157	1234	1305
Condensation in h. p. jacket	57	50	29	30
" in first receiver-jacket	61	64	69	72
" in inter. jacket	85	92	97	105
" in sec'd receiver-jacket	53	50	52	51
" in l. p. jacket	89	76	90	87
Total	1538	1489	1571	1650
Condensing water for test, lbs.	22847	22186	20244	20252
Priming, by calorimeter	0.013	0.012	0.011	0.012
Temperatures, Fahrenheit:				
Condensed steam	95.4	92.1	102.4	105.3
Condensing-water, cold	41.9	42.1	43.0	42.8
Condensing-water, hot	96.1	96.6	106.3	109.6
Pressure of the atmosphere, by the barometer, lbs. per sq. in.	14.8	14.8	14.7	14.7
Boiler pressure, lbs. per sq. in. absolute	155.3	155.5	156.9	157.7
Vacuum in condenser, inches of mercury	25.0	25.1	24.1	23.9
Events of the stroke:				
High-pressure cylinder—				
Cut-off, crank end	0.192	0.194	0.245	0.283
head end	0.215	0.205	0.271	0.305
Release, both ends	1.00	1.00	1.00	1.00
Compression, crank end	0.05	0.05	0.04	0.04
head end	0.05	0.05	0.05	0.05
Intermediate cylinder—				
Cut-off, both ends	0.29	0.29	0.29	0 29
Release, both ends	1.00	1.00	1.00	1.00
Compression, crank end	0.03	0.03	0 03	0.03
head end	0.04	0.04	0.04	0.04
Low-pressure cylinder—				
Cut-off, crank end	0.38	0.38	0.38	0.38
head end	0.39	0.39	0.39	0.39
Release, both ends	1.00	1.00	1.00	1.00

TABLE V—*Continued.*

	I.	II.	III.	IV.
Absolute pressures in the cylinder, pounds per sq. in.:				
High-pressure cylinder—				
Cut-off, crank end	145.9	145.9	138.8	138.3
head end	143.2	143.1	140.3	140.6
Release, crank end	41.3	41.5	44.7	48.4
head end	41.5	40.5	45.7	49.8
Compression, crank end	43.7	45.3	48.5	53.2
head end	48.7	47.9	54.5	62.0
Admission, crank end	64.5	68.8	72.2	81.2
head end	75.3	74.8	86.7	97.8
Intermediate cylinder—				
Cut-off, crank end	37.2	37.6	38.6	40.9
head end	35.0	35.3	39.6	42.6
Release, crank end	13.6	14.2	14.7	16.0
head end	13.4	13.8	14.9	16.0
Compression, crank end	16.3	17.3	18.2	19.0
head end	17.9	18.8	20.3	22.4
Admission, crank end	20.4	20.8	22.2	23.1
head end	21.1	22.8	24.2	26.7
Low-pressure cylinder—				
Cut-off, crank end	12.1	12.6	12.4	13.2
head end	12.0	12.4	13.1	14.0
Release, crank end	5.6	5.3	5.1	5.7
head end	5.4	5.8	5.9	6.4
Compression and admission—				
crank end	3.7	3.8	4.1	4.2
head end	4.3	4.5	4.6	4.7
Heat-equivalents of external work, B. T. U., from areas on indicator-diagram to line of absolute vacuum:				
High-pressure cylinder—				
During admission,				
$A W_a$, crank end	5.71	5.78	7.00	8.19
head end	6.61	6.37	8.42	9.50
During expansion,				
$A W_b$, crank end	10.65	10.76	10.40	10.25
head end	10.81	11.04	11.22	11.09
During exhaust,				
$A W_c$, crank end	7.73	7.89	8.44	9.02
head end	8.08	8.15	9.04	9.66
During compression,				
$A W_d$, crank end	0.48	0.60	0.49	0.50
head end	0.62	0.64	0.73	0.81
Intermediate cylinder—				
During admission,				
$A W_a'$, crank end	7.58	7.57	7.98	8.64
head end	7.43	7.55	8.46	9.10
During expansion,				
$A W_b'$, crank end	9.54	9.54	9.91	10.64
head end	9.22	9.31	10.37	11.14

TABLE V—*Continued.*

	I.	II.	III.	IV.
Intermediate cylinder—				
During exhaust,				
$A W_c'$ crank end.....	9.27	9.47	9.64	10.54
head end.....	9.27	9.47	10.18	10.84
During compression,				
$A W_d'$, crank end.....	0.39	0.43	0.57	0.46
head end.....	0.60	0.70	0.78	0.84
Low-pressure cylinder—				
During admission,				
$A W_a''$, crank end.....	7.75	7.95	8.33	8.97
head end.....	7.99	8.19	8.66	9.39
During expansion,				
$A W_b''$, crank end.....	6.83	7.10	6.86	7.45
head end.....	6.87	7.12	7.34	7.87
During exhaust,				
$A W_c''$, crank end.....	5.08	5.08	4.62	5.09
head end.....	5.08	5.16	4.81	5.00
During compression,				
$A W_d''$, crank end	0.00	0.00	0.00	0.00
head end.....	0.00	0.00	0.00	0.00
Quality of the steam in the cylinder. At admission and at compression the steam was assumed to be dry and saturated :				
High-pressure cylinder—				
At cut-off..... x_1..	0.785	0.784	0.848	0.875
At release..... x_2..	0.899	0.903	0.920	0.931
Intermediate cylinder—				
At cut-off..... x_1'..	0.899	0.912	0.906	0.908
At release ..... x_2'..	0.994	superheated.	superheated.	superheated.
Low-pressure cylinder—				
At cut-off..... x_1''.	0.978	superheated.	0.970	0.974
At release..... x_2''.	superheated.	superheated.	superheated.	superheated.
Interchanges of heat between the steam and the walls of the cylinders, in B. T. U. Quantities affected by the positive sign are absorbed by the cylinder-walls; quantities affected by the negative sign are yielded by the walls :				
High-pressure cylinder—				
Brought in by steam .. Q..	132.93	130.77	141.11	149.84
During admission..... Q_a.	23.54	23.43	17.49	14.93
During expansion..... Q_b..	− 18.69	− 19.28	− 15.33	− 14.03
During exhaust..... Q_c..	− 8.36	− 7.22	− 3 50	− 2.38
During compression... Q_d.	0.45	0.51	0.49	0.52
Supplied by jacket.... Q_j..	4.56	4.08	2.39	2.50
Lost by radiation..... Q_e..	1.50	1.52	1.54	1.54
First intermediate receiver—				
Supplied by jacket.... Q_r.	4.92	5.20	5.67	5.95
Lost by radiation. .... Q_{re}.	0.58	0.58	0.59	0.59

TABLE V—*Continued.*

	I.	II.	III.	IV.
Intermediate cylinder—				
Brought in by steam.. Q'..	131.89	129.61	137.87	146.64
During admission. ... Q_a'.	13.62	11.74	11.33	11.75
During expansion..... Q_b'.	− 18.65	− 18.84	− 20.30	− 21.88
During exhaust ... ... Q_c'.	0.22	1.57	2.88	3.41
During compression... Q_d'.	0.44	0.51	0.62	0.59
Supplied by jacket.... Q_j'.	6.82	7.50	7.97	8.64
Lost by radiation..... Q_e'.	2.45	2.48	2.50	2.51
Sec'd intermediate receiver—				
Supplied by jacket.... Q_r'.	4.20	4.04	4.27	4.22
Lost by radiation..... Q_{re}'	1.20	1.22	1.23	1.24
Low-pressure cylinder—				
Brought in by steam .. Q''.	132.14	130.50	138.61	147.33
During admission..... Q_a''	5.85	3.05	5.57	5.29
During expansion..... Q_b''	− 9.51	− 7.09	− 8.65	− 10.13
During exhaust..... Q_c''	2.53	2.23	− 1.44	− 0.11
During compression... Q_d''	0.00	0.00	0.00	0.00
Supplied by jacket .... Q_j''	7.08	6.20	7.41	7.14
Lost by radiation..... Q_e''	4.34	4.40	4.45	4.47
Total loss by radiation—				
By preliminary tests, ΣQ_e.	10.07	10.20	10.31	10.35
By equation (49).....	11.68	10.19	8.75	8.07
Power and economy:				
Heat-equivalents of works per stroke—				
H. P. cylinder..... AW..	8.44	8.34	9.17	9.52
Interm. cylinder.... AW'..	7.12	6.95	7.77	8.42
L. P. cylinder..... AW''.	9.64	10.06	10.87	11.79
Totals.....	25.20	25.35	27.81	29.73
Tot. heat furnished by jackets	27.58	27.02	27.71	28.45
Distribution of work—				
High-pressure cylinder....	1.00	1.00	1.00	1.00
Intermediate cylinder.....	0.84	0.83	0.85	0.88
Low-pressure cylinder	1.14	1.21	1.19	1.24
Horse-power.....	104.9	104.2	113.1	120.3
Steam per H. P. per hour...	14.65	14.31	13.90	13.73
B. T. U. per H. P. per minute	247	241	236	232

full boiler-pressure while the steam in the cylinder is below atmospheric pressure. The superheating was small in all cases—not more than would be accounted for by the errors of the tests. The exhaust waste Q_c'' from the low-pressure cylinder in the triple-expansion tests is very small in all cases —less than two per cent of the heat supplied to the cylinders. The apparent absurdity of a positive value for Q_c'' in two of

the tests (indicating an absorption of heat by the cylinder walls during exhaust) may properly be attributed to the unavoidable errors of the test.

In the fourth test, when the engine was developing 120.3 horse-power, there were 1305 pounds of steam supplied to the cylinders in an hour, and 345 pounds to the steam-jackets; so that the steam per horse-power per hour passing through the cylinders was

$$1305 \div 120.3 = 10.86 \text{ pounds},$$

while the condensation in the jackets was

$$345 \div 120.3 = 2.87 \text{ pounds}.$$

So that, as shown on page 245, the B. T. U. per horse-power per minute supplied to the cylinders by the entering steam was 191.1, while the jackets supplied 40.6 B. T. U., making in all 231.7 B. T. U. per horse-power per minute for the heat-consumption of the engine. In the same connection it was shown that the thermal efficiency of the engine for this test was 0.183, while the efficiency for incomplete expansion in a non-conducting cylinder corresponding to the conditions of the test was 0.222; so that the engine was running with 0.824 of the possible efficiency. In light of this satisfactory conclusion some facts with regard to the test are interesting.

It will be noted that 149.84 B. T. U. per stroke are brought in by the steam supplied to the high-pressure cylinder and that 28.45 B. T. U. per stroke are supplied by the steam-jackets; and that, further, 29.73 B. T. U. are changed into work while 10.35 are radiated. Thus it appears that the jackets furnished almost as much heat as was required to do all the work developed. Of the heat furnished by the jackets something more than a third was radiated; the other two-thirds may fairly be considered to have been changed into work, since the exhaust waste of the low-pressure cylinder was practically zero.

Table VI gives the results of tests made on a vertical

TABLE VI.

TRIPLE-EXPANSION VERTICAL PUMPING-ENGINE AT MILWAUKEE, WISCONSIN.

CYLINDER DIAMETERS, 28, 48, AND 74 INCHES; STROKE, 60 INCHES.

By Professor R. C. CARPENTER, *Trans. Am. Soc. Mech. Engrs.*, vol. XV, p. 313.

Duration, hours	24
Revolutions per minute	20.3
Pressures, pounds per square inch:	
Gauge at throttle	121.5
Vacuum	13.8
Barometric	14.5
High-pressure jackets	121.4
Intermediate and low-pressure jackets	56.5
First receiver	32.4
Second receiver	13.8
Moisture in steam, per cent	1.1
Steam used in jackets, per cent	9.3
Indicated horse-power	573.9
Pump, horse-power	521.0
Friction, per cent	9.2
Steam per horse-power per hour	11.8
B. T. U. per horse-power per minute	217.6
Hirn's analysis, interchanges of heat between the walls of the cylinder and the steam; in per cent of total heat per stroke:	
High-pressure cylinder—	
Absorbed during admission	10.5
Restored during expansion	− 7.9
Rejected during exhaust	− 4.7
Absorbed during compression	0.2
Intermediate cylinder—	
Absorbed during admission	11.8
Restored during expansion	− 6.2
Rejected during exhaust	− 7.2
Given up during compression	− 0.2
Low-pressure cylinder—	
Absorbed during admission	2.1
Restored during expansion	− 0.1
Rejected during exhaust	− 21.2
Given up during compression	− 1.2
Thermodynamic efficiency	0.194

triple-expansion pumping-engine, together with an application of Hirn's analysis, which is especially interesting as the engine was made by the builders of the experimental engines at the Massachusetts Institute of Technology, and it ran under much the same conditions. Its higher efficiency and better economy was due in part to the fact that the engine was larger and superior in certain details, but more particularly to the good vacuum attained. The principal difference in the results of the application of Hirn's analysis to the tests on the two engines is in the heat rejected by the walls during exhaust; for the experimental engine these quantities are small for all three cylinders, and in particular is practically zero for the low-pressure cylinder, while for the pumping-engine they are of considerable magnitude, and the exhaust waste for the low-pressure cylinder is large. So far as any significance can be attached to this comparison it shows that a considerable exhaust waste is not incompatible with a high economy.

Alternate Method for Compound Engines.—When the method of applying Hirn's analysis to a compound engine, which has just been explained and exemplified, can be used it is most satisfactory, as the comparison of the observed and calculated radiation losses gives an excellent check on the accuracy of the test in general. If, however, it should be impossible in any case to properly account for the heat added to or lost by the steam in the intermediate receiver, we may still make an application of Hirn's analysis by the following method.

There is no difficulty in making the application to the high-pressure cylinder by the method already explained. The trouble comes in the determination of the heat Q' supplied by the steam which enters the low-pressure cylinder. But by applying equation (292) to the low-pressure cylinder we have

$$Q_e' = Q' + Q_j' - Mq_4 - G(q_k - q_i) - A(W_a + W_b - W_c - W_d), \quad (297)$$

and consequently

$$Q' = Q_e' - Q_j' + Mq_4 + G(q_k - q_i) + A(W_a + W_b - W_c - W_d), \quad (298)$$

by aid of which equation Q' may be eliminated from the equation for Q_a', the heat absorbed by the wall of the low-pressure cylinder during admission. Therefore we have for the equations for a compound engine

$$Q_a = Q + I_0 - I_1 - AW_a; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (299)$$

$$Q_b = I_1 - I_2 - AW_b; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (300)$$

$$Q_c = I_2 - I_3 - Q - Q_j + Q_e + A(W + W_c); \quad . \quad (301)$$

$$Q_d = I_3 - I_0 + AW_d; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (302)$$

$$Q_a' = Q_e' - Q_j' + Mq_4 + G(q_k - q_i) + I_0 - I_1 + A(W_b - W_c - W_d); \quad . \quad . \quad . \quad (303)$$

$$Q_b' = I_1' - I_2' - AW_b'; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (304)$$

$$Q_c' = I_2' - I_3' - Mq_4 - G(q_k - q_i) + AW_c'; \quad . \quad (305)$$

$$Q_d' = I_3' - I_0' + AW_d'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (306)$$

Maire's Tests.—In Table VII is given a large number of tests on important engines of various types, together with the application of Hirn's analysis. Some of the engines tested, such as the Cornish engine and the Bull engine, are out of date, and the interest in some others is impaired by the low pressure of steam in the boiler. It is also unfortunate that Maire did not separate the interchanges of heat during the admission and expansion in the low-pressure cylinder, nor does his report allow us to make the separation. We have consequently for the low-pressure cylinder only the exhaust waste Q_c and the exchange Q_d for compression. Nevertheless the table is instructive and important.

The dimensions of the engines and the conditions of the tests were as follows:

The steam-consumption was determined by measuring the feed-water supplied to the boiler. The steam condensed in the steam-jackets was collected and weighed separately. The air-pump discharge was allowed to flow through an orifice under a measured head; the coefficient of discharge for the orifice was determined by direct experiments. The per cent of priming in the steam was determined by calorimetric tests.

The test 1 was made on a single-cylinder rotative pumping-engine, having a diameter of 45 inches and a stroke of 5 feet 6 inches. The sides and ends of the cylinder were jacketed with boiler steam. The steam was distributed by separate slide-valves near the ends of the cylinder, with expansion-plates adjustable by hand, on the backs of the main valves.

The tests 2, 3, and 4 were made on a Woolf beam-engine driving a flour-mill. The cylinders were $24\frac{1}{2}$ inches in diameter by 3 feet 5 inches stroke, and 38 inches diameter by 5 feet 6 inches stroke, and were steam-jacketed on the sides only. The steam was distributed to the high-pressure cylinder by a slide-valve with cut-off plates on the back, and to the low-pressure cylinder by a piston-valve, all being worked by eccentrics.

The tests 5 and 6 were made on an unjacketed horizontal Woolf engine, of a type very commonly used in factories in Lancashire. The cylinders were $15\frac{3}{4}$ and $28\frac{1}{2}$ inches in diameter by 4 feet 3 inches stroke. The piston speed was about 680 feet per minute and the load was light, so that the steam was much wire-drawn.

The test 7 was made on a compound beam receiver-engine with the cranks at right angles, working pumps directly from the beams. The cylinders were 21 and 36 inches in diameter, and the stroke was 5 feet 6 inches. Both cylinders, with the exception of the high-pressure cylinder and the receiver-covers, were jacketed with boiler steam. The steam was

TABLE VII.

APPLICATION OF HIRN'S ANALYSIS TO SIMPLE AND COMPOUND ENGINES.

By GEORGE MAIRE.

Proc. of the Inst. of Civ. Engrs., vols. LXX and LXXIX.

	Type of Engine.	Barometer, inches of Mercury.	Pressures in lbs. on the sq. inch, absolute.		Revolutions per minute.	Total number of expansions.	Percentage of priming in steam.	Percentage of steam passed through jackets.	Horse-power, Indicated.	Percentage of water in cylinder.			Steam per h. p. per hour, pounds.	B. T. U. per horse-power, per min.	Maximum theoretical efficiency.
			In boiler.	In steam-jackets.						At cut-off, h. p. cylinder.	At end of stroke, h. p. cylinder.	At end of stroke, l. p. cylinder.			
1	Single-cylinder beam.....	29.6	48	....	14.6	6.8	4	4	101.8	44	28	....	23.0	....	0.281
2	Compound Woolf beam	30.2	84.8	....	33.7	9.6	2½	7.1	267.9	37	24	17	18.4	....	0.340
3		29.8	88.1	...	34.2	9.5	2½	8.3	267.9	37	21	14	18.1	....	0.339
4		29.7	88.9	....	34.5	7.7	2½	....	267.9	34	28	35	20.1	....	0.341
5	Compound horizontal tandem.	30.6	86.0	....	80.4	11.5	4	....	120.3	42	25	26	21.9	387	0.281
6		30.6	84.8	....	81.5	11.6	4	...	149.7	43	25	26	21.5	352	0.268
7	Compound receiver beam	30.2	76	....	24	13.6	4	13.2	127.4	34	22	20	15.7	281	0.305
8	Cornish	30.0	46.4	. ..	12.8	....	3	4.5	146.0	....	....	...	25.0	448	0.257
9	Single-cylinder beam engine.	29.9	56.7	56.6	20.3	4.3	1	4.9	123.4	29	23	...	22.5	409	0.265
10		29.9	55.8	55.8	20.3	3.8	1	....	123.6	37	27	....	26.7	489	0.251
11	Single-cylinder beam engine.	29.9	59.4	59.4	20	3.1	1	4.9	119.5	22	17	. ..	21.7	396	0.268
12		29.7	59.3	59.3	20	1.9	1	4 4	122.4	15	14	....	24.0	440	0.267
13	Bull engine, double stroke	30.3	55.4	55.4	12.3	1.7	1	4.1	174.6	16	14	23	32.4	565	0.233
14	Woolf beam engine....	30.2	67.8	67.8	17.8	11.9	3	13.4	213	25	13	1	16.5	288	0.253
15		30.1	67.2	67.1	17.8	10.2	3	11.0	214.5	24	13	2	17.4	306	0.250
16	Woolf beam engine...	30.4	78.0	78.0	28.3	16.5	1	11.3	132.8	31	13	15	16.1	294	0.303
17		30.4	76.0	76.0	28.2	13.3	1	9.7	171.9	29	16	17	15.8	289	0.295
18		30.1	75.0	116.0	27.5	13.1	1	10.2	171.2	29	15	12	15.6	285	0.296

TABLE VII—*Continued.*

	Type of Engine.	Actual efficiency.	Ratio of actual to theoretic efficiency.	Interchange of heat between the steam and the walls of the cylinders in per cent of total heat supplied to cylinders.										
				Absorbed during admission, h. p. cylinder. Q_a	Restored during expansion, h. p. cylinder. Q_b	Rejected during exhaust, h. p. cylinder. Q_c	Absorbed during compression, h. p. cylinder. Q_d	Rejected during exhaust, l. p. cylinder. $Q_{c'}$	Absorbed during compression, l. p. cylinder. $Q_{d'}$	Radiation, h. p. cylinder. Q_e	Radiation, l. p. cylinder. $Q_{e'}$	Supplied by jacket, h. p. cylinder. Q_j	Supplied by jacket, l. p. cylinder. $Q_{j'}$	Error.
1	Single-cylinder beam.....	0.099	0.35	33.8	16.4	19.8	— 0.04	....	....	1.0	....	3.4	....	....
2	Compound Woolf beam	0.122	0.36	26.2	12.1	18.3	2.6	11.5	— 0.2	0.5	0.7	2.2	3.3	....
3		0.125	0.37	25.4	14.5	16.1	3.3	9.6	— 0.1	0.5	0.7	2.5	3.7	....
4		0.113	0.33	25.2	6.4	20.2	2.0	22.4	— 0.2	0.5	0.7	...	...	....
5	Compound horizontal tandem.	0.101	0.36	35.6	19.3	20.0	4.0	17.2	— 0.7	0.4	0.6	....	....	2.7
6		0.104	0.39	36.1	20.1	19.3	3.7	16.4	— 0.4	0.4	0.7	....	...	3.9
7	Compound receiver beam	0.147	0.48	19.8	13.0	15.1	5.0	11.8	— 4.5	0.9	2.1	4.3	6.4	3.7
8	Cornish	0.091	0.35	....	....	....	....	....	....	....	...	....	....	....
9	Single-cylinder beam engine.	0.099	0.37	23.2	8.8	18.0	0.3	....	....	0.7	....	4.0	....	....
10		0.083	0.33	30.8	11.0	20.3	0.8	....	....	0.6	...	....	....	....
11	Single-cylinder beam engine.	0.102	0.38	17.0	7.2	13.1	0.14	....	....	0.6	....	4.0	....	...
12		0.093	0.35	10.9	2.9	10.9	0.04	....	....	0.5	....	3.6	....	....
13	Bull engine, double stroke	0.068	0.30	13.6	3.9	12.9	2.0	20.7	— 0.3	0.5	0.5	1.8	1.7	0.6
14	Woolf beam engine ..	0.139	0.55	15.8	12.8	10.2	2.5	0.4	1.1	0.8	1.4	5.6	5.7	2.5
15		0.132	0.53	13.5	12.6	9.1	5.1	0.2	4.8	0.8	1.3	3.8	5.3	3.2
16	Woolf beam engine....	0.140	0.46	22.2	18.7	10.3	4.2	12.2	0.9	0.6	1.1	3.7	5.5	3.7
17		0.142	0.48	21.8	14.6	12.1	2.6	12.8	0.2	0.5	0.8	2.8	5.1	3.9
18		0.144	0.48	20.9	15.0	11.3	2.7	8.5	0.03	0.5	0.8	3.1	4.9	1.1

distributed by slides, one at each end of each of the cylinders, with cut-off plates adjustable by hand.

The test 8 was made on a Cornish engine working a single-acting piston-pump direct from the beam, and having the usual Cornish, double-beat, steam, equilibrium, and exhaust-valves, a single-acting air-pump, and a jet-condenser; the cylinder was 68¼ inches in diameter by 8 feet stroke, and was jacketed on the sides with boiler steam. The pump delivered its water on the up or steam stroke, so that the preponderance of weight on the pump-pole was only enough to overcome the suction lift. The valves and piston were inspected to assure their tightness before the test. The engine was doing the highest duty at the West Middlesex Waterworks, and was taken as one of the best engines of its type now working. This type of engine was developed in Cornwall, where it was used to pump water from deep mines by a pump-rod hung directly from one end of the beam while the piston was hung from the other end of the beam. It had no fly-wheel, but the pump-rod, beam, and counter-weights made in the aggregate a large reciprocating mass, that absorbed work during the first part of the stroke when the steam-pressure in the cylinder was high, and restored that work and assisted the steam to complete the stroke, after it had lost pressure through expansion, during the latter part of the stroke. Such a reciprocating mass is essential to the proper action of the engine with expansion. The pumps of the original engines were worked by the weight of the rods during the return or equilibrium stroke, at which time there was free communication between the two ends of the cylinder. The lower end of the cylinder was open to the condenser during the steam-stroke.

The tests 9 and 10 were made on a single-cylinder beam rotative pumping-engine, having a diameter of 32 inches and a stroke of 5 feet 6 inches. The cylinder sides and base were jacketed with boiler steam. Steam was distributed by slide-

valves at the top and bottom of the cylinder, with cut-off plates, adjustable by hand, on the backs of the main valves.

The tests 11 and 12 were made on a single-cylinder beam rotative engine, similar to the one just described, and taking steam from the same boilers. The cylinder was 27 inches in diameter by 6 feet stroke.

The test 13 was made on a Bull engine with a cylinder 68 inches in diameter by 10 feet stroke, driving direct a 45-inch plunger-pump, and forcing water to a height of 40 to 55 feet. The valves and gear were of the usual Cornish pattern, and the sides and base of the cylinder were steam-jacketed. This type of engine differs from the Cornish engine in not having a beam, and though the pump-rod is loaded there is seldom sufficient reciprocating mass to allow of much expansion. In the case of the engine tested only $1\frac{3}{4}$ expansions could be obtained. For convenience, the steam-stroke is detailed under the heading of the high-pressure cylinder, and the exhaust-stroke under the heading of the low-pressure cylinder.

The tests 14 and 15 were made on a Woolf beam rotative engine, working a double-acting pump. The cylinders were 29 inches diameter by 5 feet 5 inches stroke, and $47\frac{1}{2}$ inches diameter by 8 feet stroke, and jacketed with steam on the sides and ends.

The most interesting tests for our present purpose are 2, 3, and 4, since they show how a compound engine is affected by steam-jackets. It appears that in this case the interchanges of heat for the high-pressure cylinder are not much affected, though there is some increase in the heat restored during expansion, and the exhaust waste is less when steam is used in the jackets. But the exhaust waste for the low-pressure cylinder is very much reduced, and may in a large measure account for the gain in economy.

The comparison of test 7 with tests 14 and 15 is instructive, for the latter show practically no exhaust waste from the low-pressure cylinder, while the former has a considerable exhaust waste; nevertheless the economy for test 7 is notably

better than for tests 14 and 15. The advantage is probably due to the higher steam-pressure and larger number of expansions enjoyed by the engine represented by test 7. This is only one more example to show that while the exhaust waste is always a direct loss it may be unadvisable to try to reduce it to zero by the lavish application of steam-jackets or by other ways of supplying heat to steam on its way through a compound or multiple-expansion engine. It will be shown later that it is possible to overdo the application of heat in this way and so injure the economy of the engine; the best result appears to be attained by a judicious or a fortunate compromise of the gain from expansion and the loss from condensation and evaporation, together with the amelioration of the latter by the use of steam-jackets and intermediate reheaters.

Quality of Steam at Compression.—In all the work of this chapter the steam in the cylinder at compression has been considered to be dry and saturated, and it has been asserted that little if any error can arise from this assumption. It is clear that some justification for such an assumption is needed, for a relatively large weight of water in the cylinder would occupy a small volume and might well be found adhering to the cylinder walls in the form of a film or in drops; such a weight of water would entirely change our calculations of the interchanges of heat. The only valid objection to Hirn's analysis is directed against the assumption of dry steam at release. Indeed, when the analysis was first presented some critics asserted that the assumption of a proper amount of water in the cylinder is all that is required to reduce the calculated interchanges of heat to zero. It is not difficult to refute such an assertion from almost any set of analyses, but unfortunately such a refutation cannot be made to show conclusively that there is little or no water in the cylinder at compression; in every case it will show only that there must be a considerable interchange of heat.

For the several tests on the Hirn engine given in Table

III, Hallauer determined the amount of moisture in the steam in the exhaust-pipe, and found it to vary from 3 to 10 per cent. Professor Carpenter* says that the steam exhausted from the high-pressure cylinder of a compound engine showed 12 to 14 per cent of moisture. Numerous tests made in the laboratory of the Massachusetts Institute of Technology show there is never a large percentage of water in exhaust-steam. Finally, such a conclusion is evident from ordinary observation. Starting from this fact and assuming that the steam in the cylinder at release is at least as dry as the steam in the exhaust-pipe, we are easily led to the conclusion that our assumption of dry steam is proper. Professor Carpenter reports also that a calorimeter test of steam drawn from the cylinder during compression showed little or no moisture. Nevertheless there would still remain some doubt whether the assumption of dry steam at compression is really justified, were we not so fortunate as to have direct experimental knowledge of the fluctuations of temperature in the cylinder walls.

Dr. Hall's Investigations.—For the purpose of studying the temperatures of the cylinder-walls Dr. E. H. Hall used a thermo-electric couple, represented by Fig. 74. *I* is a cast-

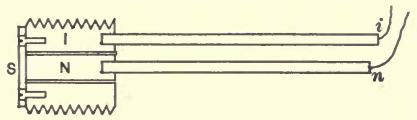

FIG. 74.

iron plug about three-quarters of an inch in diameter, which could be screwed into the hole provided for attaching an indicator-cock to the cylinder of a steam-engine. The inner end of the plug carried a thin cast-iron disk, which was assumed to act as a part of the cylinder-wall when the plug was in place. To study the temperature of the outside surface of the disk a nickel rod *N* was soldered to it, making a

* *Trans. Am. Soc. Mech. Engrs.*, vol. XII, p. 811.

thermo-electric couple. Wires from I and N led to another couple made by soldering together cast-iron and nickel, and this second couple was placed in a bath of paraffine which could be maintained at any desired temperature. In the electric circuit formed by the wires joining the two thermo-electric couples there was placed a galvanometer and a circuit-breaker. The circuit-breaker was closed by a cam on the crank-shaft, which could be set to act at any point of the revolution. If the temperature of the outside of the disk S differed from the temperature of the paraffine bath at the instant when contact was made by the cam, a current passed through the wires and was indicated by the galvanometer. By properly regulating the temperature of the bath, the current could be reduced and made to cease, and then a thermometer in the bath gave the temperature at the surface of the disk for the instant when the cam closed the electric circuit. Two points in the steam-cycle were chosen for investigation, one immediately after cut-off and the other immediately after compression, since they gave the means of investigating the heat absorbed during compression and admission of steam, and the heat given up during expansion and exhaust.

Three different disks were used: the first one half a millimetre thick, the second one millimetre thick, and a third two millimetres thick. From the fluctuations of temperature at these distances from the inside surface of the wall some idea could be obtained concerning the variations of temperature at the inner surface of the cylinder, and also how far the heating and cooling of the walls extended.

The account given here is intended only to show the general idea of the method, and does not adequately indicate the labor and difficulties of the investigation which involved many secondary investigations, such as the determination of the conductivity of nickel. Having shown conclusively that there is an energetic action of the walls of the cylinder, Dr. Hall was unable to continue his investigations.

Callendar and Nicolson's Investigations.—A very refined and complete investigation of the temperature of the cylinder walls and also of the steam in the cylinder was made by Messrs. Callendar and Nicholson * in 1895 at the McGill University, by the thermoelectric method.

The wall temperatures were determined by a thermoelectric couple of which the cylinder itself was one element and a wrought-iron wire was the other element. To make such a couple, the cylinder-wall was drilled nearly through, and the wire was soldered to the bottom of the hole. Eight such couples were established in the cylinder-head, the thickness of the unbroken wall varying from 0.01 of an inch to 0.64 of an inch. Four pairs of couples were established along the cylinder-barrel, one near the head, and the others at 4 inches, 6 inches, and 12 inches from the head. One of each pair of wall couples was bored to within 0.04 of an inch, and the other to 0.5 of an inch of the inside surface of the cylinder. Other couples were established along the side of the cylinder to study the flow of heat from the head toward the crank end.

The engine used for the investigations was a high-speed engine, with a balanced slide-valve controlled by a fly-wheel governor. During the investigations the cut-off was set at a fixed point and the speed was controlled externally. By the addition of a sufficient amount of lap to prevent the valve from taking steam at the crank end the engine was made single-acting. The normal speed of the engine was 250 revolutions per minute, but during the investigations the speed was from 40 to 90 revolutions per minute. The diameter of the cylinder was 10.5 inches and the stroke of the piston was 12 inches. The clearance was ten per cent of the piston displacement.

The indicator-diagram for a cut-off at one-fifth stroke, at which most of the investigations were made, is given by Fig. 75. From the indicated pressure the temperatures of

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* *Proceedings of the Inst. Civ. Engrs.*, vol. CXXXII.

the steam at various piston positions could be readily determined by aid of a table of the properties of saturated steam.

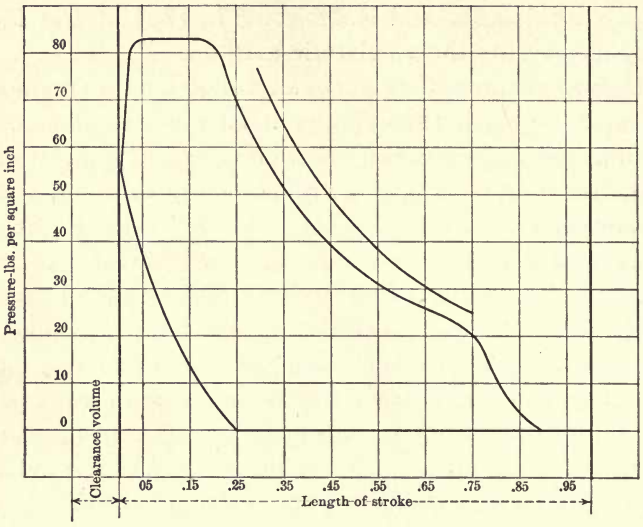

FIG. 75.

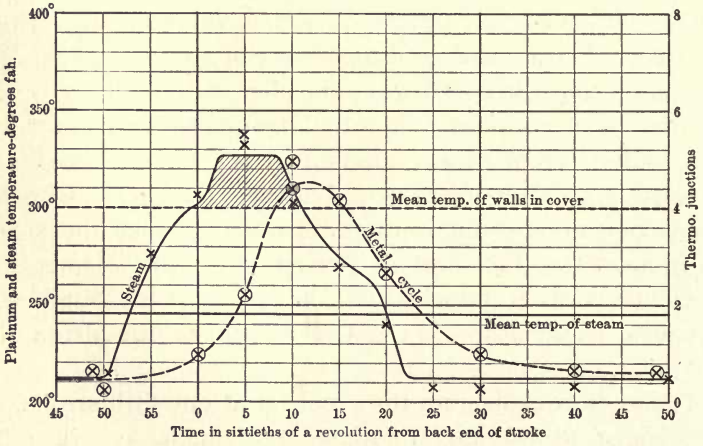

FIG. 76.

In Fig. 76 the indicated steam temperatures determined from Fig. 75 are plotted as ordinates, with sixtieths of a revolution

for abscissæ. The zero of abscissæ is at the beginning of the forward stroke and the rise of temperature corresponds with the admission of steam shown on Fig. 75. The temperature of course remains constant for the part of the revolution which corresponds to the steam line of the indicator. There is then a gradual fall of temperature corresponding to the expansion, and a rapid drop corresponding to release. After release the back-pressure temperature is constant till the beginning of compression, and during compression there is a rapid rise of temperature corresponding to the increase of pressure. The diagram Fig. 73 begins at $\frac{45}{60}$ of the revolution in order that the most interesting events shall be recorded in the middle of the diagram. The crosses in the diagram show the temperature of the steam as determined by the platinum thermometer in the piston of the engine. The divergency between the crosses and the full line representing indicated temperatures is considered by the investigators to be larger than the combined errors of the indicator and platinum thermometer. But at atmospheric pressure there are three degrees increase of temperature for each pound increase of pressure, and even at 70 pounds pressure there is one degree for a pound increase of pressure. Taking into consideration also the fact that the discrepancy is always on the side where the friction and lag of the indicator tend to place it, the discrepancy does not appear to require further explanation, unless it may be for the steam line; it is of course possible that there may be superheated steam in the middle of the cylinder during the admission of steam, together with a film of water on the cooler walls of the cylinder.

The mean temperature of the cylinder-head is represented by a dotted line just above 300° F. The mean temperature of the steam is shown by a full line at 247° F. The dotted line lettered *Metal cycle* will be explained later.

Fig. 77 gives a similar diagram of indicated steam temperatures for another test, together with a diagram of temperatures by a platinum thermometer near the surface of the cylinder

head. During all of the exhaust the steam near the cylinder head is strongly superheated, and compression carries the superheating far beyond the temperature of saturated steam during the admission. As steam comes into the cylinder the temperature near the cylinder-head falls rapidly till it passes below that of saturated steam just before cut-off. Though the temperature shown by the platinum thermometer continues to fall after cut-off it soon rises beyond that shown by

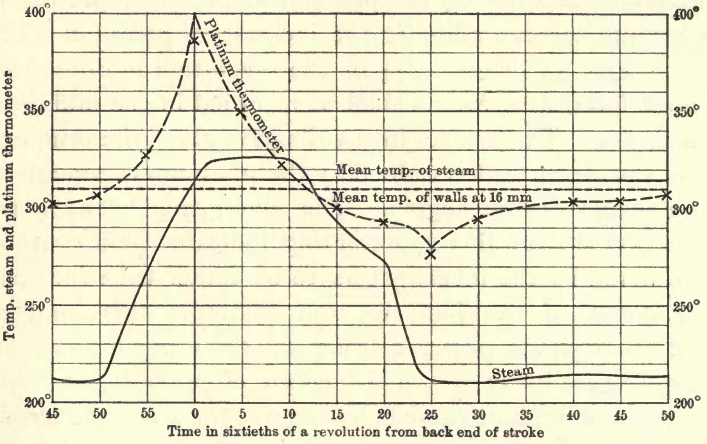

FIG. 77.

the indicator. This is considered by the investigators to show that the evaporation from the cylinder-head is probably complete soon after cut-off. Certainly the superheating during compression disposes of any assumption that there is any considerable amount of water in the cylinder at the beginning of compression.

Fig. 78 gives a similar diagram of indicated steam temperatures, and also a diagram, represented by the dotted curve, of the changes of temperature near the inner surface of the cylinder. The scale for the full line representing indicated pressures is given at the left, the scale for the changes of the temperature of the wall is given at the right. The thermoelectric couple by which the wall temperatures were deter-

mined was in the cylinder-head, and had a thickness of 0.01 of an inch of unperforated wall between it and the inner surface.

A comparison of the temperatures of the cylinder-head and of the side wall near the head is very instructive. In the first place the mean temperature of the side was 14°.3 lower than that of the wall; and secondly, the temperature of the side varied more. Thus the range of variation for the head was 4°.9, while that for the side near the head was 13°.5. The general character of the cycle of temperature

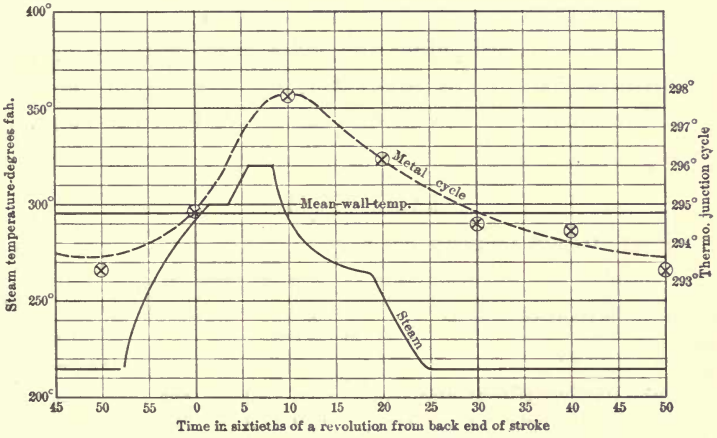

FIG. 78.

changes for the side was similar to that for the head. The mean temperature of the cylinder-wall decreased from the head toward the crank end, as might be expected for a single-acting engine; and there was a considerable flow of heat in the same direction. A double-acting engine cylinder would differ in this respect, as the flow of heat would be from both ends to the middle, provided there were any perceptible flow. The temperature cycles for different points along the side of the cylinder were in general like the cycle for the end near the head; the greatest difference for a point beyond cut-off being due to its sudden exposure to steam when it was un-

covered by the piston. Of course some difference in the cycle was produced by the fall of the mean wall temperature from the head toward the crank.

The eight thermo-electric couples in the head, with thicknesses of unbroken wall varying from 0.01 of an inch to 0.64 of an inch afforded means of determining the law of the flow of heat to and from the cylinder-wall. These and the pairs of couples along the side of the cylinder gave the means of calculating the heat absorbed by and restored by the cylinder. The methods of reducing the observations and making calculations are too intricate to be given here; it will suffice to give some of the results.

The following table gives the areas, temperatures, and the heat absorbed during a given test by the various surfaces exposed to steam at the end of the stroke, i.e., the clearance surface.

TABLE VIII.

CYCLICAL HEAT-ABSORPTION FOR CLEARANCE SURFACES.

Portions of surface considered.	Area of surface, square feet.	Mean temperature, F.	Heat absorbed B. T. U. per minute.
Cover face, 10.5 inches diameter ...	0.60	305	68
Cover side, 3.0 inches	0.70	305	79
Piston face, 10.5 inches diameter ..	0.60	295	110
Piston side, 0.5 inch	0.11	295	20
Barrel side, 3.0 inches.....	0.71	297	123
Counter bore, 0.5 inch	0.12	291	28
Ports and valves	0.90	305	102
Sums and means.....	3.74	301	530

The heat absorbed by the side of the cylinder-wall uncovered by the piston up to 0.25 of the stroke was estimated to be 55 B. T. U. per minute, which added to the above sum gives 585 B. T. U.; from which it appears that 90 per cent of the condensation is chargeable to the clearance surfaces. Further inspection shows that the condensation on the piston and the barrel is much more energetic than on the cover or

head. For example, the face of the piston absorbs 110 B. T. U. while the face of the cover absorbs only 68 B. T. U.; and the side of the cover and of the barrel, each 3 inches long, absorb 79 and 123 B. T. U., respectively. This relatively small action of the surfaces of the head indicates that less gain is to be anticipated from the application of a steam-jacket to the head than to the barrel of a steam-engine; tests on engines confirm this conclusion.

The exposed surfaces at the side of the cylinder-head and the corresponding side of the barrel are due to the use of a deeply cored head which protrudes three inches into the counterbore of the cylinder, and which has the steam-tight joint at the flange of the head. It would appear from this that a notable reduction of condensation could be obtained by the simple expedient of making a thin cylinder-head.

The final results of the investigations and the comparison of the condensation due to the heat absorbed by the walls of the cylinder are given in Table IX. Considering the intricacy and difficulty of the investigations the comparison of indicated and calculated condensations and evaporations must be con-

TABLE IX.

INFLUENCE OF THE WALLS OF THE CYLINDER.

CALLENDAR AND NICHOLSON, *Proc. Inst. Civ. Engrs.*, 1897.

	I.	II.	III.	IV.	V.	VI.	VII.
Duration, minutes	37	68	55	79	76	35	25
Revolutions per minute.....	43.8	45.7	47.7	70.4	73.4	81.7	97.0
Mean gauge-pressure.....	87.9	89.2	94.4	98.1	92.0	94.2	96.0
Gross steam per revolution.....	0.1422	0.1437	0.1483	0.1094	0.1036	0.1000	0.0856
Leakage correction	0.1004	0.0976	0.0990	0.0697	0.0627	0.0576	0.0494
Net steam per revolution.....	0.0418	0.0461	0.0493	0.0397	0.0409	0.0424	0.0362
Steam caught at compression.....	0.0107	0.0104	0.0103	0.0099	0.0098	0.0100	0.0105
Weight of mixture in cylinder ...	0.0525	0.0565	0.0596	0.0496	0.0507	0.0524	0.0467
Indicated steam at quarter stroke.	0.0407	0.0414	0.0437	0.0418	0.0394	0.0408	0.0393
Indicated steam at release	0.0466	0.0456	0.0488	0.0460	0.0436	0.0454	0.0426
Increase of indicated weight.....	0.0059	0.0042	0.0051	0.0042	0.0042	0.0046	0.0033
Adiabatic condensation..... ..	0.0019	0.0020	0.0021	0.0020	0.0019	0.0020	0.0019
Indicated evaporation	0.0078	0.0062	0.0072	0.0062	0.0061	0.0066	0.0052
Calculated evaporation.....	0.0076	0.0073	0.0072	0.0048	0.0046	0.0041	0.0035
Indicated condensation.....	0.0118	0.0151	0.0159	0.0078	0.0113	0.0116	0.0074
Calculated condensation..... . .	0.0148	0.0142	0.0136	0.0092	0.0089	0.0080	0.0067
Indicated horse-power.....	4.10	4.34	4.78	7.02	6.67	7.71	8.81
Steam per H.P. per hour, pounds.	26.8	29.1	29.5	23.8	27.1	26.9	23.8

ceded to be very satisfactory, and it must be admitted that the interchanges of heat are mainly, if not entirely, due to the metal of the cylinder-wall and not to water which remains in the cylinder from one stroke to the next.

Leakage of Valves.—Preliminary tests when the engine was at rest showed that the valve and piston were tight. The valve was further tested by running it by an electric motor when the piston was blocked, the stroke of the valve being regulated so that it did not quite open the port, whereupon it appeared that there was a perceptible but not an important leak past the valve into the cylinder. There was also found to be a small leakage past the piston from the head to the crank end.

But the most unexpected result was the large amount of leakage past the valve from the steam-chest into the exhaust. This was determined by blocking up the ports with lead and running the valve in the normal manner by an electric motor. This leakage appeared to be proportional to the difference of pressure causing the leak, and to be independent of the number of reciprocations of the valve per minute. From the tests thus made on the leakage to the exhaust, the leakage correction in Table IX was estimated. Although the investigators concluded that their experimental rate of leakage was quite definite, it would appear that much of the discrepancy between the indicated and calculated condensation and vaporization can be attributed to this correction, which was two or three times as large as the weight of steam passing through the cylinder. Under the most favorable condition (for the seventh test) the leakage was 0.0494 of a pound per stroke, and since there were 97 strokes per minute, it amounted to

$$0.0494 \times 97 \times 60 = 287.5$$

pounds per hour, or 32.6 pounds per horse-power per hour, so that the steam supplied per horse-power per hour amounted to 56.4 pounds. If it be assumed that the horse-power is proportional to the number of revolutions, then the engine

running double-acting will develop about 44 horse-power per hour, and the leakage then would be reduced to 6.5 pounds per horse-power per hour. Such a leakage would have the effect of increasing the steam-consumption from 23.5 to 30 pounds of steam per horse-power per hour.

To substantiate the conclusions just given concerning the leakage to the exhaust, the investigators made similar tests on the leakage of the valves of a quadruple-expansion engine, which had plain unbalanced slide-valves. The valves chosen were the largest and smallest; both were in good condition, the largest being absolutely tight when at rest. Allowing for the size and form of the valve and for the pressure, substantially identical results were obtained.

The following provisional equation is proposed for calculating the leakage to the exhaust for slide-valves:

$$\text{leakage} = \frac{klp}{e}, \quad . \quad . \quad . \quad . \quad . \quad (307)$$

where l is the lap and e is the perimeter of the valve, both in inches, and p is the pressure in pounds in the steam-chest in excess of the exhaust pressure. The valve of the constant in equation (307) is 0.021 for the high-speed engine used by Callendar and Nicolson, and is 0.019 for one test each of the valves for the quadruple engine, while another test on the large valve gave 0.021.

This matter of the leakage to the exhaust is worthy of further investigation. Should it be found to apply in general to slide-valve and piston-valve engines it would go far towards explaining the superior economy of engines with separate admission- and exhaust-valves, and especially of engines with automatic drop-cut-off valves which are practically at rest when closed. It may be remarked that the excessive leakage for the engine tested appears to be due to the size and form of valves. The valve was large so as to give a good port-opening when the cut-off was shortened by the fly-wheel governor, and was faced off on both sides so that it could slide

between the valve-seat and a massive cover-plate. The cover-plate was recessed opposite the steam-ports, and the valve was constructed so as to admit steam at both faces; from one the steam passed directly into the cylinder and from the other it passed into the cover-plate and thence into the steam-port. This type of valve has long been used on the Porter-Allen and the Straight-line engines; the former, however, has separate steam- and exhaust-valves. Such a valve has a very long perimeter which accounts for the very large effect of the leakage.

Messrs. Callendar and Nicolson consider that the leakage is probably in the form of water which is formed by condensation of steam on the surface of the valve-seat uncovered by the valve, and say further, that it is modified by the condition of lubrication of the valve-seat as oil hinders the leakage.

CHAPTER XV.

ECONOMY OF STEAM-ENGINES.

THE importance and the intricacy of the action of the walls of the cylinder of a steam-engine have thus far prevented any formulation of the actual economy of steam-engines. It therefore is necessary to make a study of steam-engine tests to learn the conditions favorable to economy and to determine the effects of various devices and methods employed for the purpose of improving economy.

Table X gives the economy of various types of engines, and represents the present state of the art of steam-engine construction. It must be considered that in general the various engines for which results are given in the table were carefully brought up to their best performance when these tests were made. In ordinary service these engines under favorable conditions may consume five or ten per cent more steam or heat; under unfavorable conditions the consumption may be half again or twice as much.

All the examples in the table are taken from reliable tests: a few of these tests are stated at length in the chapter on the influence of the cylinder-walls; others are taken from various series of tests which will be quoted in connection with the discussion of the effects of such conditions as steam-jacketing and compounding; the remaining tests will be given here, together with some description of the engines on which the tests were made.

The first engine named in the table is at the Chestnut Hill pumping-station for the city of Boston. Its performance is

TABLE X.

EXAMPLES OF STEAM-ENGINE ECONOMY.

Type of Engine.	Revolutions per minute.	Steam-pressure, pounds per square inch.	Horse-power.	Steam per horse-power per hour. Pounds	B. T. U. per horse-power per min.	Coal per horse-power per hour. pounds.
Triple-expansion engines:						
Leavitt pumping-engine at Chestnut Hill	50.6	176	576	11.2	204	1.15
Sulzer mill-engine at Augsburg	56	149	1823	11.3	....	1.19
Allis pumping-engine at Milwaukee	20.3	122	574	11.8	218	1.25
Willans engine	384	170	30	12.7	....	....
Experimental engine at the Massachusetts Institute of Technolgy	92	147	125	13.7	231	....
Marine engine *Iona*	61	165	645	13.4	....	1.46
do. *Meteor*	72	145	1994	15.0	....	2.01
do. *Brookline*	94	154	1136	15.5	263	2
Compound engines:						
Leavitt pumping-engine at Louisville	18.6	137	643	12.2	222	....
Leavitt pumping-engine at Boston main-drainage pumping-station	13.2	99	252	13.9	....	1.33
Mill-engine at Natick	76	159	595	13	....	....
Portable compound engine	89	92	78	16.9	....	....
Marine engine *Rush*	71	69	266	18.4	....	2.45
do. *Fusi Yama*	56	57	371	21.2	....	2.66
Simple engines, condensing:						
Corliss engine at Creusot	60	84	176	16.9	....	....
do. without jacket	59	61	150	18.1	....	....
Harris-Corliss engine at Cincinnati	76	96	145	19.4	....	....
Marine engine *Gallatin*	51	72	180	20.5	....	....
do. without jacket	60	69	259	21.9	....	....
Simple engines, non-condensing:						
Corliss engine at Creusot	61	104	178	22.1	....	....
do. without jacket	61	78	209	24.2	....	....
Harris-Corliss engine at Cincinnati	76	96	120	23.9	....	....
Hoadley portable engine	126	120	80	25.6	....	3.35
Harris-Corliss engine at the Massachusetts Institute of Technology	61	77	16	33.5	548	....
Direct-acting steam-pumps:						
Fire-pump at the Massachusetts Institute of Technology	*90	47	41	67	1110	....
do. at reduced power	*50	59	6.8	125	2070	....
Steam- and feed-pump on the *Minneapolis*	*11	....	8.8	91	....	....
do. at reduced power	*2.6	....	1.6	243	....	....

* Strokes per minute.

the best known to the writer for engines using saturated steam. Some engines using superheated steam have a less steam-consumption than this engine, but their economy is not given here for reasons which will appear in the discussion of the use of superheated steam. This engine, which was designed by Mr. E. D. Leavitt, has three vertical cylinders with their pistons connected to cranks at 120°. Each cylinder has four gridiron valves, each valve being actuated by its own cam on a common cam-shaft; the cut-off for the high-pressure cylinder is controlled by a governor. Steam-jackets are applied to the heads and barrels of each cylinder, and tubular reheaters are placed between the cylinders. Steam at boiler-pressure is supplied to all the jackets and to the tubular reheaters.

TABLE XI.

TRIPLE-EXPANSION LEAVITT PUMPING-ENGINE AT THE CHESTNUT HILL STATION, BOSTON, MASSACHUSETTS.

CYLINDER DIAMETERS 13.7, 24.375, AND 39 INCHES; STROKE 6 FEET.

By Professor E. F. MILLER, *Technology Quarterly*, vol. ix, p. 72.

Duration, hours	24
Total expansion	21
Revolutions per minute	50.6
Steam-pressure above atmosphere, pounds per square inch	175.7
Barometer, pounds per square inch	14.9
Vacuum in condenser, inches of mercury	27.25
Pressure in high and intermediate jacket and reheaters, pounds per square inch	175.7
Pressure in low-pressure jacket, pounds per square inch	99.6
Horse-power	575.7
Steam per horse-power per minute, pounds	11.2
Thermal units per horse-power per minute	204.3
Thermal efficiency of engine, per cent	20.8
Efficiency for non-conducting engine, per cent	28.0
Ratio of efficiencies, per cent	74
Coal per horse-power per hour, pounds	1.146
Duty per 1,000,000 B. T. U.	141,855,000
Efficiency of mechanism, per cent	89.5

The Sulzer engine at Augsburg has four cylinders in all, a high-pressure, an intermediate, and two low-pressure cylin-

ders. The high-pressure cylinder and one low-pressure cylinder are in line, with their pistons on one continuous rod, and the intermediate cylinder is arranged in a similar way with the other low-pressure cylinder. The engine has two cranks at right angles, between which is the fly-wheel, grooved for rope-driving. Each cylinder has four double-acting poppet-valves, actuated by eccentrics, links, and levers from a valve-shaft. The admission-valves are controlled by the governors. Four tests were made on this engine, as recorded in Table XII.

TABLE XII.

TRIPLE-EXPANSION HORIZONTAL MILL-ENGINE.

CYLINDER DIAMETERS 29.9, 44.5, AND TWO OF 51.6 INCHES; STROKE 78.7 INCHES.

Built by SULZER of Winterthur, *Zeitschrift des Vereins Deutscher Ingenieure*, vol. xl, p. 534.

	I	II	III	IV
Duration, minutes	306	322	272	327
Revolutions per minute	56.23	56.28	56.18	56.18
Steam-pressure, pounds per square inch	145.4	147.9	148.4	149.0
Vacuum, inches of mercury	27.24	27.20	27.20	27.19
Horse-power	1872	1835	1850	1823
Steam per horse-power per hour, pounds	11.53	11.49	11.49	11.33
Mean for four tests..... 11.46.....				
Coal per horse-power her hour, pounds.	1.37	1.36	1.29	1.19
Mean for four tests.....1.30.....				
Steam per pound of coal	8.78	8.49	8.97	9.62

In this connection there is given in Table XIII the details of five tests made on an engine by the same builders, but of smaller size and having but three cylinders. The high-pressure and intermediate pistons are on one continuous piston-rod and the low-pressure piston is on a separate rod. The engine has two cranks at right angles and a fly-wheel between them.

The details of the test on the Milwaukee pumping-engine are given in Table VI on page 332. The engine has three

vertical cylinders with their pistons connected to cranks at 120°. The valve-gear is of the Corliss type, controlled by a governor.

TABLE XIII.

TRIPLE-EXPANSION HORIZONTAL MILL-ENGINE AT AUGSBURG.

CYLINDER DIAMETERS 11.28, 17.75, AND 27.61 INCHES; STROKE 39.37 INCHES.

By Professor M. SCHRÖTER, *Zeitschrift des Vereins Deutscher Ingenieure*, vol. XXXIV, p. 7.

	I	II	III	IV	V
Duration, minutes	324	306	330	301	326
Revolutions per minute	70.5	70.2	70.3	76.3	70.4
Cut-off high-pressure cylinder	0.259	0.252	0.259	0.308	0.300
Total expansion	25.6	26.3	25.6	21.5	22.1
Steam-pressure, pounds per square inch above atmosphere	145.6	146.4	147.6	150.6	143.2
Barometer, pounds per square inch	13.9	13.9	13.8	13.8	13.9
Back-pressure, absolute, pounds per square inch	1.1	1.2	1.3	1.2	1.3
Condensation in jackets in per cent of total steam-consumption	16.1	20.0	18.3	17.0	18.1
Condensation withdrawn from first intermediate receiver, per cent	2.7	2.9	2.7	3.0	2.2
Horse-power	195.0	198.1	198.1	222.5	212.8
Steam per horse-power per hour, pounds	12.61	12.20	12.92	12.72	12.94
B. T. U. per horse-power per minute	226	221	230	227	230

The test on the Willans engine is taken from Table XLII on page 406. This remarkable engine, though of small size and power, gives an exceedingly good economy and is adapted to driving high-speed machinery directly. The three pistons are arranged on one rod and are single-acting. The under sides of the high-pressure and the intermediate pistons form the tops of the intermediate receiver-spaces, so that in a manner the expansion has five stages.

The test on the experimental engine at the Massachusetts Institute of Technology is quoted here because its efficiency and economy are chosen for discussion in Chapter XI. Taking its performance as a basis, it appears on page 248 that

with 150 pounds boiler-pressure and 1.5 pounds absolute back-pressure an engine may be expected to give a horse-power for 11.5 pounds of steam, from which it appears that under its conditions its performance compares favorably with the Sulzer engine or even the Leavitt engine.

TABLE XIV.

MARINE-ENGINE TRIALS.

By Professor ALEXANDER B. W. KENNEDY, *Proc. Inst. Mech. Engs.*, 1889–1892; summary by Professor H. T. BEARE, 1894, p. 33.

	Fusi Yama.	Colchester.	Ville de Douvres.	Meteor.	Iona.
Triple or compound	C.	C.	C.	T.	T.
Diameter high-pressure cylinder, inches	27.4	30	50.1	29.4	21.9
Diameter intermediate cylinder, inches	....	....	.....	44	34
Diameter low-pressure cylinder, inches	50.3	57	97.1	70.1	57
Stroke, inches	33	36	72	48	39
Duration of trial, hours	14	10.9	9	17	16
Number of expansions	6.1	6.1	5.7	10.6	19.0
Revolutions per minute	55.6	86	36	71.8	61.1
Steam-pressure above atmosphere, pounds per square inch	56.8	80.5	105.8	145.2	165
Pressure in condenser, absolute, pounds per sqare inch	2.32	2.51	4.72	2.73	0.70
Back-pressure, absolute, pounds per sq. in	3.8	3.4	6.0	3.3	1.8
Horse-power	371	1022	2977	1994	645
Steam per horse-power per hour, pounds	21.2	21.7	20.8	15.0	13.4
Thermal units per horse-power per minute	380	398	367	265	250
Coal per horse-power per hour, pounds	2.66	2.9	2.3	2.01	1.46
Steam evaporated per pound of coal	7.96	7.49	8.97	7.46	9.15
Weight of machinery per horse-power, pounds.	603	448	272	439	701

The engines of the S. S. *Iona* have an unusually large expansion and give a correspondingly good economy. The engines of the *Meteor* and of the *Brookline* give the usual economy to be expected from medium-sized marine engines. Table XIII gives details of tests on the engines of the two first ships mentioned, together with tests on compound marine engines. Table XV gives tests on the engine of the *Brookline*. It appears probable that the relatively poor

economy of marine engines compared with stationary engines is due to the smaller degree of expansion, which is accepted to avoid using large and heavy engines.

TABLE XV.

TESTS ON THE ENGINE OF THE S. S. *BROOKLINE.*

CYLINDER DIAMETERS 23, 35, AND 57 INCHES; STROKE 36 INCHES.

By F. T. MILLER and R. G. B. SHERIDAN, Thesis, 1895, M.I.T.

	I	II	III	IV	V
Duration, hours	2	2	1	$3\frac{1}{2}$	$2\frac{1}{2}$
Revolutions per minute	94.6	93.6	93.6	93	93
Steam-pressure, pounds per square inch above atmosphere	155	155	154	145	148
Vacuum, inches of mercury	21.6	21.0	22.2	21.7	20.9
Horse-power	1242	1221	1136	1137	1148
Steam per horse-power per hour, pounds	17.2	16.9	15.5	17.0	16.3
Coal per horse-power per hour, pounds	2.22	2.17	1.99	2.18	2.09
B. T. U. per horse-power per minute	292	288	263	288	277

TABLE XVI.

COMPOUND LEAVITT PUMPING-ENGINE AT LOUISVILLE, KENTUCKY.

CYLINDER DIAMETERS 27.2 AND 54.1 INCHES; STROKE 10 FEET.

By F. W. DEAN, *Trans. Am. Soc. Mech. Engs.*, vol. xvi, p. 169.

Duration, hours	144
Revolutions per minute	18.6
Pressure, pounds per square inch:	
Barometric	14.6
Boiler above atmosphere	140
At engine above atmosphere	137
Back-pressure, l. p. cylinder	0.95
Total expansions	20
Moisture in steam, per cent	0.55
Horse-power	643.4
Steam per horse-power per hour, pounds	12.2
B. T. U. per horse-power per minute	222
Thermodynamic efficiency, per cent	19
Mechanical efficiency, per cent	93

The best performance of a compound engine using saturated steam, which is known to the writer, is that reported for a pumping-engine at Louisville, Ky. This engine has

two cylinders, each jacketed with steam at boiler-pressure on barrels and heads and steam at the same pressure is used in a tubular reheater. Each cylinder has four gridiron valves actuated by as many cams on a cam-shaft. The details of the test are given in Table XVI, and Table XVII gives two tests on another compound Leavitt engine at the Boston main-drainage pumping-station. A comparison of the two tables shows the advance made in ten years from 1885 to 1895.

TABLE XVII.

LEAVITT COMPOUND PUMPING ENGINE AT THE BOSTON MAIN-DRAINAGE WORKS.

CYLINDER DIAMETERS 25.5 AND 52 INCHES; STROKE 9 FEET.

Duration, hours and minutes	24–43	24–3½
Revolutions per minute	13.17	13.42
Boiler-pressure above atmosphere, pounds per square inch	99.4	98.6
Vacuum, inches of mercury	28.1	28.0
Barometer, inches of mercury	30.18	29.81
Horse-power	251.5	290.2
Steam per horse-power per hour, pounds	13.9	14.2
Coal per horse-power per hour, pounds	1.33	1.35
Steam per pound of coal, from and at 212° F	12.12	11.83
Duty, work per 100 pounds of coal, millions of foot-pounds	122.5	122.4

TABLE XVIII.

CROSS-COMPOUND MILL-ENGINE AT NATICK, R. I.

CYLINDER DIAMETERS 18.4 AND 48.5 INCHES; STROKE 4 FEET.

By F. W. DEAN, *Trans. Am. Soc. Mech. Engs.*, vol. xvi, p. 179.

	I	II
Duration, hours	4.5	5
Revolutions per minute	76.4	76.6
Steam-pressure above atmosphere, pounds per square inch	159	158
Vacuum, inches of mercury	25.4	25.2
Total expansions	33	33.4
Moisture in steam, per cent	1.9	1.8
Horse-power	595	582
Steam per horse-power per hour, pounds	13	13.2

The compound mill-engine at Natick, R. I., has two cylinders with their pistons acting on cranks at right angles.

The valves are of the gridiron type as made by the Wheelock Engine Company. The high-pressure cylinder is jacketed on the heads and barrel; the low-pressure cylinder is jacketed on the heads only. A reheater is placed between the cylinders. A notable feature of this engine is the ratio of the volume of the cylinder,

$$\overline{48.5}^2 : \overline{18.4}^2 :: 7 : 1,$$

which gives a large number for the total expansion without requiring a very early cut-off for the high-pressure cylinder.

Table XIX gives tests on another cross-compound engine which has a cylinder ratio of three and a half to one, and which has neither steam-jackets nor reheater, and which has nevertheless nearly as good an economy.

TABLE XIX.

CROSS-COMPOUND MILL-ENGINE AT NEW BEDFORD, MASS.

CYLINDER DIAMETERS 30 AND 56 INCHES ; STROKE 72 INCHES.

By DENTON, JACOBUS, and RICE, *Trans. Am. Soc. Mech. Engs.*, vol. xv, p. 882.

Revolutions per minute	65.2
Steam-pressure above atmosphere, pounds per square inch	123.0
Vacuum, inches of mercury	25.6
Total expansions	13.4
Superheating at throttle, degrees F.	14.6
Horse-power	1592
Steam per horse-power per hour, pounds	13.5
Coal per horse-power per hour, pounds	1½
B. T. U. per horse-power per minute	247

Table XX gives tests on an engine built by the same makers as that for which tests are recorded in Table XVIII. This engine has three cylinders with a ratio of seven to one for the volumes of the largest and smallest cylinders. In two of the tests the engine was run as a compound engine using only the smallest and largest cylinders; in the other two tests the three cylinders were used as designed. The engine has jackets on the heads and barrels of the high-pressure and

intermediate cylinders and on the heads of the low-pressure cylinder. A further consideration of these tests will come in connection with the discussion of compound and triple-expansion engines.

TABLE XX.

HORIZONTAL MILL-ENGINE AT HOLYOKE, MASS.

DIMENSIONS: HIGH-PRESSURE CYLINDER, 12 IN. DIAMETER, 36 IN. STROKE; INTERMEDIATE CYLINDER, 16 IN. DIAMETER, 36 IN. STROKE; LOW-PRESSURE CYLINDER, $24\frac{13}{32}$ IN. DIAMETER, 48 IN. STROKE.

Tested by S. M. GREEN and G. I. ROCKWOOD, *Trans. Am. Soc. Mech. Engs.*, vol. xiii, p. 647.

The engine was run with large and small cylinders only, and with all three.

	I	II	III	IV
	Compound.		Triple.	
Duration, hours	5	5	5	5
Revolutions per minute	79.2	79.3	79.0	79.0
Steam-pressure above atmosphere, pounds per square inch	142	142	142	143
Per cent of steam used in jackets	13	14	16	16
Horse-power	187.1	180.7	199.1	178.2
Steam per horse-power per hour, pounds.	13.4	13.1	13.0	13.3

Table XXI gives four tests on a small portable engine which gives a very good economy for its type and size.

The details of the tests on the *Rush* and the *Fusi Yama* are given in Table XXXII and XIV on pages 256 and 358.

A remarkably complete and important series of tests was made in 1884 by M. F. Delafond. These tests are recorded in Tables XXX and XXXI, from which there are quoted in Table X four results with and without condensation and with and without steam in the jackets.

The details of the tests on the Harris-Corliss engine at Cincinnati, together with tests on two similar engines, are given in Table XXII.

TABLE XXI.

PORTABLE COMPOUND CONDENSING-ENGINE.

Tested by a Committee of the Soc. Ind. DE MULHOUSE, 1879.

Reported by ISHERWOOD, *Jour. Franklin Inst.*, vol. CXX.

	I	II	III	IV
Duration, hours	3	4	3	3.24
Revolutions per minute	89	88.5	90	88.7
Cut-off, small cylinder	0.42	0.42	0.25	0.42
Total expansions	6.26	6.26	9.64	6.26
Steam-pressure above the atmosphere, lbs. per sq. in.	91.8	91.5	91.7	92.2
Atmospheric pressure, lbs. per sq. in.	14.2	14.2	14.2	14.2
Back-pressure, lbs. per sq. in.	2 4	2.4	2.4	2.4
Horse-power, indicated	77.2	77.6	63.9	77.8
Horse-power by brake	67.7	67.5	55.7	67.5
Steam per I.H.P. per hour lbs.	17.7	17.3	16.9	16.9
B. T. U. per horse-power per hour	318	323	318	315

TABLE XXII.

AUTOMATIC CUT-OFF ENGINES.

CYLINDER DIAMETERS 18 INCHES; STROKE 4 FEET.

By J. W. HILL.

(First Millers' International Exhibition, Cincinnati, 1880.)

	Condensing.			Non-condensing.		
	R.	H.	W.	R.	H.	W.
Duration	10	10	10	9	10	10
Cut-off	0.124	0.119	0.131	0.160	0.136	0.170
Revolutions per minute	75.4	75.8	74.5	75.3	75.8	76.1
Boiler-pressure above atmos., lbs. per sq. in.	95.8	96.1	96.3	96.6	96.3	96.3
Barometer, inches of mercury	29.7	29.6	29.4	29.8	29.6	29.5
Vacuum, inches of mercury	25.5	25.7	24.0			
Back-pressure, absolute, lbs. per sq. in.	4.5	3.4	4.7	15.5	14.9	15.5
Horse-power	143.2	145.1	143.9	121.7	119.7	126.7
Steam per horse-power per hour, pounds	20.6	19.4	19.5	25.9	23.9	24.9
B. T. U. per horse-power per hour	372	349	343	433	400	415

The tests on the engine of the *Gallatin* are taken from Table XXXII on page 388.

Table XXII*a* gives the details of a test on a Hoadley portable engine which had a piston-valve controlled by a fly-wheel governor, made by Mr. Hoadley in 1876.

TABLE XXII*a*.

HOADLEY PORTABLE ENGINE.

CYLINDER DIAMETER 14.56 INCHES; STROKE 28.5 INCHES.

Duration of test	6 hrs. 2 min.
Revolutions per minute	125.96
Steam-pressure in boiler, pounds	120
Cut-off, per cent	17
Horse-power by indicator	80.29
Horse-power by brake	72.72
Friction of engine, horse-power	7.57
Horse-power by indicator without load	5.80
Steam per I.H.P. per hour, weighed, pounds	25.61
" " brake H.P. per hour, weighed, pounds	28.27
Coal per indicated horse-power per hour, pounds	3.35
" " brake horse-power per hour, pounds	3.69

TABLE XXIII.

DUPLEX DIRECT-ACTING FIRE-PUMP AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

TWO STEAM-CYLINDERS 16 INCHES DIAMETER, 12 INCHES STROKE.

Technology Quarterly, vol. viii, p. 19.

Single strokes per minute.	Length of stroke. West.	Length of stroke. East.	Steam-pressure by gauge.	Horse-power. Steam-cylinders.	Horse-power. Water-cylinders.	Steam per horse-power per hr.	B. T. U. per horse-power per min.	Duty. (Foot-pounds per 1,000,000 B. T. U.)
99	11.40	10.10	58.5	6.78	.....	125	2070	13,920,000
114	11.70	11.07	55.6	12.48	.....	101	1674	17,540,000
119	11.49	11.07	51.4	12.18	.....	109	1809	16,980,000
135	11.60	11.10	53.8	18.24	.....	92	1530	19,850,000
156	10.90	10.26	47.2	21.00	19.80	98	1619	18,280,000
193	10.09	10.31	45.6	32.95	.....	78	1291	23,730,000
175	11.77	11.79	45.6	39.55	.....	66	1083	27,980,000
180	11.74	11.66	46.5	41.20	.....	67	1110	27,030,000

The test of the small Harris-Corliss engine at the Massachusetts Institute of Technology is taken from Table II on page 318. A complete calculation for the application of Hirn's analysis to this test is given on page 313.

The two tests on the direct-acting fire-pump at the Massachusetts Institute of Technology are taken from Table XXIII, and the tests on the feed- and fire-pump on the *Minneapolis* are given in Table XXIV. Both sets of tests show the extravagant consumption of steam by such pumps when running at reduced powers. The latter table is most interesting on account of the light that it throws on the way that coal is consumed by a war-vessel when cruising at slow speeds or lying in harbor.

TABLE XXIV.

TESTS OF AUXILIARY STEAM MACHINERY OF THE U. S. S. *MINNEAPOLIS.*

By P. A. Engineer W. W. WHITE, U. S. N., *Journal Am. Soc. Naval Engs.*, vol. x.

Engine or pump tested.	Number of steam-cylinders.	Diam. of steam-cylinders, inches	Diam. of water-cylinders, inches.	Nominal stroke, inches.	Actual stroke, inches.	Double strokes or revolutions per minute.	Duration of test.	Indicated horse-power.	Steam per horse-power per hour.
Centre circulating-pump:									
Full power	2	10	36	6	6	171.6	3–7	18.9	55
Reduced power*	2	10	36	6	6	90	2–50	4.1	76
Starboard circulating-pump:									
Reduced power	2	10	36	6	6	82	3–28	2.0	125
Starboard air-pump	2	16	31.5	21	....	16.6	2–58	6.5	183
Centre air-pump†	2	16	31.5	21	....	15.2	3–2	25.2	78
Water service pump	2	7.5	4.5	10	7.5	40.9	2–59	1.04	205
Fire- and feed-pump	2	12	8.5	12	10.9	12.7	3–31	0.78	319
do.	2	12	8.5	12	12.0	37.3	1–46	6.4	156
do.	2	12	7.5	12	10	11.0	2–23	8.8	91
do.	2	12	7.5	12	10.8	2.6	3–27	1.6	243
Fire- and bilge-pump	2	14	9	12	11.2	27.7	2–2	2.5	171
Blower-engine	2	5	....	4	4	595	1–24	16.3	77
Dynamo-engine	2	10.5	....	5	5	425	1–10	22.9	65
do.	2	10.5	....	5	5	425	0–26	35.2	56
Ice-machine engine	1	7	....	10	10	73.1	5–12	6.0	70

* One cylinder only supplied with steam.

† Pump loaded with three times the power developed during official trial, when main engine indicated 7219 H. P.

Effect of Raising Steam-pressure.—A study of the examples of steam-engine economy given in Table X, and of the details of the tests given in the several tables referred to, shows that in general a gain in economy is to be obtained by increasing the steam-pressure and the total number of expan-

sions. This general statement can be held to be true only when the proper methods are used to ameliorate the effects of cylinder-condensation and reëvaporation, which have already been seen to place a very strict limit on the number of expansions to be profitably used for a simple steam-engine.

It is interesting here to investigate the effect of increasing the steam-pressure only, for a non-conducting engine, without changing the expansion. The accompanying table gives the efficiencies for such an engine for various steam-pressures calculated by equation (260), page 248, assuming in all cases that the absolute pressure at release is one-third that during admission.

TABLE XXV.

	I	II	III	IV
Pressure, pounds per square inch:				
By gauge during admission	30.3	60.3	90.3	120.3
Absolute during admission	45	75	105	135
At end of expansion	15	25	35	45
Back-pressure	14.7	14.7	14.7	14.7
Quality of steam at release	0.948	0.936	0.934	0.933
Efficiency	0.085	0.111	0.123	0.133
B. T. U. per horse-power per minute	499	382	345	319
Steam per horse-power per hour, pounds	30.4	22.6	20.7	19.0

From this calculation it appears that there is at first a notable gain in economy from increasing the steam-pressure without changing the expansion for a non-conducting engine, but that after a moderate pressure has been reached the gain for further increase is small. Now the actual engine will have a greater steam-consumption on account of the loss by external radiation, and the loss due to initial condensation, reëvaporation, and exhaust-waste. The absolute loss from external radiation increases with the steam-pressure, but the horse-power of the engine increases more rapidly; consequently the radiation per horse-power decreases as the pressure is increased. Since the changes of temperature are larger for a high than for a low pressure, it is a reason-

able inference that the condensation and reëvaporation in the cylinder, and the exhaust-waste are liable to increase as the steam-pressure is raised. We may, therefore, expect to find that tests on steam-engines will give a result similar to that calculated for a non-conducting engine, i.e., that with fixed expansion there will be at first a considerable gain from raising the steam-pressure in a simple engine, but that beyond a moderate steam-pressure the gain will be very slow. Some tests even show a loss from raising the steam-pressure too high.

A confirmation of the ideas just enunciated may be found in Table XXXVIII, page 403, which gives the results of tests on the Willans engine, using one cylinder only as a single-acting simple engine. Tests 1 to 6 were made with the cut-off at half stroke and with the steam-pressure varying from 64.7 to 20 pounds absolute, the back-pressure being in all about one pound. Increasing the pressure from 20 to 55 pounds absolute, or from about 5 to about 40 pounds above the atmosphere, gave a gain of about ten per cent, but a further increase to 64.7 pounds absolute, or about 50 pounds by the gauge, gave a distinct loss. A similar though not so pronounced effect is shown by tests 7 to 12.

A more striking confirmation is given by tests on a Corliss engine at Creusot without steam in the jackets, as represented by Tables XXX and XXXI on pages 454 and 455 and by Fig. 79. In the figure the curves lettered *C* represent tests made with a vacuum and curves lettered *N* represent tests without a vacuum. The figures used in conjunction with the curves give the approximate boiler-pressure for tests represented. The curves representing tests with condensation show a decided gain when the steam-pressure is raised from 35 pounds to 50 pounds above the atmosphere, and an appreciable gain for a further increase from 50 to 60 pounds. A further increase to 80 and 100 pounds is accompanied by a distinct loss. The tests without condensation show a large gain from raising the pressure from 50 to 75 pounds

and little if any gain from a further increase to 100 pounds above the atmosphere.

Methods of Improving Economy.—Bearing in mind the fact just established that there is little if any gain in economy to be obtained by raising the steam-pressure beyond a certain very moderate limit unless the expansion may be increased at the same time, and also the fact that the economical expansion for a simple unjacketed steam-engine is very strictly limited by initial condensation, reëvaporation, and exhaust-waste, we are led to the consideration of methods for ameliorating the action of the walls of the cylinder. Two of these have been considered in connection with the study of the influence of the cylinder-walls in the preceding chapter, namely, the use of superheated steam and the use of the steam-jacket. Another method of even more practical importance is the use of compound and multiple-expansion engines. Again, reheaters may be placed between the cylinders of compound or multiple-expansion engines to dry or superheat the steam on its way from one cylinder to another. Any two or all of these several methods may be used in conjunction. Examples of almost all of the possible combinations may be found in practice, and the advantages to be obtained from some of them can be determined from tests which will now be studied in detail.

Superheated Steam.—The most direct and effective way of improving steam-engine economy is by the use of superheated steam, and yet no permanently good results have been obtained, on account of the practical difficulties of maintaining the superheating apparatus. Some upright boilers give superheated steam, but when the superheating is forced to any very considerable degree they are likely to be troubled by wasting of the upper ends of the tubes. Whenever superheated steam has been used so as to give a notable gain in economy the superheating has been accomplished in a separate apparatus, which has taken the form of a coil of pipe exposed to the products of combustion beyond the boiler. Now it is

the accepted experience of boiler-makers that surfaces exposed to the fire must be of moderate thickness or they will rapidly waste away. Thus it is not desirable to make furnace-flues more than half an inch thick, and if they are made thicker they are liable to waste away till they are reduced to about that thickness. Plates and tubes if thin enough endure long service in a boiler when exposed to the fire because they are kept at a moderate temperature by the water in the boiler. If steam is to be superheated strongly in a coil of pipe or other device which is exposed to hot gases, the metal of the superheater must be strongly heated and is sure to waste away rapidly. There is no material that can stand long service when exposed at once to a high pressure and a high temperature. There is little risk, therefore, in predicting that all superheating devices now used will eventually be discarded for this reason.

From the considerations just stated no tests with superheated steam are given in Table X, though the economy obtained by the use of superheated steam is remarkable, both comparatively and absolutely. Several series of tests on engines using superheated steam will be given, together with a discussion of the advantages obtained from its use.

Tests on the Eutaw.—The U. S. S. *Eutaw* was built for special service in 1863–64, and had a single-cylinder inclined engine with poppet-valves and a Stevens adjustable cut-off. The steam from the boilers could be supplied directly to the engine or it could be passed through a coil superheater in the uptake. The tests were made at the dock, the speed of rotation being controlled by removing more or less of the paddles from the paddle-wheels. The details of the tests are given in Table XXVI. Four of the tests are with saturated steam and five are with superheated steam. The best result with saturated steam is obtained for a cut-off at 0.32 of the stroke, agreeing with the conclusions from tests on the *Michigan* (Table I, page 303). The tests with superheated steam show a good deal of irregularity, which cannot be satisfactorily

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accounted for either by the degree of superheating or the perfection of the vacuum. A comparison of the best result for superheated steam with the best result for saturated steam shows a gain in steam economy of

$$\frac{30.6 - 25.1}{30.6} \times 100 = 18 \text{ per cent},$$

and a gain in coal-consumption of

$$\frac{2.84 - 2.42}{2.84} \times 100 = 15 \text{ per cent}.$$

TABLE XXVI.

TESTS ON THE ENGINE OF THE U. S. S. *EUTAW*.

CYLINDER DIAMETER 4 FEET 10 INCHES; STROKE 8 FEET 9 INCHES.

By Chief Engineer ISHERWOOD, *Researches in Experimental Steam Engineering*.

	Saturated steam.				Superheated steam.				
	I	II	III	IV	V	VI	VII	VIII	IX
Duration, hours	72	72	72	72	72	72	72	72	72
Cut-off	0.24	0.32	0.50	0.58	0.29	0.32	0.50	0.50	0.58
Revolutions per minute	5.49	6.58	8.60	9.19	6.48	6.55	9.00	9.15	9.46
Initial pressure in the cylinder per square inch, absolute	40.3	40.6	40.9	38.2	41.0	41.1	41.7	41.0	41.2
Barometer, inches of mercury	30	30	30	29.7	30.1	30	30	30.1	29.9
Vacuum, inches of mercury	27	27.6	27.1	28	28	26.2	26.2	28.5	28.3
Back-pressure, pounds per square inch, absolute	1.8	1.3	1.6	0.9	1.1	2.5	1.9	0.9	0.8
Temp. of superheated steam	...	...	....	...	396	366	358	394	392
Horse-power	154.4	228.6	372.5	414.9	207.3	218.2	391.0	406.8	453.2
Steam per horse-power per hour, pounds	39.6	30.6	32.7	31.4	30.1	29.2	27.8	25.1	27.1
Combustible per horse-power per hour, pounds	3.77	2.84	2.87	2.90	2.84	2.64	2.55	2.42	2.72
Per cent of water in cylinder at release	56	37	30	24	43	37	18	21	17

There is a very considerable reduction of the water in the cylinder at release, which explains sufficiently the reason for this notable gain in economy.

Dixwell's Tests.—A small Harris-Corliss engine was fitted up for making tests on superheated steam at the Massachusetts Institute of Technology by Mr. George B. Dixwell. Six tests with superheated and saturated steam were made on this engine in 1877 in the presence of a board

of United States naval engineers. The steam was generated in cylindrical tubular boilers and was superheated in a vertical boiler with a detached brick furnace. Three different points of cut-off were tried for each condition of steam; the cut-off was controlled by the governor in the usual way when the cut-off was less than half stroke; when it was more than half stroke the valve was not released by the drop cut-off mechanism and the cut-off was produced by the lap of the valve as for a plain slide-valve.

TABLE XXVII.

DIXWELL'S TESTS ON SUPERHEATED STEAM.

CYLINDER DIAMETER 8 INCHES; STROKE 2 FEET.

Proceedings of the Society of Arts, Mass. Inst. Tech., 1887–88.

	Saturated steam.			Superheated steam.		
	I	II	III	IV	V	VI
Duration, minutes	127	83	63	180	108	75
Cut-off	0.217	0.443	0.689	0.218	0.439	0.672
Revolutions per minute	61.5	60.4	58.0	61.0	61.4	59.5
Boiler-pressure above atmosphere, pounds per square inch	50.4	50.2	50.3	50.4	50.0	50.2
Back-pressure, absolute, pounds per sq in.	15.4	15.7	15.8	15.2	15.4	15.5
Temperatures Fahrenheit:						
Near engine	302	303	303	478	441	406
In cylinder by pyrometer	278–297	279–296	282–300	313	316	315
Per cent of water in cylinder:						
At cut-off	52.2	35.9	27.9	27.4	13.6	8.9
At end of stroke	32.4	29.3	23.9	18.3	13.6	11.5
Horse-power	7.65	12.7	15.68	6.83	12.37	15.63
Steam per horse-power per hour, pounds	48.2	42.2	45.3	35.2	31.7	35.8
B. T. U. per horse-power per minute	796	696	747	631	546	621

A metallic thermometer or pyrometer was placed in a recess in the head of the cylinder. When saturated steam was used this pyrometer showed a large fluctuation, but when superheated steam was used its needle or indicator was at rest. Even if a part of the apparent change of temperature with saturated steam is attributed to the vibration of the needle and the multiplying mechanism, it is very clear that the use of superheated steam reduces the change of temperature of the cylinder-head in a remarkable manner. The effect of super-

heating on the action of the cylinder-walls is also indicated by the per cent of water in the cylinder at cut-off and release.

The apparent gain by comparing the amounts of steam used per horse-power per hour in favor of superheated steam is

$$\frac{42.2 - 31.7}{42.2} \times 100 = 25 \text{ per cent};$$

but this result is of course illusive, since the superheating required additional coal. As the coal-consumption was not determined, we must compare instead the B. T. U. per horse-power per minute, giving a real gain of

$$\frac{696 - 546}{696} \times 100 = 19 \text{ per cent.}$$

Superheated Steam in Triple Engines.—Recent tests have been made in Germany on compound and triple-expansion engines using superheated steam with extraordinary results. In Table XXVIII are given the results of tests on a triple-expansion engine at Augsburg. This engine has four cylinders, a high-pressure, an intermediate, and two low-pressure cylinders. The high-pressure cylinder and one low-pressure cylinder are in line on one side of the fly-wheel, and the intermediate cylinder and the other low-pressure cylinder are in line on the other side. The two cranks are at right angles. Each cylinder is jacketed with steam on the barrel only and has four double-acting poppet-valves. The cylinders are cast with double walls, so that there is no chance for undetected leakage from the jackets.

The steam is superheated in a coil beyond the boiler, and the gases afterwards pass through a feed-water heater or economizer, by which they are reduced to a comparatively low temperature. The coal used appears to have been of poor quality, so that while the steam- and heat-consumptions are good the coal used per horse-power per hour is large for such an engine. Comparing the best steam-consumption for

superheated and for saturated steam, the gain from superheating appears to be

$$\frac{13.2 - 12}{13.2} \times 100 = 9 \text{ per cent.}$$

The real gain, found from a comparison of the thermal units per horse-power per minute, is

$$\frac{235 - 228}{235} \times 100 = 3 \text{ per cent.}$$

TABLE XXVIII.

TRIPLE-EXPANSION HORIZONTAL MILL-ENGINE AT AUGSBURG WITH SUPERHEATED STEAM.

CYLINDER DIAMETERS 27.56, 43.31, AND TWO OF 59.06 INCHES; STROKE 63 INCHES.

By Professor M. SCHRÖTER, *Zeitschrift des Vereines Deutscher Ingenieure*, vol. xl, p. 249.

	Superheated.			Saturated.		
	I	II	III	IV	V	VI
Horse-power	1191	1167	1028	1201	977	993
Duration, minutes	416	419	415	406	416	420
Revolutions per minute	60.23	60.10	60.24	60.47	60.02	60.05
Cut-off high-pressure cylinder	39.9	39.4	32.7	38.6	28.6	28.0
Total expansions	13.6	13.1	16.2	14.4	18.9	19.3
Steam-pressure at boiler, pounds per sq. in. above atmosphere	89.7	89.4	89.2	90.7	89.4	88.9
Barometer, pounds per sq. in.	13.9	13.8	13.8	13.8	13.8	13.8
Back-pressure, absolute, pounds per sq. in.	2.0	1.9	1.7	1.9	1.7	1.6
Condensation in jackets, per cent of total steam	6.28	5.18	7.28	9.18	10.57	12.40
Temperatures Fahrenheit:						
At boiler, superheated	433	441	448	.....	.....	.....
" " saturated	330.1	329.9	329.7	330.8	329.9	329.5
At engine, superheated	414.7	418.1	420.1	.....	.....	.....
" " saturated	.....	.....	.....	325.7	326.8	327.6
Hot gases before economizer	498	527	516	.....	.....	.....
" " after economizer	288	316	300	.....	.....	.....
Steam per horse-power per hour, pounds	13.0	12.7	12.0	14.3	13.5	13.2
Coal per horse-power per hour, pounds	3.03	3.08	2.85	3.56	3.65	3.39
Pounds of water evaporated per pound of coal from and at 212° F.	4.49	4.29	4.41	3.96	4.00	3.86
B. T. U. per horse-power per min.	2.44	237	2.28	2.54	2.41	2.35

The coal used is of such inferior quality, and the consumption per horse-power per hour is so irregular that it cannot be used as a useful basis of comparison.

Schmidt's System.—Table XXIX gives a résumé of tests on various steam-engines using superheated steam according to a system devised by Wilhelm Schmidt. His system consists in part in the arrangement of the boiler for generating the steam and in part in the construction of the engine. The engine and boiler on which the tests I to IV were made may be taken as a type of the system.

The boiler is vertical, ten feet six inches high and five feet six inches in diameter. It has a corrugated fire-box and combustion-chamber about seven feet six inches high, from which the products of combustion pass through a flue 22 inches in diameter to the superheater. The upper part of the furnace or combustion-chamber is crossed by two circulation-tubes, each about 16 inches in diameter. The fire-box just above the grate and the flue at the top of the combustion-chamber are lined with fire-brick. The superheater is a continuous coil of pipe, 2.4 inches in diameter, arranged in twelve flat coils with five turns in each. Above the superheater is a feed-water heater, which is also a continuous coil of pipe, the diameter being 1.5 of an inch; it has four flat coils of five turns each. The central space inside the coils of the superheater and feed-water heater is occupied by a closed cast-iron tube 22 inches in diameter. This arrangement of boiler superheater and feed-water heater permits of a high degree of superheating together with a fair evaporative efficiency, as the gases from the superheater are cooled to about 400° F. by the feed-water heater. The superheater, like all coil superheaters, is subject to rapid wasting, for the temperature of the steam inside is about 600° F., while the gases outside are about 1200° F. In this case two such boilers are used to supply an engine which develops one hundred horse-power.

The engine has two single-acting horizontal high-pressure

TABLE XXIX.

TESTS OF THE SCHMIDT STEAM-ENGINE AND BOILER.

Recorded by Professor GUTERMUTH, *Zeitschrift des Vereines Deutscher Ingenieur*, vol. xl, p. 1390.

	Compound condensing.					Vertical compound.		Vertical high-speed.		Horizontal single-cylinder.		
Diameters of cylinders, inches	9.43 and 19.69				8.66 and 15.75	12.24 and 27.16		9.84		16.73	7.09	
Stroke, inches	19.69				13.68	19.675		6.3		31.5	11.81	
Revolutions per minute	140				175	117		250		85	200	
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Duration, hours and minutes	4–15	4–28	2–42	4–4	6–17	6–6	7–44	8	8	7–31	8	8
Pressure of steam, pounds per sq. in.	151.1	151.0	97.8	97.3	143.7	167.8	169.2	95.1	91.1	79.9	111.6	114.7
Temperatures: saturated steam	370	369.9	341.6	341.6	366.8	373.8	374.4	340.7	339.8	330.2	352.0	352.4
" superheated steam	677	640	684	677	675	604	651	572	.....	531	680	667
Indicated horse-power	110.2	119.0	72.1	71.6	68.74	71.4	75.3	16.57	14.7	67.94	22.53	22.37
Mechanical efficiency	0.917	0.923	0.907	0.909	0.886	0.865	0.826	.....	.....	.....	0.816	0.819
Steam, lbs. per horse-power per hour	10.74	10.57	10.83	11.41	9.69	10.9	10.1	19.45	39.56	21.68	15.04	15.04
Coal, lbs. per horse-power per hour	1.71	1.81	1.83	1.83	1.52	1.38	1.02	2.80	6.04	3.55	3.23	2.88
Steam per lb. of coal from and at 212° F.	7.23	6.940	6.846	7.202	.....	8.65	8.82	7.0	.....	6.43	5.99	6.93

cylinders and one double-acting vertical low-pressure cylinder. The high-pressure cylinders have a special form of piston-valve controlled by a shaft-governor; the low-pressure cylinder has a double-ported slide-valve. The low-pressure cylinder has a steam-jacket, and arrangements are made for superheating the steam between the high-pressure and the low-pressure cylinders.

Of the tests I to IV made on this engine the first was made without superheating of the steam between the two cylinders; during the second test the steam was again superheated about 100° F. above the corresponding temperature for saturated steam. The two tests III and IV were made at a reduced boiler-pressure. The third test was made with and the second without steam in the jacket of the low-pressure cylinder.

The report of the test does not give the data required for calculating the heat-consumption, so it is necessary to depend on the coal-consumption per horse-power per hour in making comparisons with other engines, since in this case the steam-consumption is not a reliable guide. The coal-consumption is very satisfactory, but is not exceptional.

The results of the tests VI and VII made by Professor Schröter on a comparatively small vertical Schmidt engine are exceptional in both the steam and coal economy; the latter, which is the only trustworthy basis of comparison, is as small as the best result given in Table X for the Leavitt pumping-engine at the Chestnut Hill station, Boston. A further comparison of the two engines shows that the Schmidt engine worked with less steam-pressure and fewer expansions. The Leavitt engine had 21 expansions while the Schmidt engine had 16 expansions in test VI and 19 in test VII; these last figures are not given in Table XXIX, but are given by Professor Schröter in his report.

The engine tested by Professor Schröter was a vertical single-acting tandem compound engine, with the small cylinder over the large cylinder. The high-pressure piston was a

hollow plunger long enough to carry the low-pressure piston at its lower end. The low-pressure piston had for its effective area the annular ring outside the plunger. The lower end of the large cylinder was closed and the space under the large piston, including the space in the hollow plunger, formed an intermediate receiver with variable volume. The high-pressure cylinder had a piston-valve, and the low-pressure cylinder had a slide-valve, cored through the body to give a double admission after the manner of the Trick valve. Both valves were on one spindle and were actuated by a single eccentric. The boiler was of the type already described.

One other test in Table XXIX is worthy of note, that is, test V on a compound condensing-engine which developed 69 horse-power for 1.52 of a pound of coal per horse-power per hour. This, though less remarkable than the economy for either test VI or test VII, is exceptionally good for an engine of that size. Other tests of less importance are recorded in the same table on a vertical high-speed non-condensing engine and on two horizontal single-cylinder engines. The vertical engine was tested with saturated and with superheated steam, and would appear to show an enormous gain from superheating; but its performance with saturated steam is too poor to serve as a basis of comparison.

With the exception of tests V, VI, and VII the performance of any engine recorded in Table XXIX may be equalled by an engine of the same power using saturated steam if we take for the basis of comparison the coal consumed per horse-power per hour. The exceptionally small steam-consumptions shown by some tests appear to be misleading.

Steam-jackets.—A comparison of the results of applications of Hirn's analysis to engines with and without steam-jackets leads to the conclusion that the beneficial effect of a steam-jacket is indirect. It appears that while some heat is supplied by the jacket during expansion (a part of which may

be changed into work) the greater part is supplied during exhaust and is carried away by the exhaust steam and lost. But the indirect effect is to maintain the inner wall of the cylinder at a higher temperature, and so reduce the initial condensation, and consequently to reduce the exhaust-waste. It further appears that the exhaust-waste is not a proper criterion of the economy of the engine, and that while a lavish use of steam-jackets on compound engines may extinguish the exhaust-waste we cannot expect to get the highest economy by that process.

It is not possible from any theoretical discussion nor from a study of all the applications of Hirn's analysis now extant to determine when or to what extent steam-jackets are advantageous. We must, consequently, go directly to comparative tests of engines with and without steam-jackets. Properly we should compare engines which otherwise are identical and which differ only in that one is made with and the other without a steam-jacket. Few if any such comparisons can be made; in general, we must be content to compare tests made on an engine with steam-jackets, some of the tests being made with steam supplied to the jackets and some without. This method of comparison is not quite fair, for it is probable that the engine which has a steam-jacket wastes more heat by radiation and conduction, both when there is and when there is not steam in the jacket, than does an unjacketed engine of the same size and power. However, if this were our only difficulty we should be fortunate, because the total radiation is in no case large for a properly lagged cylinder.

A haphazard comparison of the numerous tests that have been made of engines with and without steam in the jackets may be made to show anything, from a most extravagant and improbable gain to a positive loss. It is proper, by the way, to say that when a jacket shows no gain the engine is probably better and is certainly cheaper without. In order to arrive at any conclusion it is necessary to proceed according to a logical system.

In general, nothing can be learned from the comparison of individual tests either on a given engine with and without steam in the jackets or on separate engines. Either or both of the tests may be made under conditions which give a poor economy, whereas useful comparisons can be made only when conditions are favorable. This does not preclude a comparison when both tests give high economy.

When possible, tests should be in series, extensive enough to determine the conditions which give the best economy both with and without jackets; and in such case only the best results for each condition should be compared. The comparison is much aided by drawing curves like those in Fig. 81, page 390.

A few tests (eight or ten) may completely determine the interesting part of the curve, as, for example, the lowest curve on Fig. 81. In this case there are virtually four pairs of points and one more individual point which definitely locate the curve. The three detached points below the curve are for tests which differ in a minor particular.

In all cases the comparison should be of the heat-consumption instead of the steam-consumption; for the hot water from the jackets can and should be returned to the boiler at nearly the temperature in the boiler, or else should be used to heat the feed-water in such a way that its heat is not wasted.

Though such a method will simplify our investigations and in general lead to consistent results, we will find some discrepancies which cannot be reconciled, and which will show why there is so much difference of opinion concerning the advantage to be obtained from the use of steam-jackets.

Delafond's Tests.—In 1883 an extensive and important investigation was made by Mons. F. Delafond on a horizontal Corliss engine at Creusot to determine the conditions under which the best economy can be obtained for such an engine. The engine had a steam-jacket on the barrel, but was not jacketed on the ends. Steam was supplied to the jacket by

a branch from the main steam-pipe, and the condensed water was drained through a steam-trap into a can, so that the amount of steam used in the jacket could be determined. The engine was tested with and without steam in the jacket, both condensing and non-condensing, and at various pressures from 35 to 100 pounds above the pressure of the atmosphere. The effective power and the friction of the engine were also obtained by aid of a friction-brake on the engine-shaft.

The piping for the engine was so arranged that steam could be drawn either from a general main steam-pipe or from a special boiler used only during the test. Before making a test the engine, which had been running for a sufficient time to come to a condition of thermal equilibrium, was supplied with steam from the general supply. At the instant for beginning the test the general supply was shut off and steam was taken from the special boiler during and until the end of the test, and then the pipe from that boiler was closed. The advantage of this method was that at the beginning and end of the test the water in the boiler was quiescent and its level could be accurately determined. At the end of a test the water-level was brought to the height noted at the beginning. The water required for feeding the special boiler during the test and for adjusting the water-level at the end was measured in a calibrated tank. As the steam-pressure in the general-supply main and in the special boiler was the same, there was little danger of leakage through the valves for controlling the steam-supply; the regularity and consistency of results shown by the curves of Figs. 79 and 80 attest to the skill and accuracy with which these tests were made.

Table XXX gives the results of tests made with condensation, and Table XXXI gives the results of tests without condensation. All the tests both with and without condensation, but during which no steam was used in the jackets, are represented by the several curves of Fig. 79, while Fig. 80 represents tests made with steam in the jackets. The curves

TABLE XXX.

HORIZONTAL CORLISS ENGINE AT CREUSOT.

CYLINDER DIAMETER 21.65 INCHES; STROKE 43.31 INCHES; JACKET ON BARREL ONLY; CONDENSING.

By F. DELAFOND, *Annales du Mines*, 1884.

Number of test.	Duration, minutes.	Revolutions per minute.	Cut-off in per cent of stroke.	Steam-pressure, pounds per sq. in.	Vacuum, inches of mercury.	Steam used in jacket, per cent.	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	60	60.0	4	96.3	27.1	.....	109	23.2
2	105	58.6	6	98.8	27.1	.....	128.5	22.2
3	75	59.4	9	100	27.0	.....	161	21.4
4	36	57.7	12.5	99.1	27.0	.....	186	22.0
5	73	58.8	5.5	104	27.4	?	141	17.1
6	55	61.5	6.7	102.4	27.1	?	159.5	16.7
7	80	59.9	6.7	103.8	27.4	2.9	155	16.5
8	39	58.1	12.5	105.2	26.8	3.2	212	17.6
9	120	59.8	7.5	79.8	27.1	.....	126	21.2
10	100	59.3	8.3	81.1	27.4	.....	134	21.1
11	90	59.8	10.5	80.1	27.1	.....	150	20.8
12	55.5	58.0	14	85.5	27.1	.....	175	19.9
13	50	59.1	18	84.8	26.5	.....	194	20.4
14	94	59.6	5	85.1	27.4	3.0	112	17.7
15	102	59.6	5.5	83.3	27.6	3.1	124	17.3
16	40	59.4	11.5	84.1	27.1	1.2	176	16.9
17	40	60.0	14	84.1	27.0	1.5	193	17.5
18	91	58.3	5.9	60.5	28.0	.....	85.3	20.4
19	90	59.5	9	55.8	27.6	.....	115	19.1
20	75	59.0	15.5	61.2	27.8	.....	150	18.1
21	75	58.3	22.7	58.3	27.6	.....	172	18.4
22	31	59.2	25	61.2	27.1	.....	186	18.8
23	115	59.9	6	59.9	27.8	2.5	91.7	18.5
24	92	59.6	9	59.9	27.4	2.5	117	17.6
25	90	58.8	15.5	60.9	27.1	1.8	150	17.3
26	71	59.1	20	61.9	26.8	1.5	175	17.7
27	50	59.0	25	62.3	26.4	1.6	194	18.6
28	70	60.7	6	45.0	28.0	.....	75.6	20.7
29	80	58.8	9.5	48.9	28.1	.....	94.3	19.4
30	111	60.4	15	47.9	27.6	.....	120	18.8
31	54	58.8	21	47.8	27.6	.....	140	19.0
32	55	59.4	29	47.6	27.1	.....	165	19.8
33	98	60.3	5	45.8	28.0	2.6	68.8	19.3
34	63	57.6	10	51.6	27.6	2.3	95.5	18.5
35	60	59.7	14.3	49.1	28.1	1.4	120	18.2
36	74	60.1	22	48.6	27.8	1.4	152	18.9
37	50	59.5	29	50.2	26.8	1.2	179	19.7
38	85	60.3	18.2	33.1	27.8	.....	106	20.5
39	68	61.1	43	34.7	26.5	.....	160	22.7
40	42.5	61.0	56.7	36.3	26.0	.....	181	25.3
41	20	60.0	100	31.7	25.2	.....	182	35.9
42	73	60.7	19	32.0	27.6	1.6	111	19.8
43	80	61.9	42	33.0	26.5	1.1	162	22.1
44	40	61.1	58	35.1	26.0	0.6	180	25.4
45	25	60.4	100	34.7	25.2	0.2	199	33.0

TABLE XXXI.

HORIZONTAL CORLISS ENGINE AT CREUSOT.

CYLINDER DIAMETER 21.65 INCHES; STROKE 43.31 INCHES; JACKET ON BARREL ONLY; NON-CONDENSING.

By F. DELAFOND, *Annales des Mines*, 1884.

Number of test.	Duration, minutes.	Revolutions per minute.	Cut-off in per cent of stroke.	Steam-pressure, pounds per square inch.	Steam used in jacket, per cent.	Indicated horse-power.	Steam per horse-power per hour, pounds.
1	78	61.7	13	96.3	.....	147.5	28.4
2	55	61.4	17	100.2	.....	181.5	26.8
3	25	63.6	20	102.0	.....	217	25.8
4	80	60.8	11	98.1	2.5	143	22.8
5	60	62.0	13	103.8	3.4	177.5	22.1
6	36	62.0	16	103.0	3.1	194	22.4
7	30	62.7	20	103.5	2.0	237	21.5
8	66	62.0	15.5	73.7	.....	121	27.6
9	60	60.9	18	77.0	.....	136	26.7
10	60	60.0	24.5	76.7	.....	178	24.6
11	30	60.6	32	77.5	.....	209	24.2
12	70	61.1	16.5	77.0	1.7	137	23.7
13	50	61.6	23.5	75.8	1.2	180	21.8
14	30	60.5	30	78.0	1.3	204	22.0
15	71	61.4	24.5	50.8	.....	108	27.3
16	70	61.1	37	51.2	.....	147	27.2
17	50	60.9	58	50.5	.....	173	30.2
18	25	60.6	100	34.9	.....	145	46.8
19	70	60.5	23	52.6	1.5	108	25.3
20	60	60.5	34	51.8	1.1	141.5	25.2
21	50	60.3	58	46.2	0.7	168.5	28.7
22	30	61.1	100	33.7	0.3	147.5	46.3

are lettered to show the mean steam-pressure for the series represented and the condition, whether with or without condensation. Thus on Fig. 79 the lowest curve 60*C* represents tests made without steam in the jackets and with condensation, while the highest curve on Fig. 80 represents tests without steam in the jackets and without condensation, at 50 pounds boiler-pressure. The abscissæ for the curves are the per cents of cut-off and the ordinates are the steam-consumptions in pounds per horse-power per hour. The results for individual tests are represented by dots, through which or near

which the curves are drawn. As there are only a few tests in any series, a fair curve representing the series can be drawn through all the points in most cases. The exceptions are tests made with condensation for boiler-pressure of 80 and

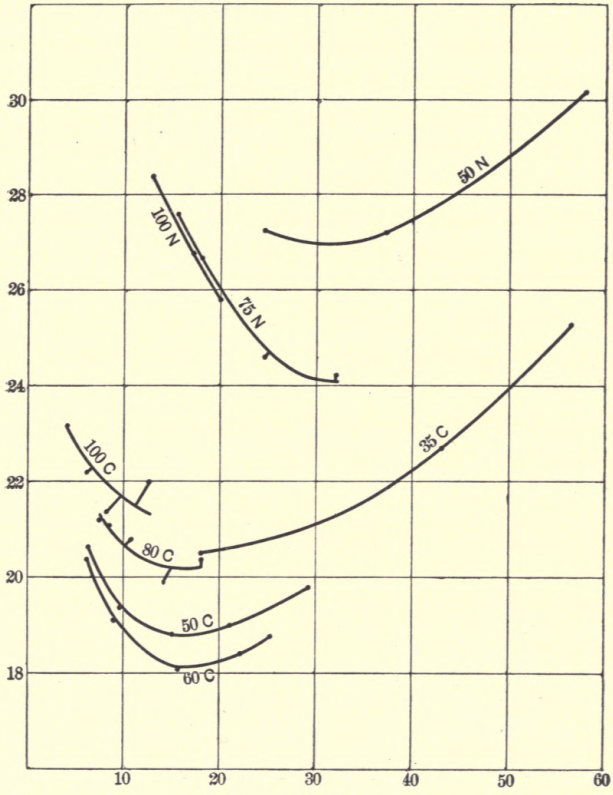

FIG. 79.

100 pounds per square inch. The forms of the curves 80*C* and 100*C*, Fig. 79, were made to correspond in a general way to the curves 50*C* and 60*C*. The discrepancies appear large on account of the large scale for ordinates, but they are not really of much importance; the largest deviation of a point from the curve 100*C* is half a pound out of about 22, which amounts to little more than two per cent. On Fig. 80 the

curve 80*C* is drawn through the points, but though its form does not differ radically from the curves 60*C* and 50*C*, so marked a minimum at so early a cut-off is at least doubtful. Considering that the probable error of determining power

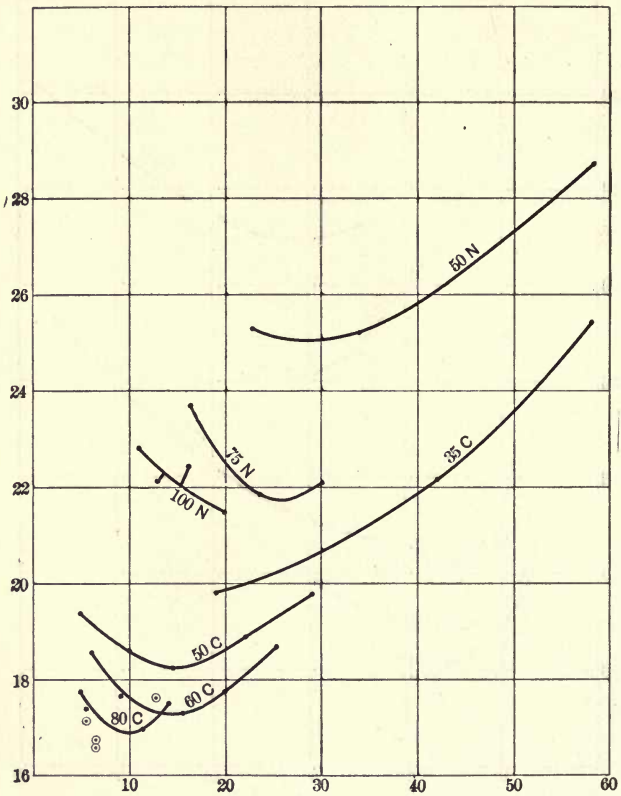

FIG. 80.

from the indicator is about two per cent, it would not be difficult to draw an acceptable curve in place of 80*C* which should correspond to the forms of 60*C* and 50*C*.

The results of the four tests made with steam in the jacket and with condensation, and which are numbered 5, 6, 7, and 8 in Table XXX, are represented by dots inside of

small circles on Fig. 80. It does not appear worth while to try to draw a curve to represent these tests.

From a comparison of the curves on Figs. 79 and 80 representing the results of tests stated in Tables XXX and XXXI, it appears that the engine when running condensing, whether with or without steam in the jackets, gave its best economy at about one-sixth cut-off. When running non-condensing the cut-off giving the best economy was at about one-third stroke. A more careful consideration of the proper cut-off for simple engines and of the total expansions for multiple-expansion engines will be given later and then somewhat fine discriminations may be attempted. The general statement given above will suffice for our present purpose.

It has already been pointed out that these tests show that the best results are obtained from this engine running condensing and without steam in the jacket for a boiler-pressure of 60 pounds. With steam in the jacket the advisable pressure is at least 80 pounds. Taking for comparison tests 16 and 20 of Table XXX, the gain from the use of the jacket is

$$\frac{18.1 - 16.9}{18.1} \times 100 = 7 \text{ per cent.}$$

All the tests with steam in the jacket show a very small percentage condensed in the jacket, so small as to raise the question whether the steam-trap for removing the condensed water could have acted properly. But we have not the data for calculating the heat-consumption in any case, and so must rest content with the comparison made.

The tests made on the engine without condensation are less complete, and on the whole it appears best to compare only the tests made at 75 pounds boiler-pressure. Comparing tests 11 and 13, Table XXXI, the gain from the use of the steam in the jacket is

$$\frac{24.2 - 21.8}{24.2} \times 100 = 10 \text{ per cent.}$$

These tests by Delafond give very good illustrations of the error and confusion that may arise when individual tests are compared to determine the advantage to be obtained from the use of a steam-jacket. Thus a comparison of tests 2 and 7 of Table XXX shows an apparent gain in steam-consumption of 25 per cent, while a comparison of tests 31 and 36 shows a slight loss from the use of steam in the jacket. Both pairs of tests standing by themselves would appear to give a fair basis for comparison; it is only by assembling several series of tests, as by aid of the curve of Figs. 79 and 80, that the real value of the steam-jacket can be determined.

Tests on Revenue Steamers.—In 1874 three steamers, the *Rush*, the *Dexter*, and the *Dallas*, were built for the United States revenue marine, which were designedly alike in all respects except that the engines were of three distinct types, and the boilers were adapted to the engines. This was done to determine which type was best adapted to the needs of the service. At about the same time another steamer, the *Gallatin*, was supplied with a new engine differing from those in the vessels just mentioned. Again, the Treasury Department had a small steamer built in 1870 for the use of the Coast Survey, named the *Bache*. These five steamers had their engines tested under the direction of Chief Engineer C. H. Loring, U.S.N., and Mr. Chas. E. Emery, Consulting Engineer U. S. R. M. During the tests the vessels were secured to the dock. One test on each engine was long enough to determine the coal-consumption; other tests were shorter and for them the steam-consumption only was determined. The feed-water was measured in a tank with two compartments, which were filled and emptied alternately, and was used as the basis for determining the steam-consumption.

The *Rush* had a compound engine with the pistons connected to cranks at right angles. The cylinders were jacketed on barrels and heads. The small cylinder had a separate cut-off valve on the back of the main valve.

The *Bache* had a compound engine with both pistons on one continuous piston-rod, the high-pressure cylinder being over the low-pressure cylinder. The large cylinder only had a steam-jacket on barrel and ends. The engine was piped to run compound or single, using the low-pressure cylinder only in the latter condition.

The *Dexter* had a single-cylinder engine without steam-jackets, and the *Dallas* had an engine of the same type, but intended for a lower steam-pressure.

The *Gallatin* had a single-cylinder engine jacketed on the barrel and heads. During tests on this engine the steam-consumption was determined by collecting and weighing the water drawn from the condenser.

The results of the tests on these several engines are given in Table XXXII. The best results, together with the thermal units per horse-power per minute, are assembled in the following supplemental table:

No. of test.	Name of engine.	Method of working.	Boiler-pressure, gauge.	Back-pressure, absolute.	Revolutions per minute.	Total number of expansions.	Indicated horse-power.	Steam per horse-power per hour.	Thermal units per horse-power per min.
6	*Bache*.....	Compound with jacket...	80.2	3.3	53.2	7.0	99.2	20.3	361
2	"	" without " ...	80.3	3.4	47.7	6.7	69.8	23.0	408
18	*Rush*	" with " ...	69.1	3.5	70.8	6.2	266.5	18.4	327
16	*Bache*.....	Simple with jacket.....	79.5	3.7	53.8	5.1	116.0	23.2	411
13	"	" without "	78.1	4.4	47.1	5.3	89.2	26 2	460
22	*Dexter*.....	" " "	68.7	3.4	56.5	4.5	185.9	23 9	423
36	*Gallatin*.	" with "	71.6	3.6	71.6	3.6	179.5	20.5	361
34	"	" without "	68.5	3.9	59.9	4.9	279.6	21.9	387

For the present we will consider only those tests which may be used to determine the advantage of using steam-jackets; the effects of using higher pressures and of compounding will be considered later.

Comparing tests 34 and 36 on the *Gallatin* with and without steam in the jacket shows a gain in steam-consumption from the use of the jacket of

$$\frac{21.9 - 20.5}{21.9} \times 100 = 7 \text{ per cent.}$$

TABLE XXXII.

TESTS ON THE U. S. REVENUE STEAMERS *RUSH*, *DEXTER*, *DALLAS*, AND *GALLATIN*, AND THE COAST-SURVEY STEAMER *BACHE*.

	Bache.	*Rush.*	*Dexter.*	*Dallas.*	*Gallatin.*
Cylinder diameters, inches...	16 & 25	24 & 28	26	36	34.1
Stroke, inches.....	24	27	36	30	30

No. of test.	Name.	Condition.	Duration, hours.	Revolutions per minute.	Total expansions.	Boiler-pressure by gauge, lbs. per sq. in.	Barometer, pounds per square inch.	Vacuum, inches of mercury.	Back-pressure, absolute, lbs. per sq. in.	Percentage of steam used in jackets.	Horse-power.	Steam per horse-power per hour, pounds.
1	*Bache.*	Comp. without jacket.	1.83	42.6	9.1	82.0	14.7	24.0	3.4	.....	55.9	23.8
2			2.07	47.7	6.7	80.3	14.7	24.3	3.4	.....	77.1	23.0
3			2.13	49.3	5.6	80.3	14.7	24.7	3.3	.....	85.8	23.2
4		Comp. with jacket.	1.73	38.9	16.9	81.4	14.9	24.5	3.7	6.5	46.4	25.1
5			2.07	48.2	9.2	80.3	14.9	26.5	3.1	7.0	77.5	20.7
6			1.93	53.2	7.0	80.2	14.9	26.5	3.3	5.1	99.2	20.3
7			1.98	56.3	5.7	80.1	14.9	26.6	3.3	4.6	110.6	20.4
8			7.07	55.6	5.7	80.0	14.7	26.1	3.4	5.4	106.0	22.0
9			15.23	53.6	5.1	79.8	14.3	24.4	3.3	5.6	102.3	22.4
10			2.00	60.6	4.2	79.1	14.9	26.5	3.3	4.0	134.5	21.2
11		Single without jacket.	1.80	37.3	11.8	81.0	14.6	24.0	4.6	.....	47.2	35.0
12			1.98	44.9	7.6	79.6	14.6	23.8	4.4	.....	71.8	29.6
13			2.05	47.1	5.3	78.1	14.6	24.2	4.4	.....	89.1	26.2
14		Single with jacket.	2.10	39.9	12.6	80.8	14.6	24.7	3.0	4.0	54.8	27.1
15			1.68	46.2	8.6	81.1	14.6	25.3	2.8	1.8	74.6	24.1
16			2.12	53.8	5.1	79.6	14.6	25.5	3.7	2.2	116.0	23.2
17			1.88	45.3	2 2	30.9	14.6	24.0	4.7	1.5	66.7	34.0
18	*Rush.*	Comp. with jacket.	55	70.8	6.2	69.1	14.8	26.5	3.5	?	266.5	18.4
19			6	55.5	4.0	36.7	14.8	26.2	3.4	?	168.7	22.1
20	*Dexter.*	Single without jacket.	2.92	56.5	4.5	68.7	14.8	25.9	3.4	.....	186.9	23.9
21			1.42	64.3	3.7	69.3	14.8	25.2	3.7	.....	228.1	24.1
22			34.5	61.1	3.5	67.1	14.8	25.5	3.6	.....	219.0	23.9
23			0.65	72.8	2.7	66.4	14.8	25.3	5.3	.....	292.4	24.3
24			1.32	50.8	2.3	40.6	14.8	26.1	3.2	.....	124.3	28.8
25			1.20	55.3	2.4	39.9	14.8	26.0	3.6	.....	161.8	28.9
26			0.92	60.7	2.1	41.9	14.8	25.5	4.3	.....	196.2	31.8
27	*Dallas.*	Single without jacket.	1.52	48.7	5.1	35.4	14.8	26.1	3.0	.....	138.0	26.7
28			1.55	56.9	3.4	35.3	14.8	26.0	3.4	.....	186.8	26.9
29			31.0	61.5	3.1	32.0	14.7	25.2	3.9	.....	221.4	26.9
30			1.60	64.5	2.9	33.7	14.8	25.4	4.1	.....	242.8	28.9
31			1.53	63.5	2.3	27.4	14.8	24.8	4.1	.....	234.3	31.0
32	*Gallatin.*	Single without jacket.	24.0	60.3	4.5	64.1	14.9	25.3	4.3	.....	247.9	24.3
33			2.05	56.0	5.6	68.2	14.6	25.1	4.7	.....	212.2	23.8
34			2.02	59.9	4.9	68.5	14.8	25.9	3.9	.....	259.0	21.9
35		Single with jacket.	24.0	61.5	4.5	65.4	14.7	25.1	4.0	3.5	260.5	22.0
36			2.22	51.1	7.3	71.6	14.8	25.7	3.6	5.1	179.5	20.5
37			1.93	68.7	4.2	67.2	14.8	25.0	4.4	3.2	324.4	21.5

A comparison of tests 13 and 16 on the *Bache*, using the large cylinder only, shows a gain of

$$\frac{26.2 - 23.2}{26.2} \times 100 = 11 \text{ per cent.}$$

Finally, a comparison of tests 2 and 6 on the *Bache* as a compound engine shows a gain of

$$\frac{23.0 - 20.3}{23.0} \times 100 = 11 \text{ per cent}$$

from jacketing the low-pressure cylinder only. Comparisons based on thermal units per horse-power per minute give the same results, because the per cent of steam used in the jackets is not large in any case.

Experimental Engine at the Massachusetts Institute of Technology.—This engine, which was added to the equipment of the laboratory of steam-engineering of the Institute in 1890, is specially arranged for giving instruction in making engine-tests. It has three horizontal cylinders and two intermediate receivers, the piping being so arranged that any cylinder may be used singly or may be combined with one or both of the other cylinders to form a compound or a triple engine. Each cylinder has steam-jackets on the barrel and the heads, and steam may be supplied to any or all of these jackets at will. The steam condensed in the jackets of any one of the cylinders is collected under pressure in a closed receptacle and measured. Originally the receivers were also provided with steam-jackets; now they are provided with tubular reheaters so divided that one-third, two-thirds, or all the surface of the reheaters can be used. The steam condensed in the reheaters is also collected and measured in a closed receptacle.

The valve-gear is of the Corliss type with vacuum dash-pots which give a very sharp cut-off. The high-pressure and intermediate cylinders have only one eccentric and wrist-plate, and consequently cannot have a longer cut-off than half stroke

under the control of the drop cut-off mechanism. The low-pressure cylinder has two eccentrics and two wrist-plates, and

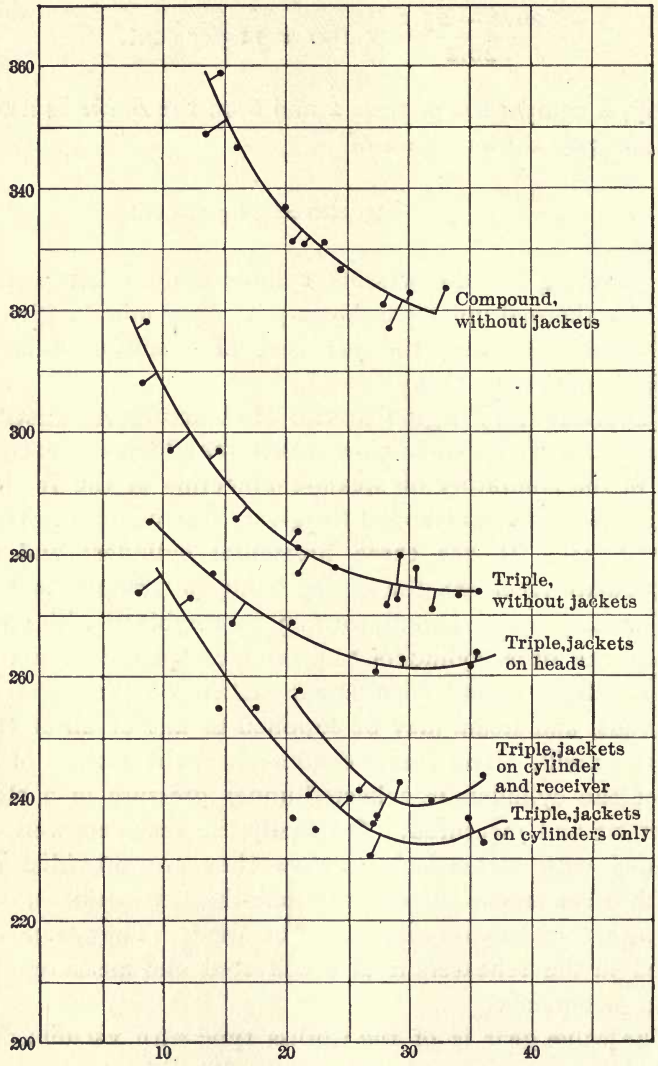

FIG. 81.

the admission-valves can be set to give a cut-off beyond half stroke. The governor is arranged to control the valves for

any or all of the cylinders. Each cylinder has also a hand-gear for controlling its valves. For experimental purposes the governor is set to control only the high-pressure valve-gear, when the engine is running compound or triple-expansion. The hand-gear is used for adjusting the cut-off for the other cylinder or cylinders; usually the cut-off for such cylinder or cylinders is set to give a very small drop between the cylinders. This arrangement throws a very small duty on the governor, so that by the aid of a large and heavy fly-wheel the engine can be made to give nearly identical indicator-diagrams for an entire test during which the load and the steam-pressure are kept constant.

The main dimensions of the engine are as follows:

Diameter of the	high-pressure	cylinder	9 inches.
" "	intermediate	"	16 "
" "	low-pressure	"	24 "
" "	piston-rods		$2\frac{3}{16}$ "
Stroke			30 "

Clearance in per cent of the piston displacements:

High-pressure	cylinder,	head end,	8.83;	crank end,	9.76
Intermediate	"	"	"	10.4	"	"	10.9
Low-pressure	"	"	"	11.25	"	"	8.84

Results of tests on the engine with the cylinders arranged in order to form a triple-expansion engine are given in Table XXXIII; those on the engine with the small and intermediate cylinders forming a compound engine are given in Table XXXIV; and other tests with the tubular reheaters in use are given in Table XXXV. The results of tests in Tables XXXIII and XXXIV are represented by the diagram Fig. 81, with the cut-off of the high-pressure cylinder for abscissæ and with the consumptions of thermal units per horse-power per minute as ordinates.

The most important investigation which has been made

TABLE XXXIII.

TRIPLE-EXPANSION EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Trans. Am. Soc. Mech. Engs., 1892–1894; *Technology Quarterly*, 1896.

	Condition.	Revolutions per minute.	Per cent of cut-off, high-pressure cylinder.	Boiler-pressure by gauge.	Vacuum in condenser, inches of mercury.	Barometer, inches of mercury.	Steam used in jackets, per cent.					Horse-power.	Steam per horse-power per hour.	B. T. U. per H. P. per minute, actual.	B. T. U. per H. P. per min. reduced to 26″ vacuum.
							High-pressure cylinder.	First receiver.	Intermediate-pressure cylinder.	Second receiver.	Low-pressure cylinder.				
1	Jackets on cylinders.	89.93	36 1	146.2	24.1	29.8	3.2	....	8.6	.....	6.3	140.8	13.8	240	233
2		90.60	35.0	147.0	24.7	30.3	2.5	...	8.8	.....	5.4	138.0	13.9	241	237
3		91.93	27.3	146.9	24.5	29.9	2.5	....	8.5	.....	7.2	125.4	13.7	237	231
4		91.55	27.0	146.7	25.4	30.1	3.2	....	9.8	.....	8.1	123.9	13.7	239	236
5		92.37	25.0	146.6	24.5	30.7	3.4	....	10.4	.....	10.1	114.7	14.3	247	240
6		84.87	21.9	145.2	24.3	30.1	3.5	....	11.3	.....	8.7	105.3	14.5	250	241
7		93.15	17.4	146.0	26.0	30.2	3.5	...	10.7	.....	11.6	103.5	14.7	255	255
8		86.70	12.0	147 0	27.4	30.5	6.1	....	15.2	.....	12.2	78.3	15.1	261	273
9		87.55	8.3	146.7	26.0	30.1	6.5	...	15.3	.....	13.0	67.4	16.0	274	274
10	Ditto.	84.23	13.5	145.2	26.1	30.0	5.3	..	11.3	.....	12.1	77.8	14.7	253	255
11		82.50	20.5	144.5	26.2	29.9	4.5	....	9.1	.....	9.9	101.9	13.5	235	237
12		82.13	23.6	145.3	26.4	30.1	3.1	....	8.5	.....	9.8	104.2	13.3	232	235
13	Jackets on cyl'drs and receivers.	91.20	36.1	143.7	24.7	30.2	2.6	4.7	6.4	5.4	5.9	154.2	14.4	249	244
14		91.40	32.8	143.6	25.0	30.2	2.9	6.4	7.1	4.3	6.4	145.1	14.1	244	240
15		91.82	29.3	143.2	25.2	30.5	3.0	5.6	7.6	4.9	6.1	137.0	14.3	246	243
16		91.83	27.5	147.1	24.7	30.3	1.4	4.7	8.9	3.1	7.3	128.8	14.1	242	237
17		92.17	25.9	145.5	25.5	30.4	3.2	4.5	8.2	4.7	5.7	125.8	14.1	243	241
18		92.57	21.9	143.7	26.4	30.6	3.4	6.8	7.1	4.1	7.7	120.2	14.6	256	256
19	Jackets on heads.	84.95	9.1	145.8	25 6	30.0	2.9	...	7.7	.....	8.7	55.9	16.6	290	285
20		84.03	13.9	144.5	26.4	29.9	2.1	...	7.2	.....	8.6	69.4	15.5	273	277
21		83.35	15.6	144.9	25.6	29.8	2.2	...	6.8	.....	8.0	72.8	15.5	273	269
22		82.40	20.7	145.3	26.7	30.3	1.4	..	6.6	.....	8.0	84.2	15.1	269	269
23		81.40	27.3	144.2	24.7	29.7	1.3	....	7.7	.....	5.6	97.4	15.2	267	261
24		81.05	29.7	143.4	25.4	29.9	1.4	...	5.3	.....	6.8	101.5	15.0	265	263
25		80.28	34.9	143.1	25.5	30.2	1.2	...	5.0	.....	6.4	109.4	15.0	265	262
26		80.32	35.6	144.0	25.0	29.9	1.1	...	4.6	.....	7.4	114.1	15.2	267	264
27	No jackets.	85.60	8.4	152.8	26.1	29.7	....	....	.....	.....	.....	53.2	17.3	318	318
28		85.62	8.3	153.3	26.1	.....	....	...	.....	.....	.....	55.7	16.9	306	308
29		85.60	10.6	152.1	26.1	29.9	...	....	.....	.....	.....	60.6	16.2	296	297
30		84.22	15.8	152.8	25.9	29.8	....	....	.....	.....	.....	74.9	15.4	287	286
31		83.03	21.3	152.0	26.3	....	....	....	.....	.....	.....	85.8	15.1	276	277
32		82.92	21.2	152.4	26.09	30.15	....	....	.....	.....	.....	86.9	15 4	281	281
33		82.55	21.0	153.0	26.02	30.0	....	....	.....	.....	.....	87.8	15.2	284	284
34		83.32	24.1	152.0	25.70	.....	....	....	.....	.....	.....	91.1	15.5	280	278
35		82.67	29.5	151.9	25.6	30.0	....	....	.....	.....	.....	99.9	15.5	283	280
36		81.78	29.1	152.0	25.7	.....	....	....	.....	.....	.....	100.5	15.2	275	273
37		82.92	28.7	152.5	26.0	29.9	....	...	.....	.....	.....	102.4	15.0	272	272
38		81.52	30.7	151.5	26.1	29.9	....	....	.....	.....	.....	106.0	15.2	278	278
39		81.57	31.8	152.0	26.0	.....	....	....	.....	.....	.....	108.2	14.9	271	271
40		81.40	35.6	152.0	26.04	30.1	...	....	.....	.....	.....	111.2	14.3	274	274
41		81.50	33.8	151.9	25.9	30.26	....	....	.....	.....	.....	112.2	15.1	274	274

TABLE XXXIV.

EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

COMPOUND; CYLINDER DIAMETERS 9 AND 24 INCHES; STROKE 30 INCHES.

Technology Quarterly, vol. xi, p. 43.

	Revolutions per minute.	Boiler-pressure by gauge.	Vacuum in condenser, inches of mercury.	Barometer, inches of mercury.	Per cent of cut-off, high-pressure cylinder.	Steam used in jackets, per cent.	Horse-power.	Steam per horse-power per hour.	B. T. U. per horse-power per minute.
1	82.72	119.3	24.7	29.56	14.4	.....	51.87	19.95	359
2	83.25	119.6	25.4	29.80	13.7	....	54.89	19.34	349
3	82.75	119.6	24.9	29.50	16.1	.....	57.00	19.27	347
4	81.88	119.2	26.0	29.50	20.0	.....	63.89	18.49	337
5	81.62	119.6	25.4	29.48	21.5	.. ....	65.36	18.30	331
6	82.40	121.0	26.4	29.32	20.4	... .	66.24	18.07	331
7	81.75	119.3	26.1	29.46	23.0	.....	70.02	18.17	331
8	81.30	119.2	25.1	29.61	24.7	.....	71.27	18.14	327
9	81.28	123.9	26.4	29.95	28.4	.....	81.41	17.46	317
10	81.00	125.5	26.3	29.74	28.1	.....	81.78	17.62	321
11	81.38	124.8	26.5	29.63	30.0	... .	82.92	17.40	323
12	80.89	119.6	24.7	29.62	33.5	.....	87.07	18.01	324
13	82.83	120.6	26.3	30.17	15.2	17.4	69.2	17.24	303
14	82.90	120.7	26.3	30.00	14.5	15.4	72.08	16.11	284
15	83.20	118.9	26.9	29.40	15.0	16.1	72.61	16.21	289
16	81.48	119.2	26.0	29.50	25.6	12.4	93.68	16.24	289
17	81.12	119.5	26.3	29.39	29.9	14.5	98.32	16.73	298
18	82.73	100.2	26.6	29.49	16.3	19.2	63.07	15.96	282
19	82.90	101.0	26.6	29.49	15.6	20.1	63.19	15.59	275
20	81.72	100.9	26.3	29.49	24.0	17.3	78.13	16.16	284
21	81.37	101.4	26.3	29.49	25.4	19.6	80.47	16.14	284

TABLE XXXV.

TRIPLE-EXPANSION EXPERIMENTAL ENGINE AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY WITH TUBULAR REHEATERS.

	Condition.	Revolutions per minute.	Per cent of cut-off, high-pressure cylinder.	Boiler-pressure by gauge.	Vacuum in condenser, inches of mercury.	Barometer, inches of mercury.	Per cent of steam used in reheaters.		Horse-power.	Steam per horse-power per hour.	B. T. U. per H. P. per minute, actual.	B. T. U. per H. P. per min. reduced to 26″ vacuum.
							First.	Second.				
1	Without steam in reheaters.	81.8	27	146.7	26.4	30.6	.....	. ....	88.6	16.0	200	288
2		81.8	27	147.5	26.1	30.4	.....	.....	87.5	16.0	291	289
3		81.6	29	147.0	25.9	30.5	.....	....	89.7	16.0	290	286
4		81.2	36	148.2	25.9	30.2	... ..	.. ..	103.3	15.5	282	281
5	Steam in first reheater.	85.5	10	147.2	25.5	30.0	.....	13	66.5	15.7	277	273
6		83.5	19	146.9	23.8	30.2	.. ...	14	84.9	15.9	277	262
7		81.4	31	146.1	25.8	30.2	.....	12	112.4	15.0	266	264
8	Steam in both reheaters.	85.0	8	147.3	26.6	30.3	10	7	61.5	15.5	269	274
9		84.5	10	146.9	26.2	30.3	12	8	74.8	14.9	261	260
10		82.4	21	147.1	25.3	30.4	10	6	95.7	14.7	258	252
11		81.9	27	147.7	25.4	30.1	6	9	105.9	14.7	259	254
12		82.0	28	146.6	25.7	30.2	7	8	107.0	14.5	256	254

on this engine is of the advantage to be obtained from the use of steam in the jackets. Four series of tests were made for this purpose: (1) with steam in all the jackets of the cylinders and receivers, (2) with steam in the jackets of the cylinders, both heads and barrels, (3) with steam in the jackets on the heads of the cylinders only, and (4) without steam in any of the jackets.

The most economical method of running the engine was with steam in all the jackets on the cylinders, but without steam in the receiver-jackets, as shown by the lowest curve on Fig. 81. There is a small but distinct disadvantage from using steam in the receiver-jackets also. This fact could not be surely determined from any pair of tests, for the difference is not more than two per cent, and is therefore not more than the probable error for such a pair of tests, but a comparison of the two curves on Fig. 81 representing tests under the two conditions gives conclusive evidence with regard to this point. It may not be improper in this connection to call attention to the three points below the lowest curve and not connected with it; they represent tests which were made after the nine tests represented by points joined to the curve, and when some additional non-conducting covering had been applied to the piping and valves of the engine. Here the slight gain from reduced radiation is made manifest, though it is too small to be taken into account in making comparisons of the different conditions of running the engine.

From the diagram Fig. 81 the best results with steam in all the jackets of the cylinders and without steam in any of the jackets are 233 and 274 B. T. U. per horse-power per minute, and the gain from the use of the steam in the jacket is

$$\frac{274 - 233}{274} \times 100 = 15 \text{ per cent.}$$

These heat-consumptions correspond to 13.8 and 15.2 pounds of steam per horse-power per hour, so that on the basis of

steam-consumption the gain from the use of steam in the jackets would appear to be only 9 per cent, instead of the actual gain of 15 per cent. This large difference is due to the large percentage of steam used in the jackets, amounting in all to 17 or 18 per cent of the total steam-consumption. The steam used in an individual jacket is, however, not excessive, being about 2.5 per cent in the jackets of the high-pressure cylinder and 7 or 8 per cent in the jackets of each of the other two cylinders.

The effect of jacketing the heads of the cylinders only is surprisingly small, as from the diagram the best result is 262 B. T. U. per horse-power per minute, which compared with the best result without steam in any of the jackets gives a gain of only

$$\frac{274 - 262}{274} \times 100 = 4 \text{ per cent.}$$

The correspondence between this result and the experiments by Callendar and Nicolson on the action of the cylinder-walls, has already been pointed out.

The tests recorded in Table XXXIV were made on this same experimental engine using only the smallest and the largest cylinders to form a compound engine. Tests 1 to 12 were made without steam in any of the jackets; tests 13 to 17 were made with steam in the jackets of the low-pressure cylinder only; while tests 18 to 21 were made with steam in the jackets of both cylinders and in the receiver jacket. The first series of twelve tests without steam in the jackets is represented by the highest curve on Fig. 81. The results of tests with steam in the jackets are not represented on the diagram to avoid confusion; they are not numerous nor important enough to warrant drawing another diagram. The curve representing tests without steam in the jackets does not show a minimum, and indicates that too large a total expansion for this method of running the engine was used. The tests with steam in all the jackets show a somewhat poorer

economy than the engine has when running triple-expanding without steam in the jackets. We cannot draw any definite conclusions concerning the use of jackets from these tests. The evidence they present with regard to the proper ratio of high- and low-pressure cylinders will be considered later.

Experimental Engine at Cornell.—The triple-expansion experimental engine in the laboratory of Sibley College, Cornell University, was built by the makers of the engine in the laboratory of the Massachusetts Institute of Technology and resembles it in all respects except that its stroke is six inches longer. The tests on the engine recorded in Table XXXVI are therefore especially valuable, as they may be compared directly with those made on the engine at the Institute.

Tests 33 to 37 were made with the engine running triple-expanding without steam in any of the jackets; tests 38 to 44 were made with steam in all the jackets of cylinders and receivers except the jackets on the low-pressure cylinder. Though differently arranged, the general effect of the jackets appears to be similar to that of the jackets on all three cylinders of the Institute engine. The steam-pressure on the Cornell engine was only 115 pounds per square inch, while that of the Institute engine was 150 pounds. Nevertheless the steam- and heat-consumption for the two engines both with and without steam in the jackets is nearly identical, and the condensation of steam in the jackets under the first condition is nearly the same percentage of the total steam-consumption. Such a result is most satisfactory, as it proves conclusively that the tests on both engines are entirely reliable and free from personal error.

The Cornell engine was also tested using the small cylinder only as a simple engine, and again using the small and intermediate cylinders to form a compound engine. With both arrangements tests were made using steam in the jackets and without steam in the jackets.

As a simple engine, the proportions of the cylinder appear

TABLE XXXVI.

EXPERIMENTAL ENGINE AT CORNELL UNIVERSITY.

CYLINDER DIAMETERS, 9, 16, AND 24 INCHES; STROKE 36 INCHES.

Trans. Am. Soc. Mech. Engs., vol. xvi, p. 913.

	Condition.	Revolutions per minute.	Number of expansions.	Steam-pressure above atmosphere, pounds per square inch.	Barometer, inches of mercury.	Vacuum, inches of mercury.	Moisture in steam, per cent.	Steam used in jackets, per cent.	Indicated horse-power.	Steam per horse-power per hour, pounds.	B. T. U. per horse-power per minute.
1		83.8	3.0	95.8	29.2	23.5	0.7	.....	73.1	25.8	
2	9 × 36	84.5	3.0	96.3	29.2	23.9	0.7	.....	62.5	24.9	
3		86.3	3.9	97.1	29.2	23.9	0.7	.....	52.4	23.5	421
4	Simple without	87.0	5.8	97.1	29.2	24.4	0.8	.....	39.9	24.9	
5	steam in	85.0	8.6	93.7	29.2	24.7	0.7	.....	30.3	25.8	
6	jackets.	87.5	12.0	100.8	29.2	24.7	0.9	.....	19.1	32.9	
7		82.0	14.2	97.4	29.2	24.7	0.8	.....	9.5	47.5	
8		83.6	2.3	95.7	29.3	24.2	0.7	3.7	71.9	25.6	
9		84.5	3.1	96.7	29.3	24.7	0.7	3.9	62.4	25.5	
10	Simple with	85.4	4.4	97.7	29.3	24.8	0.8	5.5	50.2	23.1	411
11	steam in	86.3	6.3	98.9	29.3	24.9	0.8	6.7	40.2	23.9	
12	jackets.	87.0	9.0	96.9	29.3	24.8	0.7	9.7	30.3	24.0	
13		87.7	12.5	102.4	29.3	24.8	0.8	13.4	19.9	23.4	
14		88.2	14.2	99.3	29.3	24.7	0.8	24.6	9.0	36.1	
15		83.7	7.8	106.4	29.3	21.7	0.8	.....	97.3	17.6	
16		84.6	10.2	106.0	29.3	21.5	0.8	.....	78.7	16.7	
17	Compound	85.6	16.2	105.3	29.3	22.0	0.7	.....	57.7	16.5	292
18	without steam	81.7	26.5	107.6	29.3	21.5	0.2	.....	35.7	19.2	
19	in jackets.	87.4	39.6	108.5	29.3	22.5	0.0	.....	24.6	22.8	
20		88.2	46.4	113.7	29.3	22.0	0.0	.....	11.4	35.6	
21		84.4	7.6	106.9	29.4	22.6	0 3	11.1	99.8	18.5	
22	16 × 36	85.5	11.4	106.3	29.4	22.7	0.3	14.3	77.4	18.2	316
23		86.0	17.2	112.7	29.3	21.5	0.8	14.3	58.2	18.5	
24	Compound with	87.2	32.0	112.1	29.3	21.8	0.8	19.3	35.6	20.2	
25	steam in h.p.	87.7	41.4	114.3	29.3	23.8	1.3	26.1	25.2	21.7	
26	and receiver	87.8	43.1	112.4	29.3	23.5	0.8	29.2	20.0	22.5	
27	jackets.	88.0	46.4	113.4	29.3	23.3	0.9	30.5	16.2	26.2	
28		85.0	9.0	94.5	*14.4	*11.3	0.7	18.9	76.9	18.0	
29	Compound,	85 3	10.1	95.2	14.5	11.6	0.7	14.6	71.4	17.3	
30	steam in all	85.1	10.3	94.5	.... ..	.....	0.8	15.5	76.2	17.4	
31	jackets.	85.4	12.5	95.4	14.4	11.6	0.5	13.8	72.4	16.3	284
32		85.7	13.6	103.0	14.2	11.2	0.9	18.0	65.9	18.2	
33		84.7	23.3	114.1	29.3	23.4	.....	.....	88.6	15.3	275
34	Triple without	85.9	36.6	115.4	29.3	22.7	.....	.....	66.1	18.0	
35	jackets.	86.9	53.4	114.7	29.3	22.7	. ..	. ..	46.1	19 9	
36		86.8	62.1	115.0	29.3	22.6	.....	. ...	35.5	24.1	
37		88.2	92.1	114.1	29.3	22.7	...	.. ...	22.7	27.5	
38		83.8	15.8	116.3	29.3	24.2	.....	16.7	141.4	15.3	
39		85.0	21.9	116.1	29.3	24.3	.....	21.8	112.7	13.7	237
40	Triple with	85.5	29.4	114.1	29.3	23.3	.....	24.4	89.8	14.9	
41	steam in h.p.	86.2	48.5	115.4	29.3	24.1	.....	27.9	63.0	16.8	
42	and int. and	87.0	69.0	114.7	29.3	22.9	.....	31.2	45.6	17.7	
43	receiver	87.5	88.1	115.0	29.3	22.1	... ..	33.8	33.9	21.1	
44	jackets.	88.0	96.7	114.1	29.2	21.6	.....	36.8	26.2	24.1	

* Pounds per square inch.

to be unfortunate, for the economy, measured either in terms of steam per horse-power per hour, or in terms of B. T. U. per horse-power per minute, is poor, even allowing for the low vacuum in the condenser. The condensation in the jackets appears to be normal, showing that the jackets were active, and were not using an excessive amount of steam, and yet the gain from using steam in the jackets is trivial, amounting to only

$$\frac{421 - 411}{421} \times 100 = 3 \text{ per cent.}$$

The detailed report by Professor Carpenter shows that the simple engine has nearly as much condensation and re-evaporation with steam in the jackets as without, except for tests with a very early cut-off. As the jackets did not have much effect on the action of the cylinder-walls they could not be expected to reduce the steam-consumption. For tests with an early cut-off, however, the jackets did reduce condensation and improved the economy. Thus tests 6 and 13 appear to show a large gain from the use of the jackets; but a similar gain can be obtained by lengthening the cut-off. This gives another illustration of the futility of trying to find the advantage of using steam-jackets from a pair of tests with and without steam in the jackets.

Three series of tests were made on the engine running compound, using the small and intermediate cylinders. Tests 15 to 20 were made without steam in any of the jackets and tests 28 to 32 were made with steam in all the jackets. Again, tests 21 to 27 were made with steam in the jackets of the high-pressure cylinder and the intermediate receiver, but with much worse results than when no steam was used in any of the jackets. It is a question whether the jacketing was not overdone in both series of tests with steam in the jackets, for the percentage of condensation of steam in the jackets is large, if not excessive. If the steam-consump-

tion is taken as the basis of comparison there appears to be little gain from the use of steam in the jackets; for example, the steam-consumption for test 17 is 16.5 pounds per horse-power per hour, and it is 16.3 for test 31. But a comparison of the thermal units per horse-power per minute for these same tests shows a gain of

$$\frac{292 - 275}{292} \times 100 = 6 \text{ per cent.}$$

It is probable that a better result could be obtained by jacketing the cylinders and not the intermediate receiver, or perhaps by jacketing the low-pressure cylinder only.

Tests on Pumping-engine at Laketown. — Table XXXVII gives the results of tests made on a triple-expansion pumping-engine at Laketown, Indiana, to show the effect of using steam in jackets on the cylinders and in reheaters placed in the intermediate receiver. Taking the steam-consumption as the basis of comparison, there appears to be only a trivial gain from the use of steam in the jackets. But when the comparison is made using thermal units per horse-power per minute the gain is

$$\frac{264 - 253}{264} \times 100 = 4 \text{ per cent.}$$

It appears that as good a result is obtained when steam is used in the jackets of the intermediate and low-pressure cylinders as when steam is used in all the jackets and the reheaters.

But the economy of this engine is poor—no better than that of the experimental engine at the Massachusetts Institute of Technology, which develops only half the power and is only a fifth as large. It is difficult to account for this poor performance, for, though the stroke is short, the valves, which are of the Corliss type, are in the cylinder-heads, so that the clearance is not excessive.

TABLE XXXVII.

TRIPLE-EXPANSION HORIZONTAL PUMPING-ENGINE AT LAKETOWN, INDIANA.

CYLINDER DIAMETERS 24½, 34, AND 54 INCHES; STROKE 36 INCHES.

By Professor J. E. DENTON, *Trans. Am. Soc. Mech. Engs.*, vol. xiii, p. 1340.

	1	2	3	4	5	6	7	8
	Steam in all jackets and in reheaters.					Steam in i. and l. jackets and reheaters.	Steam in i. and l. jackets.	No jackets or reheaters.
Duration	8	5	5	6	6	8	12	3
Revolutions per minute	27.8	27.6	28.3	27.3	27.7	28.0	27.9	27.7
Cut-off, h. p. cylinder, per cent	36	21	22	23	23	22	23	31
int. do. do.	39	41	43	40	40	43	43	40
l. p. do. do.	44	47	53	47	47	39	52	45
Pressures, pounds per square inch above atmosphere:								
Boiler	113	151	151	151	150	152	151	151
High-pressure jacket	113	151	151	151	150	.....	.....	.....
Intermediate- and low-pressure jackets and receivers	113	135	72	67	43	62	75	....
Vacuum, inches of mercury	26.2	26.1	26.2	26.1	26.2	26.2	26.2	26.1
Barometer, inches of mercury	29.8	28.8	29.4	28.7	29.3	29.1	29.3	29.1
Temperature jacket-water, degrees F.	336	356	308	300	280	299	310	.....
Per cent of moisture in steam	2.2	2	2	2.5	2.5	2.4	2	2
Percentage of condensation in jackets and reheaters	18	28	23	1.9	19	23	21	.....
Efficiency of mechanism	0.94	0.92	0.95	0.94	0.94	0.95	0.93	0.91
Horse-power, indicated	322	323	327	317	322	313	323	328
Steam per horse-power per hour, pounds	14.3	13.8	14.0	13.7	13 8	14.1	13.8	14.1
B. T. U. per horse-power per minute	260	248	253	250	253	256	252	264

Gain from Using Steam-jackets.—Reviewing all the tests on engines with and without steam in the jackets, bearing in mind the discrepancies which have been pointed out and not satisfactorily explained, it appears to be conservative to say that from 5 to 10 per cent may be saved by jacketing simple condensing-engines and compound condensing-engines, and that from 10 to 15 per cent may be saved by jacketing triple-expansion engines, provided that these conclusions shall not apply to engines of more than 300 horse-power. Most of the tests quoted are on engines which do not develop more than 150 horse-power.

The saving on massive engines of 1000 horse-power or

more is likely to be smaller, and very large engines may derive no benefit whatever from steam-jackets.

The saving from the use of jackets on small engines of five or ten horse-power may amount to 25 per cent or more. Isherwood found a gain of about 30 per cent from using steam in the jackets on an engine 5 inches in diameter by 10 inches stroke and developing one and a half horse-power. Such engines are seldom if ever provided with jackets, as the total fuel-consumption is of little importance, and simplicity and low first cost are more considered than economy of steam-consumption.

Intermediate Reheaters.—Many compound and triple-expansion engines have some method of reheating the steam on its way from one cylinder to another. Notable examples are the Leavitt pumping-engines, for which results are given in Table X. The fact that these engines give the best economies recorded for engines using saturated steam lead to the inference that such reheaters may be used to advantage. The only direct evidence, however, is not so favorable, for, as has been pointed out on page 394, there was found a small but distinct disadvantage from using steam in double walls or jackets on the intermediate receivers of the experimental engine at the Massachusetts Institute of Technology. It appears that this engine gives the best economy when steam is supplied to the jackets on the cylinders and not to the jackets on the reheaters, and, further, that when steam is used in the receiver-jackets the steam in the low-pressure cylinder shows signs of superheating, which may be considered to indicate that the use of the steam-jacket is carried too far.

After the tests referred to were finished the engine has been furnished with reheaters made of corrugated-copper tubing, so arranged that one-third, two-thirds, or all of the reheating-surface can be used, when desired. Table XXXV, page 393, gives the results of tests made on the engine with and without steam in the reheaters; in these tests the entire reheating-surface was used when steam was supplied to a

reheater; the effect of using part of the reheating-surface remains to be determined.

For some reason the heat-consumption when no steam was used in the reheaters is somewhat greater than that given in Table XXXIII for the engine without steam in any of the jackets; the difference, however, is not more than two or two and a half per cent and cannot be considered of much importance. It is clear from the table that there is advantage from using one reheater and still more from using two. If the heat-consumption for the engine without steam in the jackets and without steam in the reheaters (taken from Table XXXIII) is assumed to be 274 B. T. U. per minute, then the gain from using the reheaters appears to be

$$\frac{274 - 252}{274} \times 100 = 8 \text{ per cent},$$

which is scarcely more than half the gain from using steam in the jackets. These tests cannot be considered conclusive, as they are too few and refer only to one engine.

Willans's Tests.—Tables XXXVIII to XLII give the results of a very extensive investigation by Mr. Peter Willans on a peculiar engine, which he designed to run at high speed and give a good economy. His success is shown by the fact that one of his engines developing only 30 horse-power used 12.7 pounds of steam per horse-power per hour, as given in Table X.

Willans's tests covered a wide range of conditions, including variations of steam-pressures, number of expansions, speed of rotation, both with and without condensation, for his engine when run single, compound, or triple-expanding; it is consequently convenient to describe his engine and discuss his tests before considering the effects of the various conditions named on the economy of steam-engines.

The engine is a single-acting, vertical, three-cylinder, triple-expansion engine, with three pistons on a continuous piston-rod. The piston-rod is made hollow and serves as a

TABLE XXXVIII.

TESTS ON WILLANS ENGINE.

SIMPLE NON-CONDENSING.

	Duration, minutes.	Revolutions per minute.	Cut-off, per cent of stroke.	Pressures, pounds per square inch.			Horse-power.	Steam per horse-power per hour, pounds.		Percentage efficiency.	Per cent of water in cylinder.	
				Barometer.	Boiler above atmosphere.	Back-pressure, absolute.		Actual.	Required by perfect engine.		At cut-off.	At end of stroke.
1	180	393	0.60	14.5	36.3	14.8	16.5	42.8	34.7	81	12	10
2	270	408	0.44	14.5	51.0	14.5	20.0	36.0	28.6	80	19	18
3	242	409	0.34	14.6	74.0	15.4	25.5	32.6	23.0	71	27	19
4	169	403	0.30	14.7	85.0	15.4	26.8	29.7	21.2	72	24	19
5	180	400	0.26	14.8	97.0	15.6	31.6	26.9	19.2	72	25	19
6	176	398	0.24	14.6	110.0	15.4	31.5	27.8	18.7	67	31	23
7	298	406	0.22	14.7	122.0	15.6	33.6	26.0	17.9	69	30	22
8	270	408	0.44	14.5	51.0	14.5	19.8	36.0	28.6	80	19	18
9	153	201	0.44	14.6	44.0	15.2	9.3	41.8	29.2	70	24	18
10	122	111	0.44	14.4	40.2	14.7	6.5	46.0	29.4	64	35	29
11	242	409	0.34	14.6	74.0	15.4	25.5	32.6	23.0	71	27	19
12	152	205	0.34	14.5	66.5	15.1	14.1	34.4	22.5	65	34	23
13	127	113	0.34	14.4	62.0	14.7	7.9	40.7	22.9	56	46	33
14	180	401	0.26	14.8	97.0	15.6	31.6	26.9	19.2	72	25	19
15	118	223	0.26	14.7	84.7	15.5	16.8	27.8	19.7	71	25	20
16	178	123	0.26	15.0	80.0	15.4	10.0	34.1	19.6	58	43	29
17	298	406	0.22	14.7	122.0	15.0	33.6	26.0	17.9	69	30	21
18	119	224	0.22	15.0	112.0	15.5	20.5	30.2	17.7	58	42	26
19	123	138	0.22	14.7	105.4	15.5	13.1	31.2	17.7	57	45	31

TABLE XXXIX.

TESTS ON WILLANS ENGINE.

SIMPLE CONDENSING.

	Duration, minutes.	Revolutions per minute.	Cut-off.	Pressures, pounds per square inch.			Indicated horse-power.	Steam per horse-power per hour, pounds.		Percentage efficiency.	Per cent of water in cylinder.	
				Barometer.	At steam-chest, absolute.	Back-pressure, absolute.		Actual.	Required by perfect engine.		At cut-off.	At end of stroke.
1	139	382	0.5	14.6	64.7	1.0	31.6	25.7	11.2	44	16	20
2	110	380	0.5	15.1	55.1	1.0	27.2	25.2	11.5	46	17	15
3	173	381	0.5	14.7	44.8	1.0	21.9	26.7	11.2	42	21	19
4	162	382	0.5	15.0	35.0	0.9	16.1	28.9	12.1	42	24	20
5	181	385	0.5	15.0	25.0	0.9	11.5	30.0	12.8	43	25	18
6	191	378	0.5	15.0	20.0	0.8	9.1	29.4	13.4	46	23	23
7	128	383	0.29	15.1	85.1	1.0	33.2	22.2	10.3	47	25	19
8	186	382	0.29	15.1	75.1	1.0	29.0	23.4	10.5	45	28	23
9	159	380	0.29	15.1	65.1	0.9	24.8	24.0	10.6	44	28	22
10	130	378	0.29	15.1	45.0	0.8	16.8	26.2	11.0	42	33	23
11	146	382	0.29	15.1	25.1	0.8	9.18	28.2	12.0	43	39	30
12	138	380	0.29	15.1	20.1	0.9	6.9	30.0	13.2	44	39	24

TABLE XL.

TESTS ON WILLANS ENGINE.

COMPOUND NON-CONDENSING.

	Duration, minutes.	Revolutions per minute.	Point of cut-off.		Pressures, pounds per square inch.			Indicated horse-power.	Steam per horse-power per hour, pounds.		Percentage of efficiency.	Per cent of water in cylinder.		
			High-pressure cylinder.	Low-pressure cylinder.	Barometer.	Boiler above atmosphere.	Back-pressure, absolute.		Actual.	Required by perfect engine.		Cut-off, h. p. cylinder.	Cut-off, l. p. cylinder.	Release, l. p. cylinder.
1	180	400	0.6	0.6	15.0	80.2	15.8	24.9	26.2	21.2	81	5	15	14
2	123	402	0.6	0.6	15.0	90.5	15.4	29.1	24.2	19.9	82	5	15	14
3	177	398	0.52	0.6	14.6	90.4	15.6	26.1	24.5	19.6	80	8	17	15
4	118	402	0.52	0.6	14.6	100.5	15.1	30.0	23.0	18.6	81	6	21	17
5	364	405	0.47	0.6	14.7	104.6	15.0	28.7	23.8	18.7	82	10	16	15
6	123	403	0.47	0.6	14.9	113.9	15.7	33.0	21.4	17.7	83	10	16	17
7	121	403	0.43	0.6	15.0	113.0	15.7	31.0	21.4	17.8	83	11	13	17
8	181	403	0.43	0.6	15.0	124.5	15.4	34.7	20.8	16.8	81	11	20	18
9	124	404	0.39	0.6	14.9	124.8	15.8	32.3	21.3	16.9	79	13	20	16
10	189	406	0.39	0.6	14.8	135.4	15.4	36.3	20.4	16.3	80	12	19	18
11	306	402	0.36	0.6	14.7	133.6	15.2	33.3	20.3	16.3	80	14	17	17
12	184	399	0.36	0.6	14.6	143.7	15.4	36.6	20.0	15.7	79	14	20	18
13	120	405	0.33	0.6	14.4	145.2	15.2	36.4	19.7	15.6	79	15	20	18
14	179	404	0.33	0.6	14.1	155.0	15.0	38.6	19.5	15.2	78	15	21	20
15	307	402	0.31	0.6	.....	..	.. ..	37.0	19.6	.....	.. ..	19	.....	....
16	177	401	0.31	0.6	14.5	165.0	14.9	39.6	19.2	14.9	77	17	21	20
17	186	407	0.47	0.6	15.0	134.8	15.5	40.0	20.8	16.3	79	9	20	19
18	182	405	0.43	0.6	15.0	135.0	15.0	38.0	20.5	16.3	80	11	20	19
19	189	406	0.39	0.6	14.8	135.4	15.4	36.3	20.3	16.3	80	12	19	18
20	306	402	0.36	0.6	14.7	133.7	15.1	33.3	20.3	16.3	80	14	17	17
21	182	403	0.34	0.6	14.4	135.0	15.4	32.8	20.0	16.2	81	14	16	13
22	180	400	0.31	0.6	14.2	135.0	14.8	31.2	20.3	16.3	80	18	19	17
23	200	404	0.23	0.6	14.8	135.5	15.4	23.0	23.1	16.4	71	25	18	14
24	123	401	0.60	0.6	14.9	90.5	15.4	29.1	24.2	19.9	82	5	15	14
25	123	210	0.60	0.6	14.8	85.2	15.5	16.7	25.3	19.5	77	13	25	19
26	150	122	0.60	0.6	14.6	83.5	15.4	10.0	27.0	19.5	72	20	31	27
27	123	402	0.47	0.6	14.9	113.9	15.7	33.0	21.4	17.7	83	10	17	16
28	125	212	0.47	0.6	14.6	105.0	15.2	18.3	23.1	17.7	76	20	27	23
29	116	124	0.47	0.6	14.6	102.6	15.0	11.1	24.7	17.5	71	25	31	28
30	189	406	0.39	0.6	14.8	135.4	15.4	36.3	20.4	16.3	80	12	19	18
31	124	216	0.39	0.6	14.6	124.0	15 6	20.3	21.3	16.3	77	19	26	23
32	150	131	0.39	0.6	14.6	120.0	15.0	12.7	23.7	16.3	69	30	33	27

passage for the steam to and from the cylinders. The distribution of steam is accomplished by three single-acting piston-valves on one continuous valve-spindle inside the hollow piston-rod. The ports are cut through the side of the hollow piston-rod at the proper places above and below each piston. The cut-off is produced by allowing the admission-port to run into the packing-ring in the cylinder-head through which the piston-rod works, and is consequently very sharp.

TABLE XLI.

TESTS ON WILLANS ENGINE, COMPOUND CONDENSING.

	Duration, minutes.	Revolutions per minute.	Point of cut-off.		Pressures, pounds per square inch.			Indicated horse-power.	Steam per horse-power per hour, pounds.		Percentage of efficiency.	Per cent of water in cylinder.			
			High-pressure cylinder.	Low-pressure cylinder.	Barometer.	Boiler, absolute.	Back-pressure, absolute.		Actual.	Required by perfect engine.		Cut-off, h. p. cylinder.	Release, h. p. cylinder.	Cut-off, l. p. cylinder.	Release, l. p. cylinder.
1	178	402	0.58	0.45	14.7	134.7	1.1	40.1	16.7	9.5	57	11	10	28	18
2	189	405	0.58	0.45	14.7	114.7	1.1	33.2	17.0	9.8	58	11	8	28	21
3	175	401	0.58	0.45	15.1	90.1	1.1	25.6	17.3	10.1	58	9	9	30	21
4	177	404	0.58	0.45	14.6	64.6	1.0	18.7	18.0	10.4	58	12	10	27	17
5	151	399	0.58	0.45	14.6	42.6	1.0	10.8	20.3	10.8	53	12	10	32	15
6	137	311	0.58	0.45	14.8	134.8	1.0	31.0	16.3	9.3	57	11	7	29	21
7	147	311	0.58	0.45	14.6	114.6	0.9	25.7	16.9	9.2	55	11	8	30	23
8	139	301	0.58	0.45	14.5	89.5	0.9	19.5	17.6	9.7	55	12	9	34	22
9	184	302	0.58	0.45	15.2	65.2	1.1	14.0	18.5	10.3	55	13	11	36	24
10	179	300	0.58	0.45	14.7	39.7	1.1	7.9	22.0	11.2	51	17	12	40	21
11	121	203	0.58	0.45	14.6	129.6	0.9	19.9	17.1	8.6	50	12	10	38	27
12	120	198	0.58	0 45	14.8	89.8	0.9	13.3	19.0	9.0	48	18	13	45	31
13	156	203	0.58	0.45	14.7	64.7	1.0	9.4	20.1	9.7	48	18	12	46	33
14	171	196	0.58	0.45	14.8	39.8	1.0	5.3	23.8	10.9	46	26	16	48	30
15	132	115	0.58	0.45	14.6	119.6	1.0	9.0	19.7	8.3	42	19	16	51	43
16	194	116	0.58	0.45	14.8	89.8	0.9	6.7	20.1	8.7	43	21	16	51	37
17	170	112	0.58	0.45	14.8	42.8	1.1	2.9	27.0	10.6	39	32	24	59	39
18	123	397	0.28	0.45	14.6	164.6	0.9	33.2	14.8	11.0	57	26	17	32	27
19	175	399	0.28	0.45	14.6	139.6	1.0	27.1	15.2	11.7	57	23	15	33	25
20	171	402	0 28	0.45	14.7	114.7	0.8	22.1	15.8	12.3	55	22	16	38	26
21	114	396	0.28	0.45	14.8	64 8	0.9	11.7	18.2	12.4	52	32	16	45	30
22	155	394	0.28	0.45	14.9	65.9	0 8	11.9	18.3	13.1	51	28	15	42	27
23	163	296	0.28	0.45	14.7	139.7	0.9	22.1	15.2	10.9	56	28	16	37	28
24	173	199	0.28	0.45	14.7	139.7	0.9	14.8	16.5	11.4	49	31	19	48	39
25	166	118	0.28	0.45	14.8	163.8	0.9	8.8	17.2	11.3	46	34	25	52	43
26	152	402	0.18	0.45	14.9	174.9	0.9	27.5	14.3	9.9	58	31	14	36	25
27	121	398	0.18	0.45	14.9	139.9	0.8	21.6	15.0	10.3	56	31	19	37	27
28	169	407	0.18	0.45	15.0	93.0	0 8	13.2	17.2	10.4	51	40	18	44	30
29	157	304	0.18	0.45	15.1	165.1	0.7	19.9	15.2	9.7	52	36	16	43	33
30	174	300	0.18	0.45	15.2	93.2	0.8	10.6	17.8	11.0	49	38	23	49	41
31	157	203	0.18	0.45	15.2	164.2	1.0	13.5	16.8	9.7	47	42	26	48	40
32	164	199	0.18	0.45	15.2	80.2	1.0	6.0	21.5	10.4	42	51	23	56	46
33	211	404	0.14	0.45	14.7	186.7	1.2	24.9	14.7	9.2	57	37	16	35	26
34	158	396	0.14	0.45	14.6	119.6	1.0	16.0	15.5	9.4	57	39	22	39	29

TABLE XLII.

TESTS ON WILLANS ENGINE. TRIPLE-EXPANSION.

Condition.		Duration, minutes.	Revolutions per minute.	Point of cut-off.			Pressures, pounds per square inch.			Indicated horse-power.	Steam per horse-power per hour, pounds.		Percentage efficiency.	Per cent of water in cylinder.					
				High-pressure cylinder.	Intermediate cylinder.	Low-pressure cylinder.	Barometer.	Steam, absolute.	Back-pressure, absolute.		Actual.	Required by perfect engine.		Cut-off, h. p. cylinder.	Release, h. p. cylinder.	Cut-off, int. cylinder.	Release, int. cylinder.	Cut-off, l. p. cylinder.	Release, l. p. cylinder.
Non-condensing	1	467	406	0.69	0.6	0.6	14.8	164.9	15.4	35.7	19.7	15.2	77	5	.....	15	.....	22	18
	2	241	409	0.65	0.6	0.6	14.5	166.4	14.8	34.6	19.4	15.3	79	5	.....	12	.....	20	18
	3	300	401	0.65	0.6	0.6	15.0	175.0	15.3	36.4	19.1	14.6	76	4	.....	16	.....	23	19
	4	300	408	0.61	0.6	0.6	14.5	175.3	14.8	35.6	19.2	14.9	78	7	.....	12	.....	22	19
	5	240	415	0.61	0.6	0.6	14.7	186.7	.....	40.4	18.7	.....	.....	10	.....	11	.....	21	15
	6	184	400	0.01	0.6	0.6	15.0	187.5	15.4	38.5	18.5	14.4	78	5	.....	15	.....	24	19
	7	243	416	0.61	0.6	0.6	14.9	.....	15.3	40.4	18.4	.....	78	.....	.....	.....	.....	.....	.....
Condensing	8	165	379	0.5	0.58	0.45	14.5	184.5	0.7	29.5	13.0	7.8	60	13	12	20	16	35	26
	9	181	384	0.5	0.58	0.45	14.6	184.6	0.8	29.8	12.7	8.0	63	14	10	17	15	34	23
	10	184	381	0.5	0.58	0.45	14.6	164.6	0.7	26.7	12.9	8.0	62	13	11	17	14	35	26
	11	176	384	0.5	0.58	0.45	14.6	134.6	0.8	21.3	13.4	8.2	62	15	9	17	13	36	26
Condensing	12	180	302	0.5	0.58	0.45	14.6	184.6	0.7	23.1	12.9	7.8	60	16	14	19	17	41	33
	13	166	300	0.5	0.58	0.45	14.7	139.7	0.7	16.8	14.0	7.9	56	19	14	24	19	48	40
	14	117	302	0.5	0.58	0.45	14.8	58.8	0.7	6.7	17.0	9.3	55	29	17	34	25	49	33
Condensing	15	147	375	0.3	0.58	0.45	14.7	184.7	0.7	22.3	12.7	7.7	60	26	17	24	20	42	34
	16	211	380	0.3	0.58	0.45	14.7	149.8	1.0	18.3	13.1	8.3	63	27	16	23	20	43	30
	17	149	376	0.3	0.58	0.45	14.8	79.8	0.7	9.1	15.3	8.7	57	30	18	30	25	43	29

Steam is admitted to the upper sides of the pistons and exhausted on the down stroke; on the up stroke the steam is transferred to the under sides of the pistons. The spaces below the high-pressure and intermediate pistons have considerable volume and serve as receiver-spaces; but, as the volumes of these spaces increase during the up stroke and as work is done on the under side of the pistons on that stroke, each receiver-space gives an intermediate expansion, and there are in reality five stages of expansion for the triple engine. To insure a downward thrust at all times on the connecting-rod, an air-cushion is provided at the lower end of the hollow piston-rod. For this purpose, and also to provide a guide for the lower end of the piston-rod, there is a trunk or plunger working in a cylinder which is closed at the top and filled with air that is always at a pressure somewhat greater than that of the atmosphere; on the up stroke this air is compressed and it expands again on the down stroke.

Two or three of these engines are placed side by side on the same bed; two engines with their cranks opposite are equivalent to an ordinary double-acting engine; three engines with their cranks at 120° form a very smoothly running combination.

Mr. Willans first made tests in 1887 on one of his engines which was designed to run without condensation, and four years later made further tests on a condensing-engine which had somewhat different proportions, to accord with the greater range of pressures and temperatures available with a vacuum. The dimensions of these engines are as follows:

	Non-condensing.	Condensing.
Diameters, inches :		
High-pressure cylinder	7	6
Intermediate cylinder	10	8.5
Low-pressure cylinder	14	14
Stroke, inches	6	6

One single-acting engine was used for the tests in both cases to make the work simpler and easier; two or three engines running in combination would of course do two or

three times as much work and use two or three times as much steam. During the tests the speed of the engine was limited to 400 revolutions per minute for the sake of getting clearer indicator-diagrams; in practice the engines tested run at 500 revolutions per minute.

In the tests on the non-condensing engine the steam was drawn from a special boiler, and the steam-consumption was determined by weighing the feed-water, special precautions being taken to have the water-level and the rate of vaporization in the boiler the same at the beginning and end of a test; Mr. Willans estimated the error in the determination of the steam used for a test at one fourth of one per cent. During the tests on the condensing engine steam was drawn from a general supply, and the exhaust-steam was condensed in a surface-condenser, and was collected and weighed in a closed tank to avoid surface evaporation. In both cases the steam was found, by proper tests, to be dry and saturated near the engine.

By removing the upper piston and supplying steam to the intermediate cylinder directly, the engine was run as a compound engine; but as the steam was transferred to the space under the piston and did work on it before going to the low-pressure cylinder, the engine when running compound had really three stages of expansion. Again, by removing the two upper pistons the engine was run as a simple engine.

The only quantities in the table which require explanation are those in the columns headed *Steam per horse-power per hour required by a perfect engine* and *Percentage of efficiency*. The first was calculated by a method which is nearly equivalent to that given on page 235, and the efficiency is calculated by dividing the steam-consumption for a non-conducting engine by the actual steam-consumption. Since all the steam passed through the cylinders of the engine, this method is proper for this engine. The per cent of water in the cylinder is a rough index of the extent of the influence of the walls of the cylinder.

Compounding.—The most efficacious method which has been devised to increase the amount of expansion of steam in an engine, and at the same time to avoid excessive cylinder-condensation, is compounding; that is, passing the steam in succession through two or more cylinders of increasing size. An engine with two cylinders, a small or high-pressure cylinder and a large or low-pressure cylinder, is called a compound engine. An engine with three cylinders, a high-pressure cylinder, an intermediate cylinder, and a low-pressure cylinder, is called a triple-expansion engine. A quadruple engine has a high-pressure cylinder, a first and a second intermediate cylinder, and a low-pressure cylinder. Any cylinder of a compound or multiple-expansion engine may be duplicated, that is, may be replaced by two cylinders which are usually of the same size. Thus, at one time a compound engine with one high-pressure and two low-pressure cylinders was much used for large steamships. Many triple engines have two low-pressure cylinders, which with the high-pressure and the intermediate cylinders make four in all. Again, some triple engines have two high-pressure cylinders and two low-pressure cylinders and one intermediate cylinder, making five in all.

Two main questions are presented for investigation: (1) under what conditions should compounding be resorted to, and (2) how much gain may be expected? The answer to either question will be modified by the type of engine considered, and in only a few cases can explicit conclusions be drawn. The most complete investigation of these questions is that by Mr. Willans; our only regret is that we must hesitate to apply conclusions from tests on so small and so peculiar an engine to large engines of more common construction. And yet the good economy shown by his engine under all conditions gives an importance to the conclusions from his tests which can seldom be attributed to tests on small engines.

Taking first the tests on the non-condensing Willans engine, we may compare the first seven tests in Table

XXXVIII, the first sixteen in Table XL, and the first seven in Table XLII, to determine the advantage of using a compound or a triple-expansion engine, without a vacuum. Mr. Willans chose the number of expansions (and the cut-off for the cylinder taking steam from the boiler) by a quasi-theoretical method, which is too intricate to be given here. It is enough to say that it gave a terminal pressure, in the cylinder exhausting into the atmosphere, of about five pounds above the atmosphere. One series of tests, namely, 17 to 23, Table XL, gives direct evidence on this matter, and shows that for a boiler-pressure of 135 pounds the cut-off may be varied from 0.31 to 0.43 of the stroke of the high-pressure cylinder without serious effect on the economy. For the remainder of the tests on the non-condensing engine we have no other direct evidence regarding the effect of cut-off, than the fact that the steam-consumption for any test is as good as the conditions would lead us to expect for an engine of the size, and in general the results are surprisingly good.

The results of tests on the non-condensing engine, when running simple or compound, are plotted on Fig. 82, using the absolute boiler-pressure for abscissæ and the steam-consumption for ordinates. The results of tests on the engine when running triple-expanding are not plotted on the diagram to avoid confusion, and because it does not appear necessary. The curve for compound tests is well located and shows conclusively that little if any gain is to be expected from increasing the boiler-pressure above 180 pounds; indeed it appears wise to limit the pressure to 165 pounds absolute or 150 pounds above the atmosphere. An inspection of the results of tests 1 to 7 of Table XLII shows that there is no advantage in increasing the boiler-pressure for the non-condensing triple engine above 185 pounds absolute or 170 pounds above the atmosphere. Neither the results of tests in Table XXXVIII nor the curve on Fig. 82 show conclusively the maximum economical steam-pressure for simple engines, but it is doubtful whether there is any real advantage in raising the steam-

pressure above 125 pounds absolute or 110 pounds above the atmosphere. These limits of 110 pounds for a simple engine, 150 pounds for a compound engine, and 170 pounds for a triple engine can apply only to non-condensing engines and

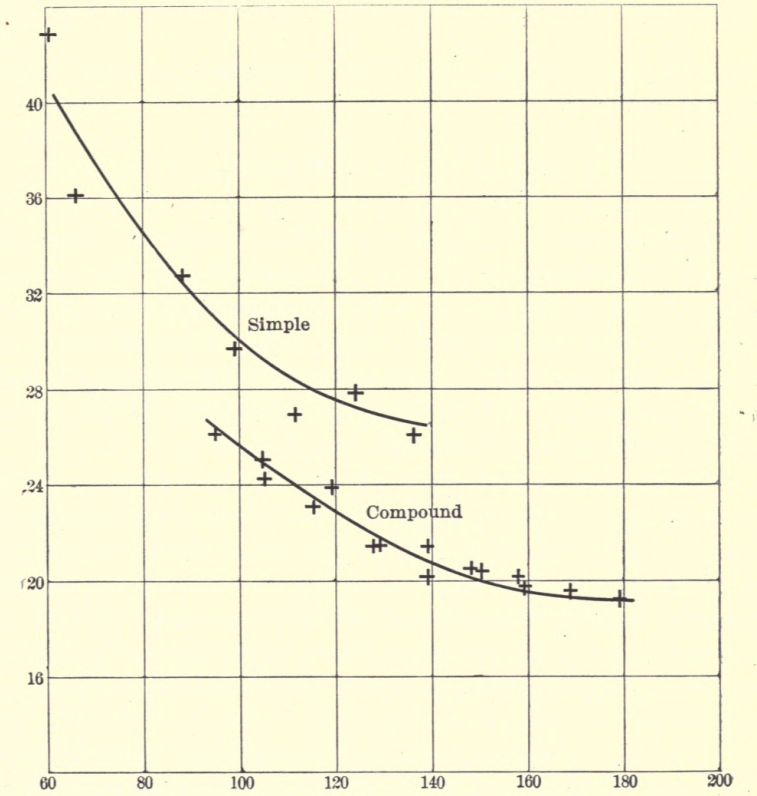

FIG. 82.

should be restricted to engines of the Willans type or of similar types.

To determine the gain from compounding we may consider that the steam-consumption for the simple engine is 27 pounds per horse-power per hour with a boiler-pressure of 125 pounds absolute, that the consumption for the compound engine is 19.5 pounds for a boiler-pressure of 165 pounds

absolute, and that the consumption for the triple engine is 18.5 pounds for a boiler-pressure of 185 pounds absolute.

The gain from compounding is therefore

$$\frac{27 - 19.5}{27} \times 100 = 28 \text{ per cent.}$$

The gain from using the triple engine instead of the compound engine is

$$\frac{19.5 - 18.5}{19.5} \times 100 = 5 \text{ per cent.}$$

Coming now to Mr. Willans' tests on his condensing engine, we find that for each arrangement of the engine (simple, compound, and triple) he made several series of tests with constant cut-off and varying steam-pressure for each series. Two such series were made for both the simple and the triple engine when running at full speed, and four series at full speed were made for the compound engine.

As has already been pointed out, the series of simple tests with the cut-off at 0.5 of the stroke showed no advantage in raising the pressure beyond 55 pounds absolute; the series with the cut-off at 0.29 of the stroke showed that the steam-pressure could be advantageously raised to at least 85 pounds absolute, and under these conditions the steam-consumption of 22.2 pounds per horse-power per hour was probably a minimum.

The engine running compound-condensing gives its best economy (14.3 pounds of steam per horse-power per hour) when running at full speed with 175 pounds absolute steam-pressure, and with the cut-off for the high-pressure cylinder at 0.18 of the stroke, but nearly as good results are attained for a cut-off at 0.14 and for a cut-off at 0.28 of the stroke. The two series for the triple engine with the cut-off for the high-pressure cylinder at 0.3 and at 0.5 of the stroke show the same economy of 12.7 pounds of steam per horse-power per hour with the steam-pressure of 185 pounds absolute.

The gain from compounding is therefore

$$\frac{22.2 - 14.3}{22.2} \times 100 = 35 \text{ per cent};$$

and the gain from using a triple instead of a compound engine is

$$\frac{14.3 - 12.7}{14.3} \times 100 = 11 \text{ per cent.}$$

It is very interesting to consider the effect of the increase of steam-pressure on the percentage of efficiency of the engine in these tests on Mr. Willans' engine. This percentage is greater for the non-condensing than for the condensing engine, and is in general greater for low than for high pressure, which can readily be accounted for by the greater initial condensation and re-evaporation accompanying a wider range of pressure and temperature. It is, however, notable that the compound and triple engines maintain the efficiency better than the simple engine does, and that the percentage of efficiency is high for those engines at all pressures. The non-condensing engine shows about the same ratio of efficiency when running compound and when running triple; but the condensing engine has a decided advantage in this regard when running triple. This agrees very well with the fact that there is a gain of 11 per cent for the condensing engine, and only 5 per cent for the non-condensing engine, to be attained by running triple instead of compound.

The tests on this engine at reduced speeds will be referred to later.

Gain from Compounding.—In considering the gain to be attained by compounding, it has sometimes been considered that the engines compared should have the same steam-pressure, in order that the comparison should be fair to the simple engine; and a similar condition has been claimed for comparisons of compound and triple engines. But the object of compounding is the ability to use high pressures and large

ratios of expansion to advantage. Each engine should therefore be run under its best condition, using as high a pressure and as large an expansion as may be profitable. Of course each engine should have the advantage of using steam-jackets or superheated steam, or else both should be without these aids. In general, comparisons should be made only between engines which show a good economy for their respective types.

Let us take for example tests on the experimental engine at Cornell University using all three cylinders to form a triple engine, the two smaller cylinders to form a compound engine, or the small cylinder as a simple engine.

CORNELL EXPERIMENTAL ENGINE.

Data and Results.	Simple.	Compound.	Triple.
Revolutions per minute	85.4	85.4	85.0
Steam-pressure above atmosphere	97.7	95.4	116.1
Total expansion	3.9	12.5	21.9
Steam per horse-power per hour, pounds	23.1	16.3	13.7
B. T. U. per horse-power per minute	411	284	237

It is probable that the simple engine has too high steam-pressure and the triple engine has too low pressure. However, making comparisons as the results stand, we have as the gain from compounding

$$\frac{411 - 284}{411} \times 100 = 31 \text{ per cent},$$

and from using a triple instead of a compound engine

$$\frac{284 - 237}{284} \times 100 = 20 \text{ per cent}.$$

On the other hand, a comparison of the results of tests on the engines of the *Rush* and the *Gallatin* (Table XXXII, page 388, tests 18 and 36) shows a gain of only

$$\frac{20.5 - 18.4}{20.5} \times 100 = 10 \text{ per cent};$$

but it is very clear that the steam-pressure and the expansion are too small for the best economy of a compound engine. And again, while the *Gallatin* shows a fair economy for a simple engine, the economy of the *Rush* is poor.

Taking now the best results of tests on simple, compound, and triple engines from Table X, all being supplied with steam-jackets, and the compound and triple engines with intermediate reheaters, we have the following results:

Data and Results.	Simple Corliss at Creusot.	Compound Leavitt at Louisville.	Triple Leavitt at Chestnut Hill.
Revolutions per minute	60	18.6	50.6
Steam-pressure above atmosphere, pounds	84	137	176
Total expansion	9	20	21
Steam per horse-power per hour, pounds	16.9	12.2	11.2
B. T. U. per horse-power per minute	.....	222	204

Using the steam-consumption as the basis of comparison, we have for the gain from compounding

$$\frac{16.9 - 12.2}{16.9} \times 100 = 28 \text{ per cent};$$

and for the gain from using a triple instead of a compound engine

$$\frac{12.2 - 11.2}{12.2} \times 100 = 8 \text{ per cent.}$$

The total expansion for the two Leavitt engines is nearly the same, but is obtained by different means. The ratio of the large to the small cylinder of the compound engine is a trifle less than four, and the cut-off for the high-pressure cylinder is a little less than one-fifth stroke. The triple engine has a little more than eight for the extreme ratio of the cylinders, and has the cut-off for the high-pressure cylinder at a little more than four tenths. In the design of a compound engine the desired expansion may be attained either by using a small ratio for the cylinders and a short cut-off for the high-pressure

cylinder, or by using a large ratio for the cylinders and a long cut-off on the high-pressure cylinder. The compound pumping-engine at Louisville now under discussion was designed by the first method, giving remarkable results. Some designers have, on the other hand, claimed an exceptional economy for certain compound engines which have a large ratio of large to small cylinders, and have preferred them to triple engines. In confirmation we have the results of tests in Table XX, page 362, on an engine which was run triple, using three cylinders, and compound, using only the large and small cylinders, omitting the intermediate cylinder. These tests show as good an economy for the compound engine as for the triple engine, and tests given in Table XXIII on an engine with a larger ratio of cylinders and a very large total expansion show even better economy. But the result (13 pounds of steam per horse-power per hour), though good, is not so low as that for the Louisville engine, and is but little better than is given in Table XIX for a compound mill-engine at New Bedford, which has a moderate ratio of cylinders and much smaller expansion. The New Bedford engine was supplied with slightly superheated steam, but had no steam-jackets. Finally the indicator diagrams from the high-pressure cylinder of the Holyoke engine (Table XX) when running triple showed clearly that the cut-off for the intermediate cylinder was too short and the back-pressure for the high-pressure cylinder was too high, because the expansion line reached the back-pressure line at about two-thirds stroke. It cannot therefore be determined which method of obtaining the desired expansion for a compound engine is the better.

Marine engines will not run smoothly with a short cut-off, and consequently a large number of expansions can be obtained only by using a large ratio of cylinders. But even for triple engines the ratio of the large to the small cylinder is commonly five or six and the total expansions are correspondingly small. The advantage of using a larger ratio, when allowable, is shown by comparing the tests in Table

XIV, page 358, on the *Meteor* and on the *Iona;* the engine of the *Iona* had the further advantage of higher steam-pressure and a relatively early cut-off on the high-pressure cylinder.

In conclusion, it may be said that for condensing engines there is no advantage in using more than 80 pounds steam-pressure, while compound engines may advantageously have the pressure raised to 135 pounds above the atmosphere. The gain from higher steam-pressure and compounding will be 25 to 30 per cent. The best pressure for triple engines cannot now be determined from experiments; it is, however, doubtful if there is any advantage in using more than 175 pounds above the atmosphere. Such a further increase of pressure and the use of a triple instead of a compound engine may be expected to give 8 or 10 per cent better economy.

For a simple non-condensing engine the steam-pressure may be 100 to 115 pounds above the atmosphere, and for a compound engine the pressure may be 150 pounds, while for a triple engine the pressure may be 175 pounds or possibly somewhat more. The gain from compounding will be 20 to 30 per cent, and the gain from using a triple instead of a compound engine will be 5 per cent or perhaps a little more.

These conclusions apply only when the engine is run at full power and at the best point of cut-off or the most economical total expansion. In general the compound engine will suffer more loss of economy when running at a reduced load than a simple engine will; and a triple engine will suffer even more than a compound engine from the same cause.

Cut-off and Expansion.—It has already been pointed out on page 385 in connection with Delafond's tests that the best point of cut-off for a simple engine, whether jacketed or not, is about one-third stroke when the engine is non-condensing and it is about one-sixth stroke when condensing. In general, other tests on simple engines such as those on the *Bache*, *Dexter*, and *Gallatin* (Table XXXII, page 388), and on the small Corliss engine at the Massachusetts Institute of Technology (Tables II and XXVII, pages 318 and 371), confirm

these conclusions. Tests on the experimental engine at Cornell (Table XXXVI, page 397) indicate that a longer cut-off than one-sixth may be used to advantage, but these tests were made with a poor vacuum and consequently show a poor economy for all grades of cut-off.

The term *total expansion* for a compound or a triple engine can properly have only a conventional significance; it is usually taken to be the product of the ratio of the large to the small cylinder by the reciprocal of the fraction of the stroke at cut-off for the high-pressure cylinder. This conventional total expansion is about 20 for all the tests on triple engines quoted in Table X, page 354, except those on marine engines, which show a relatively poor economy. It may therefore be concluded that it is not advisable to use much more expansion for any triple engine, and that less expansion should be used only when the conditions of service (for example, at sea) prevent the use of large expansion.

The case is not so clear for compound engines. The Louisville engine, which gives the best economy of any compound engine quoted in Table X, has 20 expansions. The Natick engine has 33 expansions, which are clearly excessive, and the engine at New Bedford (page 361) has only 13.4. The Cornell engine (page 397) gives its best economy when running compound with 12 or 16 expansions. We may conclude that in general 15 expansions may be used to advantage; marine engines and other engines which cannot readily be designed for so much expansion may be expected to give a less economy in consequence.

Variation of Load.—In general an engine should be so designed that it may give a fair economy for a considerable range of load or power. Very commonly the engine will have sufficient range of power with good economy if designed to give the best economy at the normal load. In general, however, it is well to assign a less expansion and consequently a longer cut-off to the engine than would be determined from a consideration of the steam- (or heat-) consumption alone. For,

in the first place, the best brake or dynamic economy is always attained for a little longer cut-off than that which gives the best indicated economy, and in the second place the economy is less affected by lengthening than by shortening the cut-off. The first comes from the fact that the frictional losses of the engine increase less rapidly than the power, as will be shown in the next chapter; and the second is evident from consideration of curves of steam-consumption as given by Fig. 81, page 390, and Figs. 79 and 80, pages 383–4.

The allowable range of power for a simple engine is greater than for a compound or a triple engine. Comparisons for a simple and a triple engine may be made by aid of Figs. 80 and 81. The Corliss engine at Creusot when supplied with steam at 60 pounds pressure, with condensation and with steam in the jacket, developed 150 horse-power and used 17.3 pounds of steam per horse-power per hour. If the increase be limited to 10 per cent of the best economy, that is, to 19 pounds per horse-power per hour, the horse-power may be reduced to about 92, giving a reduction of nearly 40 per cent from the normal power. The triple engine at the Massachusetts Institute of Technology with steam at 150 pounds pressure and using steam in all the cylinder-jackets developed 140 horse-power and used 233 B. T. U. per horse-power per minute. Again, limiting the increased consumption to 10 per cent or to 254 B. T. U., the power may be reduced to about 104 horse-power, giving a reduction of 26 per cent from the normal power. The effect of increasing power for these engines cannot be well shown from the tests made on them, but there is reason to believe that the simple engine would preserve its advantage if a comparison could be made. Though the tests which we have on compound engines do not allow us to make a similar investigation of the effect of changing load, there is no doubt that it is intermediate in this respect between the simple and the triple engine.

When the power developed by a compound engine is reduced by shortening the cut-off of the high-pressure cylin-

der, the cut-off of the low-pressure cylinder must be shortened at the same time to preserve a proper distribution of power and division of the range of temperature between the cylinders. If this is not done the work will be developed mainly in the high-pressure cylinder, which will be subjected to a large fluctuation of temperature, and the engine will lose the advantages sought from compounding. A compound non-condensing engine, if the cut-off for the large cylinder is fixed, is likely to have a loop on the low-pressure indicator-diagram due to expansion below the atmosphere, if the power is reduced by shortening the cut-off of the high-pressure cylinder. Such a loop is always accompanied by a large loss of economy; if the loop is large the engine may be more wasteful than a simple engine, for the high-pressure piston develops nearly all the power and may have to drag the low-pressure piston, which is then worse than useless.

There is seldom much difficulty in running a simple engine at any desired reduced power by shortening the cut-off or reducing the steam-pressure, or by a combination of the two methods. But a compound engine sometimes gives trouble when run at very low power (even when attention is given to the cut-off of the low-pressure cylinder), which usually takes the form just discussed; i.e., the power is developed mainly in the high-pressure cylinder. Triple engines are even more troublesome in this way. A compound or triple engine running at much reduced power is subject not only to loss of economy and to irregular action, but the inside surface of the low-pressure cylinder is liable to be cut or abraded.

Automatic and Throttle Engines.—The power of an engine may be regulated by (1) controlling the steam-pressure or (2) by adjusting the cut-off. Usually these two methods are used separately, but in some instances they are used in combination. Thus a locomotive-driver may reduce the power of his engine either by shortening the cut-off or by partially closing the throttle-valve, or he may do both at once. Stationary engines are usually run at a fixed speed and are con-

trolled by mechanical governors, which commonly consist of revolving weights that are urged away from the axis of revolution by centrifugal force and are restrained by the attraction of gravity or by the tension of springs.

The earliest and simplest steam-engine governor, invented by Watt, has a pair of revolving pendulums (balls on the ends of rods that are hinged to a vertical spindle at their upper ends) which are urged out by centrifugal force and are drawn down by gravity. When the engine is running steadily at a given speed the forces acting on the governor are in equilibrium and the balls revolve in a certain horizontal plane. If the load on the engine is reduced the engine speeds up and the balls move outward and upward until a new position of equilibrium is found with the balls revolving in a higher horizontal plane. Through a proper system of links and levers the upward motion of the balls is made to partially close a throttle-valve in the pipe which supplies steam to the engine and thus adjusts the work of the engine to the load.

Shaft-governors have large revolving-weights whose centrifugal forces are balanced by strong springs. They are powerful enough to control the distribution or the cut-off valve of the engine, which, however, must be balanced so that it may move easily.

Automatic engines, like the Corliss engines, have four valves, two for admission and two for exhaust of steam. The admission, release, and compression are fixed, but the cut-off is controlled by the governor. Usually an admission-valve is attached to the actuating mechanism by a latch or similar device, which can be opened by the governor, and then the valve is closed by gravity, by a spring, or by some other independent device. The office of the governor is to control the position of a stop against which the latch strikes and by which it is opened to release the valve.

Corliss and other automatic engines have long had a deserved reputation for economy, which is commonly attribu-

ted to their method of regulation. It is true that the valve-gears of such engines are adapted to give an early cut-off, which is one of the elements of the design of an economical simple engine, but their advantage over some other engines is to be largely attributed to the small clearance which the use of four valves makes convenient, and to the fact that the exhaust-steam is led immediately away from the engine, without having a chance to abstract heat after it leaves the cylinder. These engines also are free from the loss which Callendar and Nicolson attribute to direct leakage from the steam to the exhaust side of slide-valves, and to valves of similar construction. And yet the Hoadley engine (Table XXII, page 363), which was of the second type having a piston-valve controlled by a shaft-governor, compares very favorably with the Corliss engine at Creusot (Table XXXI, page 382), though it must be admitted that the performance of the Hoadley engine is exceptionally good for its type.

Every steam-engine should have a reserve of power in excess of its normal power; and again it is convenient if not essential that a single-cylinder engine should be able to carry steam through the greater part of its stroke in starting. These conditions, together with the fact that it is somewhat difficult to design a plain slide-valve engine to give an early cut-off, have led to the use of a long cut-off for engines controlled by a throttle-governor. The tests on the Corliss engine at Creusot (Tables XXX and XXXI, pp. 381 and 382) show clearly the disadvantage of using a long cut-off for simple engines. It has already been pointed out that a non-condensing engine should have the cut-off at about one-third stroke. With cut-off at that point and with 75 lbs. steam-pressure the engine developed 209 horse-power and used 24.2 lbs. of steam per horse-power per hour when running without steam in the jacket and without condensation. If the steam-pressure is reduced to 50 lbs. and the cut-off is lengthened to 58 per cent of the stroke, the steam-consumption is increased

to 30.2 lbs. per horse-power per hour, the horse-power being then 173. The gain from using the shorter cut-off is

$$\frac{30.2 - 24.2}{30.2} \times 100 = 20 \text{ per cent.}$$

A similar comparison for the same engine running with a vacuum and with steam in the jacket shows even a larger difference. Thus in test 16 the steam-pressure is 84 lbs. and the cut-off is at 11.5 per cent of the stroke, the horse-power is 176, and the steam-consumption per horse-power per hour is 16.9 lbs., while the consumption for about the same power in test 44 is 25.4 lbs. of steam per horse-power per hour, the steam-pressure being 35 and the cut-off at 58 per cent of the stroke; here the gain from using the shorter cut-off is

$$\frac{25.4 - 16.9}{25.4} \times 100 = 33 \text{ per cent.}$$

Considering also that automatic engines are usually well built and carefully attended to, while throttling-engines are often cheaply built and neglected, the good reputation of the one and the bad reputation of the other are easily accounted for.

It is, however, far from certain that an automatic engine will have a decided advantage over a throttle-engine, provided the latter is skilfully designed, well built and cared for, and arranged to run at the proper cut-off. Considering the rapid increase in steam-consumption per horse-power per hour when the cut-off is unduly shortened, it is not unreasonable to expect as good if not better results from a simple throttling-engine than from an automatic engine when both are run for a large part of the time at reduced power.

The disadvantage of running a compound or a triple engine with too little expansion can be seen by comparing the two tests on the engine of the *Rush* (Table XXXII, p. 388); on the other hand the great disadvantage of too much expansion is made evident from the tests on the engine in the lab-

oratory of the Massachusetts Institute of Technology (Table XXXIII, p. 392). Considering that the allowable variation from the most economical cut-off is more limited for a compound or a triple engine, it appears that there is less reason for using an automatic governor instead of a throttling governor for compound and triple engines than there is with simple engines. Nevertheless the most economical engines (simple, compound, or triple) are automatic engines.

Effect of Speed on Condensation.—Though the condensation of steam on the walls of the cylinder of an engine is very rapid, it is not instantaneous. It appears reasonable that the amount of condensation, and consequently the influence of the cylinder-walls, may be reduced by increasing the speed of revolution. This conclusion is confirmed by the results of the investigations by Callendar and Nicolson, and also by the tests on the Willans engine. Table IX, page 349, shows that the condensation per stroke was reduced from 0.0159 to 0.0074 by increasing the revolutions from 46 to 97 per minute. It must be borne in mind in this connection that the effect of external radiation and convection is nearly proportional to the time, and that, consequently, its influence is more pronounced at low speeds than at high speeds. Since the engine used by Callendar and Nicolson was run at relatively low speeds and small powers, the influence of external radiation and conduction must have been abnormally large, and consequently an increase of speed had notable effect in the manner just explained.

Table XXXVIII, page 403, gives four sets of three tests each on the simple non-condensing Willans engine; the three tests of a set were intended to be made at 100, 200, and 400 revolutions, all other conditions for the set being the same. Each set shows a notable gain in economy and a notable reduction in condensation on account of the increase of speed. Thus a comparison of tests 8 and 10 shows that the condensation is reduced by nearly one half, and that there is a gain of more than 20 per cent in steam-consumption, by in-

creasing the speed from 111 to 408 revolutions per minute. The other sets of tests show similar results, as do also three sets of speed tests at the bottom of Table XL on the compound non-condensing engine.

In the development of applications of electricity there has been a tendency to use high-speed engines, which can either be connected directly to the electric generators, or connected to them by belting without an intermediate shaft. In general the engines used have consumed more steam per horse-power per hour than slow-speed engines running under similar conditions. This result may be attributed to the type of engine, which is virtually a plain slide-valve engine controlled by a shaft-governor. In order that the valve may be controlled directly by the governor it must move freely, and consequently some form of balanced valve or piston-valve must be used. Such valves may be tight when properly adjusted and in good condition, but they are likely to leak in ordinary service. And again, in order that the engine may run with a comparatively early cut-off, the engine must have a large clearance or else the compression will be excessive. Compound high-speed engines have been developed to meet the requirements of this service, and have given satisfaction when run at full power. At reduced power certain difficulties arise, as stated on page 420, especially when the cut-off for the low-pressure cylinder is fixed.

Steam-turbines.—Many attempts have been made from time to time to devise some form of impulse or reaction wheel that can be driven by the direct action of steam. The fundamental difficulty in devising such a wheel is the high velocity of steam when flowing out of an orifice, for the linear velocity of the wheel must have a proper relation to the velocity of the steam in order to obtain economical results.

Fig. 83 represents the essential parts of a Laval steam-turbine, consisting of a wheel which carries a large number of curved steel cups or buckets to receive the jet of steam from a nozzle set at an angle. The figure shows only one nozzle,

but a number may be used, the wheel in question having four. Three of the nozzles have a diameter of 0.138 of an inch at the throat and one a diameter of 0.157 of an inch. All the nozzles are diverging toward the exit end and are intended to give an adiabatic expansion of the steam to the pressure of the atmosphere, so as to develop the maximum kinetic energy

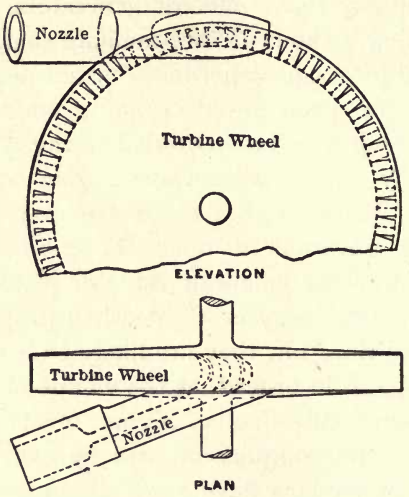

FIG. 83.

at the exit. The wheel runs at approximately 24,000 revolutions per minute, and as it has a diameter of about five inches at the middle of the buckets, their velocity is about 30,000 feet per minute or 500 feet per second. The velocity of steam flowing from a pressure of 125 pounds by the gauge or 140 pounds absolute, with continuous expansion to the pressure of the atmosphere, is about 2800 feet per second.

The results of tests on this turbine by Professor Goss are given in Table XLIII. It is reported that a test of a 64 horse-power Laval turbine by Prof. J. E. Cederblom gave a steam-consumption of 19.7 pounds per brake horse-power per hour when supplied with steam at about 100 pounds by the gauge, and exhausting into a vacuum of 26 inches of mercury.

TABLE XLIII.

TESTS ON A LAVAL STEAM-TURBINE.

By Prof. W. F. M. Goss, *Trans. Am. Soc. Mech. Engs.*, vol. xvii, p. 81.

		Revolutions per minute.	Brake horse-power.	Steam-pressure by gauge.		Steam per brake horse-power per hour, pounds.
				At boiler.	Near engine.	
Four nozzles in action, three having a diameter of 0.138 of an inch and one 0.157 of an inch.	1	21180	0.00	130	17.1	.....
	2	25210	1.63	"	42.2	128.6
	3	20190	2.36	"	48.5	99.8
	4	20980	2.97	"	55.6	85.7
	5	18990	3.46	"	61.9	79.6
	6	20520	4.38	"	70.8	71.5
	7	21080	5.10	"	76.9	64.4
	8	25520	7.52	"	99.6	53.6
	9	24300	8.24	"	104.4	51.3
	10	23880	10.33	"	126.3	47.8
Three nozzles in action, two 0.138 of an inch and one 0.157 of an inch in diameter.	11	17960	0.00	"	31.3	.....
	12	20920	3.95	"	83.6	67.8
	13	21050	4.77	"	93.4	60.0
	14	24660	6.50	"	111.7	53.3
Two nozzles in use, each 0.138 of an inch in diameter.	15	25220	0.00	"	42.2	.....
	16	20300	1.95	"	83.5	83.4
	17	18920	3.43	"	121.1	65.0
	18	23900	3.87	"	127.0	59.3

The Parsons steam-turbine has a number of turbines arranged in series through which the steam passes in succession, thus breaking up the difference of pressure between the supply and the exhaust into a number of steps; consequently the velocity of the steam impinging onto any set of vanes is comparatively low. Instead of a few nozzles, each turbine has a series of guides extending entirely around the wheel, thus supplying steam to all the vanes of the movable wheels at the same time. Table XLIV gives the results of tests on a Parsons turbine having seven separate turbines and seven corresponding steps in the reduction of the steam-pressure from the supply to the exhaust; the low-pressure turbine was

made double, thus giving eight moving wheels in all. Later turbines have been given a large number of separate turbines so that a large number of expansions of steam is attained, and it is claimed that their economy is as good as that of the best steam-engines.

TABLE XLIV.

TESTS ON PARSON'S STEAM-TURBINE.

SIX HIGH-PRESSURE SINGLE DISKS ; ONE LOW-PRESSURE DOUBLE DISK.

By Prof. ALEX. B. W. KENNEDY, *Engineering*, vol. lvi. p. 126.

Duration, minutes	72	93	122	46.5
Steam-pressure, pounds per square inch above atmosphere	97	94.6	94.5	97.1
Corresponding temperature of saturated steam.	336	334	334	336
Temperature of steam near engine	356	371	401	391
Vacuum, pounds per square inch	14.3	14.3	14.0	14.1
Revolutions per minute	4480	4475	4655	4625
Kilowatts	27.9	68.8	110.8	123
Electrical horse-power	37.5	92.2	148.5	164.9
Steam per kilowatt per hour, pounds	44.1	31.2	27.9	27.2
Steam per electrical horse-power per hour, pounds	32.9	23.3	20.8	20.3

CHAPTER XVI.

FRICTION OF ENGINES.

THE efficiency and economy of steam-engines are commonly based on the indicated horse-power, because that power is a definite quantity that may be readily determined. On the other hand, it is usually difficult and sometimes impossible to make a satisfactory determination of the power actually delivered by the engine. A common way of determining the work consumed by friction in the engine itself is to disconnect the driving-belt, or other gear for transmitting power from the engine, and to place a friction-brake on the main shaft; the power developed is then determined by aid of indicators, and the power delivered is measured by the brake, the difference being the power consumed by friction. Such a determination for a large engine involves much trouble and expense, and may be unsatisfactory since the engine-friction may depend largely on the gear for transmitting power from the engine, especially when belts or ropes are used for that purpose.

The friction of a pumping-engine may be determined from a comparison of the indicated power of the steam-cylinders with the indicated work of the pumps, or, better, with the work done in lifting water from the well and delivering it to the forcing-main. But the friction thus determined is the friction of both the engine and the pump. Air-compressors and refrigerating machines may be treated in the same way to determine the friction of both engine and compressor. Again, the combined friction of an engine and a directly connected electric generator may be determined by comparing the indicated power of the engine with the electric

output of the generator, allowing for electricity consumed or wasted in the generator itself.

The friction of a steam-engine may consume from 5 to 15 per cent of the indicated horse-power, depending on the type and condition of the engine. The power required to drive the air-pump (when connected to the engine) is commonly charged to the friction of the engine. It is usual to consider that seven per cent of the indicated power of the engine is expended on the air-pump. Independent air-pumps which can be driven at the best speed consume much less power; those of some United States naval vessels used only one or two per cent of the power of the main engines. But as independent air-pumps are usually direct-acting steam-pumps, much of the apparent advantage just pointed out is lost on account of the excessive steam-consumption of such pumps.

Mechanical Efficiency.—The ratio of the power delivered by an engine to the power generated in the cylinder is the mechanical efficiency; or it may be taken as the ratio of the brake to the indicated power. The mechanical efficiency of engines varies from 0.85 to 0.95, corresponding to the per cent of friction given above.

The following table gives the mechanical efficiencies of a

TABLE XLV.

MECHANICAL EFFICIENCIES OF ENGINES.

Kind of engine.	Horse-power.	Efficiency.
Simple engines:		
Horizontal portable	24	0.86
" " Hoadley	80	0.91
High-speed, straight-line	56	0.96
Corliss condensing	160	0.81
" non-condensing	100	0.86
Compound:		
Portable	78	0.88
Semi-portable	60	0.88
Horizontal	59	0.90
Horizontal mill-engine	288	0.86
Schmidt superheated steam	110	0.92
Leavitt pumping-engine	643	0.93
Triple-expansion Leavitt pumping-engine	576	0.90

number of engines, determined by brake tests, or, in case of the pumping-engines, by measuring the work done in pumping waters.

Initial Friction and Load Friction.—A part of the friction of an engine, such as the friction of the piston-rings and at the stuffing-boxes of piston-rods and valve-rods, may be expected to remain constant for all powers. The friction at the cross-head guides and crank-pins is due mainly to the thrust or pull of the steam-pressure, and will be nearly proportional to the mean effective pressure. Friction at other places, such as the main bearings, will be due in part to weight and in part to steam-pressure. On the whole it appears probable that the friction may be divided into two parts, of which one is independent of the load on the engine, and the other is proportional to the load. The first may be called the initial friction and the second, the load friction. Progressive brake tests at increasing loads confirm this conclusion.

Table XLVI gives the results of tests made by Walther-Meunier and Ludwig* to determine the friction of a horizontal-receiver compound engine, with cranks at right angles and with a fly-wheel, grooved for rope-driving, between the cranks. The piston-rod of each piston extended through the cylinder-cover and was carried by a cross-head on guides, and the air-pump was worked from the high-pressure piston-rod. The cylinders each had four plain slide-valves, two for admission and two for exhaust; the exhaust-valves had a fixed motion, but the admission-valves were moved by a cam so that the cut-off was determined by the governor.

The main dimensions of the engine were:

Stroke	1.1	metres.
Diameter: small piston	0.536	"
large piston	0.800	"
piston-rods	0.080	"
Diameter, air-pump pistons	0.360	"
Stroke, air-pump	0.476	"
Diameter, fly-wheel	6.610	"

* *Bulletin de la Soc. Ind. de Mulhouse*, vol. lvii, p. 140.

TABLE XLVI.

FRICTION OF COMPOUND ENGINE.

WALTHER-MEUNIER and LUDWIG, *Bulletin de la Soc. Ind. de Mulhouse*, vol. lvii, p. 140.

	Condition.	Horse-powers—Chevaux aux vapeur.			Friction.	Efficiency.
		Indicated.	Effective.	Absorbed by engine.		
1	Compound condensing with air-pump.	288.5	249.0	39.5	0.137	0.863
2		276.9	238.9	38.0	0.138	0.862
3		265.6	228.9	36.7	0.139	0.861
4		243.7	208.8	34.9	0.144	0.856
5		222.7	188.7	34.0	0.153	0.847
6		201.5	168.6	32.9	0.164	0.836
7		180.4	148.5	31.9	0.178	0 822
8		158.1	128.4	29.7	0.189	0.811
9		136.1	108.3	27.8	0.205	0.795
10	High-pressure cylinder only. Condensing with air-pump.	153.1	128.4	24.7	0.161	0.839
11		142.0	118.3	23.7	0.167	0.833
12		130.9	108 3	22.6	0.173	0.827
13		120.1	98.2	21.9	0.182	0.818
14		109.0	88.2	20.8	0.191	0.809
15		97.5	78.1	19.4	0.199	0.801
16		86.3	68.1	18.3	0.212	0.788
17		75.7	58.0	17.7	0.234	0.766
18		65.5	48.0	17.5	0.267	0.733
19		55.2	37.9	17.3	0.313	0.687
20	High-pressure cylinder only. Non-condensing, no air-pump.	145.9	128.4	17.5	0.120	0.880
21		135.7	118.3	17.4	0.129	0.871
22		125 2	108.3	16.9	0.135	0.865
23		114.4	98.2	16.2	0.142	0.858
24		103.9	88.2	15.7	0.152	0.848
25		93.0	78.1	14.9	0.160	0.840
26		82.0	68.1	13 9	0.170	0.830
27		71.7	58.0	13.7	0.191	0.809
28		61.6	48.0	13.6	0.221	0.779
29		51.3	37.9	13.4	0.262	0.738

The engine during the experiments made 58 revolutions per minute. The air-pump had two single-acting vertical pistons.

Each experiment lasted 10 or 20 minutes, during which the load on the brake was maintained constant, and indicator-diagrams were taken. The experiments with small load on

the brake (numbers 9, 18, 19, 28, and 29) were difficult and uncertain.

The first nine tests were made with the engine working compound. Tests 10 to 19 were made with the high-pressure cylinder only in action and with condensation, the low-pressure connecting-rod being disconnected. Tests 20 to 29 were made with the high-pressure cylinder in action, without condensation.

The results of these tests are plotted on Fig. 84, using the

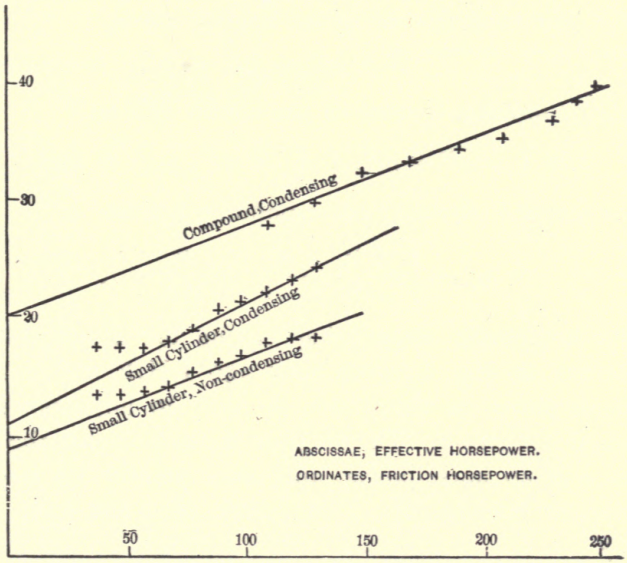

FIG. 84.

effective horse-power for abscissæ and the brake horse-powers for ordinates. Omitting tests with small powers (for which the brake ran unsteadily), it appears that each series of tests can be represented by a straight line which crosses the axis of ordinates above the origin; thus affording a confirmation of the assumption that an engine has a constant initial friction, and a load friction which is proportional to the load.

Now the initial friction which depends on the size and construction of the engine may be assumed to be propor-

tional to the normal net or brake horse-power, P_n, which the engine is designed to deliver, and may be represented by

$$aP_n,$$

where a is a constant to be determined from a diagram like Fig. 84. If P is the net horse-power delivered by the engine at any time, then the load friction corresponding is

$$bP,$$

where b is a second constant to be determined from experiments. The total friction of the engine will be

$$F = aP_n + bP, \quad . \quad . \quad . \quad . \quad . \quad (308)$$

so that the indicated power of the engine will be

$$\text{I.H.P.} = P + aP_n + bP = aP_n + (1 + b)P. \quad (309)$$

The mechanical efficiency corresponding will be

$$\eta_m = \frac{\text{I.H.P.} - F}{\text{I.H.P.}} = \frac{P}{\text{I.H.P.}}. \quad . \quad . \quad . \quad (310)$$

The compound condensing engine for which tests are represented by Fig. 84 developed 290 I.H.P. and delivered 250 horse-power to the brake, so that 40 horse-power was consumed in friction. The diagram shows also that the initial friction was 20 horse-power, and consequently the load friction was 20 horse-power. The values of a and b are consequently

$$a = 20 \div 250 = 0.07;$$
$$b = (40 - 20) \div 250 = 0.07.$$

The indicated horse-power for a given load P is

$$\text{I.H.P.} = 0.07P_n + 1.07P. \quad . \quad . \quad . \quad (311)$$

Similar equations can be deduced for the engine with steam supplied to the small cylinder only; but as the engine is not then in normal condition they are not very useful.

The maximum efficiency of this engine is

$$250 \div 290 = 0.86;$$

but at half load (125 horse-power) the indicated horse-power is

$$\text{I.H.P.} = 0.07 \times 250 + 1.07 \times 125 = 151,$$

and the efficiency is

$$125 \div 151 = 0.83.$$

TABLE XLVII.

FRICTION OF CORLISS ENGINE AT CREUSOT.

By F. DELAFOND, *Annales des Mines*, 1884.

Condensing with air-pump, tests 1–33.
Non-condensing without air-pump, tests 34–46.

	Cut-off fraction of stroke.	Pressure at cut-off, kilos per sq. cm.	Revolutions per minute.	Horse-power—Cheval à vapeur.		
				Indicated.	Effective.	Absorbed by engine.
1	0.039	0.64	64.0	27.8	16.3	11.5
2	0.044	2.40	68.5	60.0	37.6	22.4
3	0.044	2.90	65.0	67.2	45.2	22.0
4	0.065	4.90	64.0	117.0	88.7	28.3
5	0.065	6.20	61.0	138.5	106.3	32.2
6	0.065	7.10	64.0	163.2	129.2	34.0
7	0.065	7.60	64.0	185.0	144.6	40.4
8	0.100	0.16	58.0	21.0	10.6	10.4
9	0.106	1.55	60.0	61.9	42.3	19.6
10	0.100	2.82	57.3	82.7	61.0	21.7
11	0.096	4.80	58.3	135.3	106.7	28.6
12	0.128	4.82	58.3	154.5	124.8	29.7
13	0.142	0.76	62.0	42.3	28.4	13.9
14	0.137	0.71	60.6	44.3	28.7	15.6
15	0.132	2.50	54.0	79.5	59.8	19.7
16	0.147	2.60	61.6	100.0	78.2	21.8
17	0.155	4.65	60.0	177.2	145.0	32.2
18	0.167	0.22	61.0	40.2	27.9	12.3
19	0.197	2.55	57.2	110.8	83.3	27.5
20	0.273	0.40	62.3	50.2	33.8	16.4
21	0.264	1.57	63.3	89.1	61.8	27.3
22	0.240	1.64	62.0	87.2	63.1	24.1
23	0.245	3.25	56.0	145.0	116.0	29.0
24	0.260	4.76	58.0	209.4	178.0	31.4
25	0.335	0.25	59.0	47.2	32.5	14.7
26	0.339	1.94	58.3	111.7	90.0	21.7
27	0.338	2.97	61.0	161.8	133.0	28.8
28	1	0.47	59.3	81.3	67.2	14.1
29	1	0.47	61.0	80.8	67.9	12.9
30	1	1.60	61.6	148.5	128.4	20.1
31	1	2.70	61.5	216.5	191.0	25.5
32	1	2.70	61.5	215.5	191.0	24.5
33	0.50	0.70	61.5	15.8	0.0	15.8
34	0.120	6.00	60.0	132.5	107.5	25.0
35	0.106	7.00	53.0	125.0	103.0	22.0
36	0.120	7.50	62.0	172.0	148.0	24.0
37	0.150	4.57	55.0	102.3	86.5	15.8
38	0.262	4.50	59.0	149.2	132.3	16.9
39	0.293	4.55	59.0	171.8	153.8	18.0
40	0.371	4.40	60.0	195.3	177.2	18.1
41	0.348	2.75	58.0	85.1	73.1	12.0
42	0.348	2.75	58.5	84.8	71.1	13.7
43	0.440	3.48	62.0	151.0	134.3	16.7
44	0.111	3.30	62.0	12.8	0.0	12.8
45	0.50	1.20	62.0	12.3	0.0	12.3
46	1	0.50	62.0	10.45	0.0	10.45

Table XLVII gives the results of a large number of brake tests made on a Corliss engine at Creusot by M. F. Delafond, both with and without a vacuum, and with varying steam-pressures and cut-off. The tests with a vacuum are plotted on Fig. 85, and those without a vacuum are given in Fig. 86. In both figures the abscissæ are the indicated horse-powers, and the ordinates are the friction horse-powers. Most of the tests are represented by dots; those tests which were made with the most economical cut-off (one sixth for the engine with

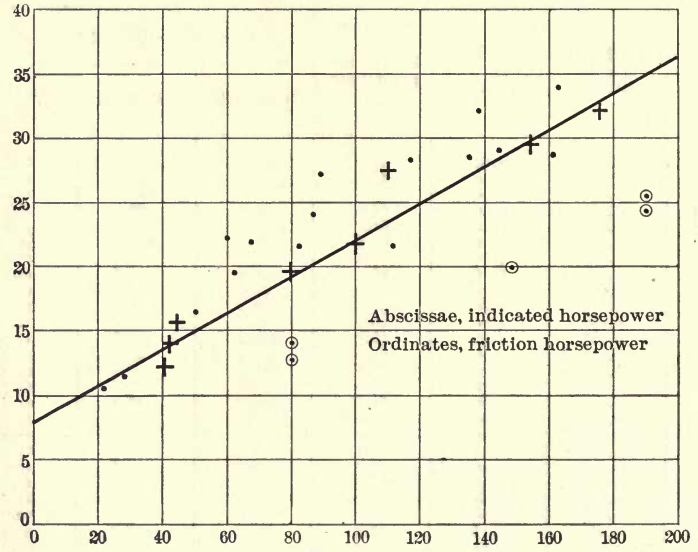

FIG. 85.

condensation and one third without) are represented by crosses. A few tests with very long cut-off, on Fig. 84, are represented by circles. The straight lines on both figures are drawn to represent the tests indicated by crosses. In general the points representing tests with short cut-off and high steam-pressure lie above the lines, and points representing tests with long cut-off and low steam-pressure lie below the lines, though there are some notable exceptions to this rule. The circles on Fig. 84, representing tests with cut-off near the

end of the stroke, show much less friction than the other tests. The tests on this engine show clearly that both initial and load friction are affected by the cut-off and the steam-pressure, and that friction tests should be made at the cut-off which the engine is expected to have in service.

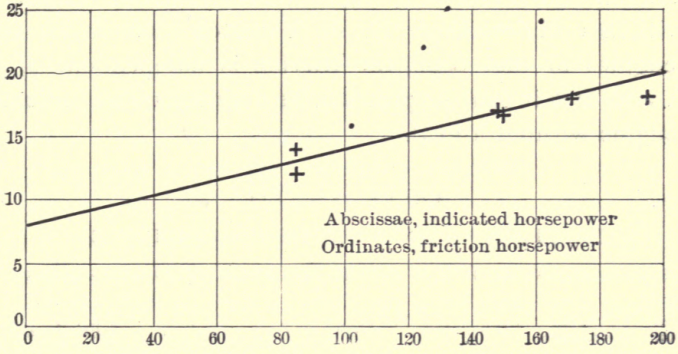

FIG. 86.

The initial friction was eight horse-power both with and without condensation. But Table XXX shows that the engine with condensation gave the best economy when it indicated 160 horse-power; the friction was then 30 horse-power, so that the net horse-power was 130, which will be taken for the normal horse-power P_n. Consequently

$$a = 8 \div 130 = 0.06;$$
$$b = (30 - 8) \div 130 = 0.17.$$
$$\therefore \quad \text{I.H.P.} = 0.062F_n + 1.17P. \quad . \quad . \quad (312)$$

In like manner Table XLVII shows the best economy without condensation, for about 100 indicated horse-power, for which the friction is 14 horse-power, leaving 86 for the normal power of the engine. Consequently

$$a = 8 \div 86 = 0.09;$$
$$b = (14 - 8) \div 86 = 0.07.$$
$$\therefore \quad \text{I.H.P.} = 0.09P_n + 1.07P. \quad . \quad . \quad . \quad (313)$$

The increased value of a for the non-condensing engine is due directly to the reduction in the economical power of the engine; on the other hand, the reduction in the value of b is due to the omission of the air-pump.

Thurston's Experiments.—As a result of a large number of tests on non-condensing engines, made under his direction or with his advice, Prof. R. H. Thurston* concludes that, for engines of that type, the friction is independent of the load, and that it can, in practice, be determined by indicating the engine without a load.

TABLE XLVIII.

FRICTION OF NON-CONDENSING ENGINE.

STRAIGHT-LINE ENGINE, 8 INCHES DIAMETER, 14 INCHES STROKE.

No. of Diagram.	Boiler-pressure.	Revolutions.	Brake H. P.	I. H. P.	Frictional H. P.
1	50	332	4.06	7.41	3.35
2	65	229	4.98	7.58	2.60
3	63	230	6.00	10.00	4.00
4	69	230	7.00	10.27	3.27
5	73	230	8.10	11.75	3.65
6	77	230	9.00	12.70	3.70
7	75	230	10.00	14.02	4.02
8	80	230	11.00	14.78	3.78
9	80	230	12.00	15.17	3.17
10	85	230	13.00	15.96	2.96
11	75	230	14.00	16.86	2.86
12	70	230	15.00	17.80	2.80
13	72	231	20.10	22.07	1.97
14	75	230	25.00	28.31	3.31
15	60	229	29.55	33.04	3.40
16	58	229	34.86	37.20	2.34
17	70	229	39.85	43.04	3.19
18	85	230	45.00	47.79	2.78
19	90	230	50.00	52.60	2.60
20	85	230	55.00	57.54	2.54

Table XLVIII gives the details of one series of tests. The friction horse-power is small in all the tests, and the variations are small and irregular, and appear to depend on the state of lubrication and other minor causes rather than on the change of load.

* *Trans. of the Am. Soc. of Mech. Engrs.*, vols. viii, ix, and x.

Distribution of Friction.—As a consequence of his conclusion in the preceding section, Professor Thurston says that the friction of an engine may be found by driving it from some external source of power, with the engine in substantially the same condition as when running as usual, but without steam in its cylinder, and by measuring the power required to drive it by aid of a transmission dynamometer. Extending the principle, the distribution of friction among the several members of the engine may be found by disconnecting the several members, one after another, and measuring the power required to run the remaining members.

The summary of a number of tests of this sort, made by Prof. R. C. Carpenter and Mr. G. B. Preston, are given in Table XLIX. Preliminary tests under normal conditions showed that the friction of the several engines was practically the same at all loads and speeds.

The most remarkable feature in this table is the friction of the main bearings, which in all cases is large, both relatively and absolutely. The coefficient of friction for the main bearings, calculated by the formula

$$f = \frac{33000 \text{ H.P.}}{pcn},$$

is given in Table L. p is the pressure on the bearings in pounds for the engines light, and *plus* the mean pressure on the piston for the engines loaded; c is the circumference of the bearings in feet; n is the number of revolutions per minute; and H.P. is the horse-power required to overcome the friction of the bearings.

The large amount of work absorbed by the main bearings and the large coefficient of friction appear the more remarkable from the fact that the coefficient of friction for car-axle journals is often as low as one-tenth of one per cent, the difference being probably due to the difference in the methods of lubrication.

TABLE XLIX.

DISTRIBUTION OF FRICTION.

Parts of Engine.	Percentages of Total Friction.				
	Straight-line 6×12 Balanced Valve.	Straight-line 6″×12″ Unbalanced Valve.	7″×10″ Lansing Iron Works—Traction-Locomotive Valve-gear.	12″×18″ Lansing Iron Works—Automatic Balanced Valve.	21″×20″ Lansing Iron Works—Condensing Balanced Valve.
Main Bearings	47.0	35.4	35.0	41.6	46.0
Piston and Rod	32.9	25.0	21.0		
Crank Pin	6.8	5.1	13.0	49.1	21.8
Cross Head and Wrist Pin.	5.4	4.1			
Valve and Rod	2.5	26.4	22.0	9.3	21.0
Eccentric Strap	5.3	4.0			
Link and Eccentric			9.0		
Air Pump					12.0
Total	100.0	100.0	100.0	100.0	100.0

TABLE L.

COEFFICIENT OF FRICTION FOR THE MAIN BEARINGS OF STEAM-ENGINES.

Engine.	F. H. P. due to Main Journals.	Weight on Journals in Pounds.	Diameter of Journal in Inches	Coefficient of Friction, Engine Light.	Coefficient of Friction, Engine Loaded.	Revolutions of Journal per Minute.
6″ × 12″ Straight-line	0.85	1500	3	.10	.06	230
*12″ × 18″ Automatic (L. I. W.)	3.70	2600	5	.19	.05	190
7″ × 10″ Traction (L. I. W.)	0.68	500	$2\frac{3}{4}$	.31	.08	200
21″ × 20″ Condensing (L. I. W.)	3.30	4000	$5\frac{1}{2}$	.09	.04	206

*The 12″ × 18″ automatic engine was new, and gave, throughout, an excessive amount of friction as compared with the older engines of the same class and make.

The second and obvious conclusion from Table L is that the valve should be balanced, and that nine tenths of the friction of an unbalanced slide-valve is unnecessary waste.

The friction of the piston and piston-rod is always considerable, but it varies much with the type of the engine, and with differences in handling. It is quite possible to change the effective power of an engine by screwing up the piston-rod stuffing-box too tightly. The packing of both piston and rod should be no tighter than is necessary to prevent perceptible leakage, and is more likely to be too tight than too loose.

CHAPTER XVII.

COMPRESSED AIR.

COMPRESSED air is used for transmitting power, for storing energy, and for producing refrigeration. Air at moderate pressure, produced by blowing-engines, is used in the production of iron and steel; and currents of air at slightly higher pressure than that of the atmosphere (produced by centrifugal fan-blowers) are used to ventilate mines, buildings, and ships, and for producing forced draught for steam-boilers. Attention will be given mainly to the transmission and storage of energy. The production and use of ventilating currents require and are susceptible of but little theoretical treatment. Refrigeration will be reserved for another chapter.

A treatment of the transmission of power by compressed air involves the discussion of air-compressors, of the flow of air through pipes, and of compressed-air engines or motors. The storage of energy differs from the transmission of power in that the compressed air, which is forced into a reservoir at high pressure, is used at a much lower pressure at the air-motor.

Air-compressors.—There are three types of machines used for compressing or moving air: (1) piston air-compressors, (2) rotary blowers, (3) centrifugal blowers or fans.

The piston air-compressor is always used for producing high pressures. It consists of a piston moving in a cylinder with inlet- and exit-valves at each end. Commonly the valves are actuated by the air itself, but some compressors have their valves moved mechanically. Blowing-engines are usually

piston-compressors, though the pressures produced are only ten or twenty pounds per square inch.

Rotary blowers have one or more rotating parts, so arranged that as they rotate, chambers of varying capacity are formed which receive air at atmospheric pressure, compress it, and deliver it against a higher pressure. They are simple and compact, but are wasteful of power on account of friction and leakage, and are used only for moderate pressures.

Fan-blowers consist of a number of radial plates or vanes fixed to a horizontal axis and enclosed in a case. When the axis and the vanes attached to it are rotated at a high speed air is drawn in through openings near the axis and is driven by centrifugal force into the case, from which it flows into the delivery-main or duct. Only low pressures, suitable for ventilation and forced draught, can be produced in this way. But little has been done in the development of the theory or the determination of the practical efficiency of fan-blowers. Some ventilating-fans have their axes parallel to the direction of the air-current, and the vanes have a more or less helicoidal form, so that they may force the air by direct pressure; they are in effect the converse of a windmill, producing instead of being driven by the current of air. They are useful rather for moving air than for producing a pressure.

Fluid Piston-compressors.—It will be shown that the effect of clearance is to diminish the capacity of the compressor; consequently the clearance should be made as small as possible. With this in view the valves of compressors and blowers are commonly set in the cylinder-heads. Single-acting compressors with vertical cylinders have been made with a layer of water or some other fluid on top of the piston, which entirely fills the clearance-space when the piston is at the end of the stroke. An extension of this principle gives what are known as fluid piston-compressors. Such a compressor commonly has a double-acting piston in a horizontal cylinder much longer than the stroke of the piston, thus giving a large clearance at each end. The clearance-spaces extend upward

to a considerable height, and the admission- and exhaust-valves are placed at or near the top, and the entire clearance-space is filled with water. The spaces and heights must be so arranged that when the piston is at one end of its stroke the water at that end shall fill the clearance and cover the valves, and at the other end the water shall not fall to the level of the top of the cylinder. There are consequently two vertical fluid pistons actuated by a double-acting horizontal piston. It is essential that the spaces in which the fluid pistons act shall give no places in which air may be caught as in a pocket, and that there are no projecting ribs or other irregularities to break the surface of the water; and, further, the compressor must be run at a moderate speed. The water forming the fluid pistons becomes heated and saturated with air by continuous use, and should be renewed.

Air-pumps used with condensing-engines or for other purposes may be made with fluid pistons which are renewed by the water coming with the air or vapor. In case the water thus supplied is insufficient, water from without may be admitted, or water from the delivery may be allowed to flow back to the admission side of the pump.

Displacement-compressors.—When a supply of water under sufficient head is available, air may be compressed in suitably arranged cylinders or compressors by direct action of the water on air, compressing it and expelling it by displacement. Such compressors are very wasteful of power, and in general it is better to use water-power for driving piston-compressors, properly geared to turbine-wheels or other motors.

Cooling during Compression.—There is always a considerable rise of temperature due to compressing air in a piston air-compressor, which is liable to give trouble by heating the cylinder and interfering with lubrication. Blowing-engines which produce only moderate pressures usually have their cylinders lubricated with graphite, and no attempt is made to cool them. All compressors which produce high

pressures have their cylinders cooled either by a water-jacket or by injecting water, or by both methods.

Since the air after compression is cooled either purposely or unavoidably, there would be a great advantage in cooling the air during compression, and thereby reducing the work of compression. Attempts have been made to cool the air by spraying water into the cylinder, but experience has shown that the work of compression is not much affected by so doing. The only effective way of reducing the work of compression is to use a compound compressor, and to cool the air on the way from the first to the second cylinder. Three-stage compressors are used for very high pressures. It is, however, found that air which has been compressed to a high pressure and great density is more readily cooled during compression.

Moisture in the Cylinder.—If water is not injected into the cylinder of an air-compressor the moisture in the air will depend on the hygroscopic condition of the atmosphere. But even if the air were saturated with moisture the absolute and the relative weight of water in the cylinder would be insignificant. Thus at 60° F. the pressure of saturated steam is about one-fourth of a pound per square inch, and the weight of one cubic foot is about 0.0008 of a pound, while the weight of one cubic foot of air is about 0.08 of a pound. If the atmosphere is not saturated, then the watery vapor drawn into the compressor with the air will follow the laws of superheated steam. Now the adiabatic equations for air and for superheated steam are

$$pv^{1.405} = p_1 v_1^{1.405} \quad \text{and} \quad pv^{\frac{4}{3}} = p_1 v_1^{\frac{4}{3}},$$

so that the only effect of moisture in the air will be to slightly reduce the exponent of the adiabatic equation. This conclusion probably holds when the cylinder is cooled by a water-jacket.

When water is sprayed into the cylinder of a compressor the temperature of the air and the amount of vapor mixed

with it vary, and there is no ready way of determining its condition. But, as has been stated, the spraying of water into the cylinder does not much reduce the work of compression, and consequently it is probable we can assume that the compression always follows the law expressed by an exponential equation; such as

$$pv^n = p_1 v_1^n.$$

The value to be given to n is not well known; it may be as small as 1.2 for a fluid piston-compressor, and it may approach 1.4 when the cooling of the air is ineffective, as is usually the case.

Power Expended.—The indicator-diagram of an air-compressor with no clearance-space is represented by Fig. 87.

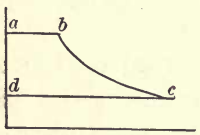

FIG. 87.

Air is drawn in at atmospheric pressure in the part of the cycle of operations represented by dc; in the part represented by cb the air is compressed, and in the part represented by ba it is expelled against the higher pressure.

If p_1 is the specific pressure and v_1 the specific volume of one pound of air at atmospheric pressure, and p_2 and v_2 corresponding quantities at the higher pressure, then the work done by the atmosphere on the piston of the compressor while air is drawn in is $p_1 v_1$. Assuming that the compression curve cb may be represented by an exponential curve having the form

$$pv^n = p_1 v_1^n = \text{const.},$$

then the work of compression is

$$\int p\,dv = p_1 v_1^n \int_{v_2}^{v_1} \frac{dv}{v^n} = \frac{p_1 v_1}{n-1}\left\{\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right\}$$

$$= \frac{p_1 v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}.$$

The work of expulsion from b to a is

$$p_2 v_2 = p_2 v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = p_1 v_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}.$$

The effective work of the cycle is therefore

$$W = \frac{p_1 v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_1 v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - p_1 v_1;$$

$$\therefore \quad W = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (314)$$

Equation (314) gives the work done to compress one pound of air, p_1 and p_2 being specific pressures (in pounds per square foot), and v_1 the specific volume, which may be calculated by aid of the equation

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0},$$

in which the subscripts refer to the normal properties of air at freezing-point and at atmospheric pressure.

If, instead of the specific volume v_1, we use the volume V_1 of air drawn into the compressor we may readily transform equation (314) to give the horse-power directly, obtaining

$$\text{H. P.} = \frac{144 p_1 V_1 n}{33000(n-1)}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}, \quad . \quad . \quad (315)$$

where p_1 is the pressure of the atmosphere in pounds per square inch, and n is the exponent of the equation representing the compression curve, which may vary from 1.4 for dry-air compressors to 1.2 for fluid piston-compressors.

Effect of Clearance.—The indicator-diagram of an air-compressor with clearance may be represented by Fig. 88. The end of the stroke expelling air is at *a*, and the air remaining in the cylinder expands from *a* to *d*, till the pressure becomes equal to the pressure of the atmosphere before the next supply of air is drawn in. The expansion curve *ad* may commonly be represented by an exponential equation having the same exponent as the compression curve *cb*, in which case the air

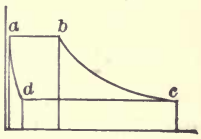

FIG. 88.

in the clearance acts as a cushion which stores and restores energy, but does not affect the work done on the air passing through the cylinder. The work of compressing one unit of weight of air in such a compressor may be calculated by aid of equation (312), but the equation (315) for the horse-power cannot be used directly.

The principal effect of clearance is to increase the size of the cylinder required for a certain duty in the ratio of the entire length of the diagram in Fig. 88 to the length of the line *dc*.

Let the clearance be $\frac{1}{m}$ part of the piston displacement. At the beginning of the filling stroke, represented by the point *a*, the volume will be filled with air at the pressure p_2. After the expansion represented by *ad* the air in the clearance will have the pressure p_1, and, assuming that the expansion follows the law expressed by the exponential equation

$$pv^n = p_1 v_1^n,$$

its volume will be

$$\frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

part of the piston displacement. The ratio of the line *dc* to the length of the diagram will consequently be

$$\frac{dc}{ac} = 1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m}; \quad . \quad . \quad . \quad . \quad . \quad (316)$$

and this is the factor by which the piston displacement calculated without clearance must be divided to find the actual piston displacement.

Temperature at the End of Compression.—When the air in the compressor-cylinder is dry or contains only the moisture brought in with it, it may be assumed that the mixture of air and vapor follows the law of perfect gases,

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1},$$

which, combined with the exponential equation

$$pv^n = p_1 v_1^n,$$

gives

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (317)$$

from which the final temperature T_2 at the end of compression may be determined when T_1 is known. When water is used freely in the cylinder of a compressor the final temperature cannot be determined by calculation, but must be determined from tests on compressors.

Contraction after Compression.—Ordinarily compressed air loses both pressure and temperature on the way from the compressor to the place where it is to be used. The loss of pressure will be discussed under the head of the flow of air in long pipes; it should not be large, unless the air is carried a long distance. The loss of temperature causes a contraction of volume in two ways: first, the volume of the air at a given pressure is inversely as the absolute temperature; second, the moisture in the air (whether brought in by the air or supplied in the condenser) in excess of that which will saturate the air at the lowest temperature in the conduit, is condensed. Provision must be made for draining off the condensed water. The method of estimating the contraction of volume due to the condensation of moisture will be exhibited later in the calculation of a special problem.

Interchange of Heat.—The interchanges of heat between the air in the cylinder of an air-compressor and the walls of the cylinder are the converse of those taking place between the steam and the walls of the cylinder of a steam-engine, and are much less in amount. The walls of the cylinder are never so cool as the incoming air nor so warm as the air expelled; consequently the air receives heat during admission and the beginning of compression, and yields heat during the latter part of compression and during expulsion. The presence of moisture in the air increases this effect.

Volume of the Compressor-cylinder.—Let a compressor making n revolutions per minute be required to deliver V_2 cubic feet of air at the temperature t_2° F., or T_2° absolute, and at the absolute pressure p_2 pounds per square inch, at the place where the air is to be used. Assuming that the air is dry when it is delivered and that the atmosphere is dry when it is taken into the compressor, then the volume drawn into the compressor per minute at the temperature T_1 and the pressure p_1 will be

$$V_1 = V_2 \frac{p_2 T_1}{p_1 T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (318)$$

cubic feet; and this expression will be correct whatever may be the intermediate temperatures, pressures, or condition of saturation of the air.

If the compressor has no clearance the piston displacement will be

$$\frac{V_1}{2n}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (319)$$

if the clearance is $\frac{1}{m}$ part of the piston displacement, dividing by the factor (316) gives for the piston displacement

$$\frac{V_1}{2n} \div \left\{ 1 - \frac{1}{m} \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} + \frac{1}{m} \right\}, \quad . \quad . \quad . \quad . \quad (320)$$

expressed in cubic feet.

The pressure in the compressor-cylinder when air is drawn in is always less than the pressure of the atmosphere, and when the air is expelled it is greater than the pressure against which it is delivered. From these causes and from other imperfections the compressor will not deliver the quantity of air calculated from its dimensions, and consequently the volume of the cylinder as calculated, whether with or without clearance, must be increased by an amount to be determined by experiment.

Compound Compressors.—When air is to be compressed from the pressure p_1 to the pressure p_2, but is to be delivered at the initial temperature t_1, the work of compression may be reduced by dividing it between two cylinders, one of which takes the air at atmospheric pressure and delivers it at an intermediate pressure p' to a reservoir, from which the other cylinder takes it and delivers it at the required pressure p_2, provided that the air be cooled, at the pressure p', between the two cylinders.

The proper method of dividing the pressures and of proportioning the volumes of the cylinders so that the work of compression may be reduced to a minimum may be deduced from equation (314) when there is no clearance or when the clearance is neglected.

The work of compressing one pound of air from the pressure p_1 to the pressure p' is

$$W_1 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad (321)$$

The work of compressing one pound from the pressure p' to p_2 is

$$W_2 = p'v'\frac{n}{n-1}\left\{\left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 1\right\} = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 1\right\}, \quad (322)$$

because the air after compression in the first cylinder is cooled to the temperature t_1 before it is supplied to the second cylinder, and consequently $p'v' = p_1 v_1$. The total work of compression is

$$W = W_1 + W_2 = p_1 v_1 \frac{n}{n-1}\left\{\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}} - 2\right\}, \quad (323)$$

and this becomes a minimum when

$$\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p'}\right)^{\frac{n-1}{n}}$$

becomes a minimum. Differentiating with regard to p', and equating the first differential coefficient to zero, gives

$$p' = \sqrt{p_1 p_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (324)$$

Since the air is supplied to each cylinder at the temperature t_1, their volumes should be inversely as the absolute pressures p_1 and p'. This method also leads to an equal distribution of work between the two cylinders, for if the value of p' from equation (324) is introduced into equations (322) and (323) we shall obtain

$$W_1 = W_2 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1} \right)^{\frac{n-1}{2n}} - 1 \right\}; \quad . \quad . \quad (325)$$

and the total work of compression is

$$W = 2 p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1} \right)^{\frac{n-1}{2n}} - 1 \right\}. \quad . \quad . \quad . \quad . \quad (326)$$

Three-stage Compressors.—When very high pressures are required, as where air is used for storing energy, it is customary to use a compressor with a series of three cylinders, through which the air is passed in succession, and to cool the air on the way from one cylinder to the next. If the initial and final pressures are p_1 and p_2, and if p' and p'' are the pressures in the intermediate receivers in which the air is cooled, the conditions for most economical compression may be deduced in the following way:

The work of compressing one pound of air in the several cylinders will be

$$W_1 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}; \quad . \quad . \quad . \quad (327)$$

$$W_2 = p' v' \frac{n}{n-1} \left\{ \left(\frac{p''}{p'} \right)^{\frac{n-1}{n}} - 1 \right\}; \quad . \quad . \quad . \quad (328)$$

$$W_3 = p'' v'' \frac{n}{n-1} \left\{ \left(\frac{p_2}{p''} \right)^{\frac{n-1}{n}} - 1 \right\}. \quad . \quad . \quad (329)$$

But since the air is cooled to the initial temperature on its way from one cylinder to the other so that

$$p_1 v_1 = p' v' = p'' v''; \quad . \quad . \quad . \quad . \quad . \quad (330)$$

consequently the total work of compressing one pound of air will be

$$W = W_1 + W_2 + W_3$$
$$= p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p''}{p'}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p''}\right)^{\frac{n-1}{n}} - 3 \right\}. \quad (331)$$

This expression will be a minimum when

$$\left(\frac{p'}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p''}{p'}\right)^{\frac{n-1}{n}} + \left(\frac{p_2}{p''}\right)^{\frac{n-1}{n}}$$

becomes a minimum; that is, when

$$\left(\frac{dW}{dp'}\right)_{p''} = \frac{n-1}{n} \frac{p'^{-\frac{1}{n}}}{p_1^{\frac{n-1}{n}}} - \frac{n-1}{n} \frac{p''^{\frac{n-1}{n}}}{p'^{\frac{2n-1}{n}}} = 0, \quad . \quad (332)$$

and

$$\left(\frac{dW}{dp''}\right)_{p'} = \frac{n-1}{n} \frac{p''^{-\frac{1}{n}}}{p'^{\frac{n-1}{n}}} - \frac{n-1}{n} \frac{p_2^{\frac{n-1}{n}}}{p''^{\frac{2n-1}{n}}} = 0. \quad . \quad (333)$$

Equations (328) and (329) lead to

$$p'^2 = p_1 p'', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (334)$$

$$p''^2 = p' p_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (335)$$

from which by elimination we have

$$p' = \sqrt[3]{p_1^2 p_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (336)$$

and

$$p'' = \sqrt[3]{p_1 p_2^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (337)$$

Since the temperature is the same at the admission to each of the three cylinders, the volumes of the cylinders should be inversely proportional to the absolute pressures p_1, p', and p''. As with the compound compressors, this method of arranging a three-stage compressor leads to an equal distribution of work between the cylinders. For, if the values of p' and p'' from equations (336) and (337) are introduced into equations (327) to (329), taking account also of the equation (330) we shall have

$$W_1 = W_2 = W_3 = p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{3n}} - 1 \right\}; \quad . \quad (338)$$

and consequently the total work of compression is

$$W = 3 p_1 v_1 \frac{n}{n-1} \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{3n}} - 1 \right\}. \quad . \quad . \quad . \quad . \quad . \quad (339)$$

Friction and Imperfections.—The discussion has thus far taken no account of friction of the compressor nor of imperfections due to delay in the action of the valves and to heating the air as it enters the cylinder of the compressor.

From comparisons of indicator-diagrams taken from the steam- and the air-cylinders of certain combined steam-engines and air-compressors at Paris, Professor Kennedy found a mechanical efficiency of 0.845. Professor Gutermuth found an efficiency of 0.87 for a new Riedler compressor. It will be fair to assume an efficiency of 0.85 for compressors which are driven by steam-engines; compressors driven by turbines will probably be affected to a like extent by friction.

The following table given by Professor Unwin * shows the effect of imperfect valve-action and of heating the entering

* *Development and Transmission of Power*, p. 182.

air as deduced from tests on a Dubois-François compressor which had a diameter of 18 inches and a stroke of 48 inches:

RATIO OF ACTUAL AND APPARENT CAPACITIES OF AN AIR-COMPRESSOR.

Piston speed, feet per minute.	Revolutions per minute.	Ratio of air delivered at atmospheric pressure and temperature to volume displaced by piston.
80	10	0.94
160	20	0.92
200	25	0.90
240	30	0.86
280	35	0.78

This table does not take account of the effect of clearance, nor is the clearance for the compressor stated. It is probable that five or ten per cent will be enough to allow for imperfect valve-action after the effect of clearance is properly calculated. The effect of clearance is to require a larger volume of cylinder than would be needed without clearance. The effect of imperfect valve-action and of heating of the entering air is to require an additional increase in the size of the cylinder of the air-compressor and also to increase the work of compression.

Efficiency of Compression.—If air could be so cooled during compression that the temperature should not rise the compression line *cd*, Fig. 89, would be an isothermal line and the work of compressing one pound of air would be

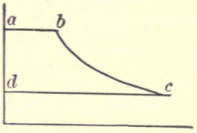

FIG. 89.

$$W = p_2 v_2 + p_1 v_1 \log_e \frac{v_1}{v_2} - p_1 v_1 ;$$

but $p_1 v_1 = p_2 v_2$ for an isothermal change, and consequently

$$W = p_1 v_1 \log_e \frac{p_2}{p_1} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (340)$$

Some investigators have taken the work of isothermal compression, represented by equation (340), as a basis of comparison for compressors and have considered its ratio to the actual work of compression as the efficiency of compression. This throws together into one factor the effect of heating during compression and the effect of imperfect valve-action.

Professor Riedler* obtained indicator-diagrams from the cylinders of a number of air-compressors and drew upon them the diagrams which would represent the work of isothermal compression, without clearance or valve losses. A comparison of the areas of the isothermal and the actual diagrams gave the arbitrary efficiency of compression just described. The following table gives his results:

ARBITRARY EFFICIENCY OF COMPRESSION.

Type of compressor.	Pressures in main, atmospheres.	Lost work in per cent of useful work.	Arbitrary efficiency.
Colladon, St. Gothard	6	105.0	0.488
do.	6	92.0	0.521
Sturgeon	3	94.3	0.515
Colladon	4	38.15	0.772
Slide-valve	5	49.3	0.670
Paxman	6	42.7	0.701
Cockerill	6	40.2	0.713
Riedler two-stage	6	12.07	0.892

A similar comparison for a fluid piston-compressor showed an efficiency of 0.84.

There are three notable conclusions that may be drawn from this table: (1) there is much difference between compressors working at the same pressures, (2) a simple compressor loses efficiency rapidly as the pressure rises, and (3) the compound or two-stage compressor shows a great advantage over a simple compressor.

* *Development and Distribution of Power*, Unwin.

Test of a Blowing-engine.—Pernolet * gives the following test of a blowing-engine used to produce the blast for Bessemer converters at Creusot. The engine was a two-cylinder horizontal engine, with the cranks at right angles. The piston-rod for each cylinder extended through the cylinder-head and actuated a double-acting compressor. The dimensions were:

Diameter, steam-pistons	1.2 metres.
" air-pistons	1.5 "
Stroke	1.8 "
Diameter of fly-wheel	8.0 "

At 28 revolutions per minute the following results were obtained:

Indicated horse-power of steam-cylinders	1082
" " " air-cylinders	999
Efficiency	0.92
Temperature of air admitted	10° C.
" " " delivered	60° C.
Pressure of air delivered, metres of mercury above the atmosphere	1.21
Pressure of air in supply-pipe, metres of mercury below the atmosphere	0.023

At 25 revolutions there was no sensible depression of pressure in the supply-pipe.

The air from such a blowing-engine probably suffers little loss of temperature after compression.

Air-pumps.—The feed-water supplied to a steam-boiler usually contains air in solution, which passes from the boiler with the steam to the engine and thence to the condenser. In like manner the injection-water supplied to a jet-condenser brings in air in solution. Also there is more or less leakage of air into the cylinder communicating with the condenser

* *L'Air Comprimé*, 1876.

and into the exhaust-pipe or the condenser itself. An air-pump must therefore be provided to remove this air and to maintain the vacuum. The air-pump also removes the condensed steam from a surface-condenser, and the mingled condensed steam and injection-water from a jet-condenser. If no air were brought into the condenser the vacuum would be maintained by the condensation of the steam by the injection, or the cooling water, and it would be sufficient to remove the water by a common pump, which, with a surface-condenser, might be the feed-pump.

The weight of injection-water per pound of steam, calculated by the method on page 250, will usually be less than 20 pounds, but it is customary to provide 30 pounds of injection-water per pound of steam, with some method of regulating the quantity delivered.

It may be assumed that the injection-water will bring in with it one-twentieth of its volume of air at atmospheric pressure, and that this air will expand in the condenser to a volume inversely proportional to the absolute pressure in the condenser. The capacity of the air-pump must be sufficient to remove this air and the condensed steam and injection-water.

An air-pump for use with a surface-condenser may be smaller than one used with a jet-condenser. In marine work it is common to provide a method of changing a surface- into a jet-condenser, and to make the air-pump large enough to give a fair vacuum in case such a change should become advisable in an emergency.

Seaton* states that the efficiency of a vertical single-acting air-pump varies from 0.4 to 0.6, and that of a double-acting horizontal air-pump from 0.3 to 0.5, depending on the design and condition; that is, the volume of air and water actually discharged will bear such ratios to the displacement of the pump.

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* *Manual of Marine Engineering.*

He also gives the following table of ratios of capacity of air-pump cylinders to the volume of the engine cylinder or cylinders discharging steam into the condenser:

RATIO OF ENGINE AND AIR-PUMP CYLINDERS.

Description of pump.	Description of engine.	Ratio.
Single-acting vertical	Jet-condensing, expansion 1½ to 2	6 to 8
" "	Surface- " " 1½ to 2	8 to 10
" "	Jet- " " 3 to 5	10 to 12
" "	Surface- " " 3 to 5	12 to 15
" "	" " compound.....	15 to 18
Double-acting horizontal...	Jet-condensing, expansion 1½ to 2	10 to 13
" " ...	Surface- " " 1½ to 2	13 to 16
" " ...	Jet- " " 3 to 5	16 to 19
" " ...	Surface- " " 3 to 5	19 to 24
" " ...	" " compound.....	24 to 28

Calculation for an Air-compressor.—Let it be required to find the dimensions of an air-compressor to deliver 300 cubic feet of air per minute at 100 pounds per square inch by the gauge, and also the horse-power required to drive it.

If it is assumed that the air is forced into the delivery-pipe at the temperature of the atmosphere, and, further, that there is no loss of pressure between the compressor and the delivery-pipe, equation (318) for finding the volume drawn into the compressor will be reduced to

$$V_1 = V_2 \frac{p_2}{p_1} = 300 \times \frac{114.7}{14.7} = 2341 \text{ cubic feet.}$$

If now we allow five per cent for imperfect valve-action and for heating the air as it is drawn into the compressor the apparent capacity of the compressor will be

$$2341 \div 0.95 = 2464 \text{ cubic feet;}$$

this is the volume on which the power for the compressor must be calculated.

If the clearance of the compressor is 0.02 of the piston

displacement, then the factor for allowing for clearance will be

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{114.7^{\frac{1}{1.4}}}{14.7}\right) + \frac{2}{100} = 0.9332$$

if the exponent of the equation representing the expansion of the air in the clearance is 1.4. Consequently the volume on which the dimensions of the compressor must be based is

$$2464 \div 0.9332 = 2640 \text{ cubic feet.}$$

At 80 revolutions per minute the mean piston displacement will be

$$2640 \div (2 \times 80) = 16.5 \text{ cubic feet.}$$

Assuming a stroke of 3 feet, the mean area of the piston must be

$$(144 \times 16.5) \div 3 = 792 \text{ square inches.}$$

Allowing 16 square inches for a piston-rod $4\frac{1}{2}$ inches in diameter gives a mean area of 800 square inches for the piston, which corresponds very nearly to 32 inches for the diameter of the piston.

The power expended in the compressor-cylinder may be calculated by equation (315), using for V_1 the apparent capacity of the compressor, giving

$$\text{H. P.} = \frac{144 \times 14.7 \times 2464 \times 1.4}{33000 \times (1.4 - 1)}\left\{\left(\frac{114.7}{14.7}\right)^{\frac{1.4-1}{1.4}} - 1\right\} = 442.$$

If the friction of the combined steam-engine and compressor is assumed to be 15 per cent the horse-power of the steam-cylinder must be

$$442 \div 0.85 = 520.$$

If the temperature of the atmosphere drawn into the

compressor is 70° F., then by an equation like (80), page 67, the delivery temperature will be

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (460.7 + 70)\left(\frac{114.7}{14.7}\right)^{\frac{1.4-1}{1.4}} = 954°.4$$

absolute, or about 494° F.

The calculation has been carried on for a simple compressor, but there will be a decided advantage in using a compound compressor for such work. Such a compressor should have for the pressure in the intermediate reservoir

$$p' = \sqrt{p_1 p_2} = \sqrt{114.7 \times 14.7} = 41.06 \text{ pounds.}$$

The factor for allowing for clearance of the low-pressure cylinder will now be

$$1 - \frac{1}{m}\left(\frac{p'}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{41.06}{14.7}\right)^{\frac{1}{1.4}} + \frac{2}{100} = 0.9784.$$

The loss from imperfect action of the valves and for heating of the air as it enters the compressor will be less for a compound than for a simple compressor, but we will here retain the value 2464 cubic feet, previously found for the apparent capacity of the compressor. The volume from which the dimensions of the condenser will be found will now be

$$2464 \div 0.9784 = 2518 \text{ cubic feet,}$$

which with 80 revolutions per minute will give 15.74 cubic feet for the piston displacement, and 755.5 square inches for the effective piston area, if the stroke is made 3 feet, as before. Adding 16 inches for the piston-rod, which will be assumed to pass entirely through the cylinder, will give for the diameter of the low-pressure cylinder $31\frac{5}{8}$ inches.

Since the pressure p' is a mean proportional between p_1 and p_2, the clearance factor for the high-pressure cylinder will be the same as that for the low-pressure cylinder, and, as the

volumes are inversely proportional to the pressures p_1 and p', the high-pressure piston displacement will be

$$(15.74 \times 14.7) \div 41.06 = 5.64 \text{ cubic feet.}$$

If we allow 8 inches for a rod $4\frac{1}{2}$ inches in diameter at one side of the piston, then the mean area of the piston will be 278.7 square inches, which corresponds to a diameter of $18\frac{7}{8}$ inches for the high-pressure cylinder. In reality the piston-rod for the compound compressor may have a less diameter than the rod for a simple compressor, because the maximum pressure on both pistons will be less than that for the piston of the simple compressor. Again, the rod which extends from the large to the small piston may be reduced in size. But details like these which depend on the calculation of strength cannot properly receive much attention at this place.

The power required to drive the compressor may be derived from equation (236), replacing v_1, the specific volume, by V_1, the apparent capacity of the low-pressure cylinder. Using the apparent capacity already obtained, 2464 cubic feet, we have for the power expended in the air-cylinders

$$\text{H. P.} = \frac{2 \times 144 \times 14.7 \times 2464 \times 1.4}{33000 \times (1.4 - 1)} \left\{ \left(\frac{41.06}{14.7} \right)^{\frac{1.4-1}{1.4}} - 1 \right\} = 377;$$

and, as before, allowing 15 per cent for friction of the engine and compressor, we have for the indicated horse-power of the steam-engine

$$377 \div 0.85 = 444.$$

The temperature at the delivery from the low-pressure cylinder will be for 70° F. atmospheric temperature

$$(460.7 + 70) \left(\frac{41.06}{14.7} \right)^{\frac{1.4-1}{1.4}} = 711°.7$$

absolute, or 251° F. Since p' is a mean proportional between p_1 and p_2, this will also be the temperature of the air delivered by the high-pressure cylinder.

Friction of Air in Pipes.—The resistance to the flow of a liquid through a pipe is represented in works on hydraulics by an expression having the form

$$\zeta \frac{u^2}{2g} \frac{l}{m}, \quad \ldots \ldots \ldots \quad (341)$$

in which ζ is an experimental coefficient, u is the velocity in feet per second, g is the acceleration due to gravity, l is the length of the pipe in feet, and m is the hydraulic mean depth, which last term is obtained by dividing the area of the pipe by its perimeter. For a cylindrical pipe we have consequently

$$m = \tfrac{1}{4}\pi d^2 \div \pi d = \tfrac{1}{4}d. \quad \ldots \ldots \quad (342)$$

The expression represents the head of liquid required to overcome the resistance of friction in the pipe when the velocity of flow is u feet per second. Such an expression cannot properly be applied to flow of air through a pipe when there is an appreciable loss of pressure, for the accompanying increase in volume necessitates an increase of velocity, whereas the expression treats the velocity as a constant. If, however, we consider the flow through an infinitesimal length of pipe, for which the velocity may be treated as constant, we may write for the loss of head due to friction

$$\zeta \frac{u^2}{2g} \frac{dl}{m}. \quad \ldots \ldots \ldots \quad (343)$$

This loss of head is the vertical distance through which the air must fall to produce the work expended in overcoming friction, and the total work thus expended may be found by multiplying the loss of head by the weight of air flowing through the pipe. It is convenient to deal with one pound of air, so that the expression for the loss of head also represents the work expended.

The air flowing through a long pipe soon attains the temperature of the pipe and thereafter remains at a constant temperature, so that our discussion for the resistance of fric-

tion may be made under the assumption of constant temperature, which much simplifies our work, because the intrinsic energy of the air remains constant. Again, the work done by the air on entering a given length dl will be equal to the work done by the air when it leaves that section, because the product of the pressure by the volume is constant.

Since there is a continual increase of volume corresponding to the loss of pressure to overcome friction, and consequently a continual increase of velocity from the entrance to the exit end of the pipe, there is also a continual gain of kinetic energy. But the velocity of air in long pipes is small and the changes of kinetic energy can be neglected.

The air expands by the amount dv as it passes through the length dl of pipe, and each pound does the work pdv. This work must be supplied by the loss of head, and, since there is no other expenditure of energy, the work expended in the loss of head is equal to the work done by expansion; consequently

$$pdv = \zeta \frac{u^2}{2g} \frac{dl}{m}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (343)$$

But from the characteristic equation

$$pv = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (344)$$

we have

$$dv = -\frac{RT}{p^2} dp,$$

which substituted in equation (343) gives

$$\zeta \frac{u^2}{2g} \frac{dl}{m} = -\frac{RT}{p} dp. \quad . \quad . \quad . \quad . \quad . \quad (345)$$

If the area of the pipe is A square feet, and if W pounds of air flow through it per second, then

$$u = \frac{Wv}{A} = \frac{WRT}{Ap}, \quad . \quad . \quad . \quad . \quad . \quad (346)$$

in which v is the specific volume, for which a value may be derived from equation (344). Replacing u in equation (345) by the value just derived, we have

$$\zeta \frac{W^2 T^2 R^2 dl}{2gA^2 p^2 m} = -\frac{RT}{p} dp;$$

$$\therefore \quad \zeta \frac{W^2 dl}{2gA^2 m} = -\frac{p}{RT} dp. \quad \ldots \quad (347)$$

Integrating between the limits L and 0, and p_2 and p_1, we have

$$\zeta \frac{W^2 L}{gA^2 m} = \frac{p_1^2 - p_2^2}{RT}. \quad \ldots \quad (348)$$

But from equation (346) the velocity at the entrance to the pipe where the pressure is p_1 will be

$$u_1 = \frac{WRT}{Ap_1} \quad \text{and} \quad W = \frac{Ap_1 u_1}{RT},$$

so that equation (348) may be reduced to

$$\zeta \frac{A^2 p_1^2 u_1^2 L}{gA^2 m R^2 T^2} = \frac{p_1^2 - p_2^2}{RT};$$

$$\therefore \quad \zeta \frac{u_1^2 L}{gRTm} = \frac{p_1^2 - p_2^2}{p_1^2}. \quad \ldots \quad (349)$$

Equation (349 may be solved as follows:

$$u_1 = \left\{ \frac{gRTm}{\zeta L} \frac{p_1^2 - p_2^2}{p_1^2} \right\}^{\frac{1}{2}}; \quad \ldots \quad (350)$$

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{gRTm} \right\}^{\frac{1}{2}}; \quad \ldots \quad (351)$$

$$\zeta = \frac{gRTm}{u_1^2 L} \frac{p_1^2 - p_2^2}{p_1^2}. \quad \ldots \quad (352)$$

The first two forms allow us to calculate either the velocity or the loss of pressure; the last form may be used to calculate values of ζ from experiments on the flow through pipes.

From experiments made by Riedler and Gutermuth* Professor Unwin † deduces the following values for ζ:

Diameter of pipe, feet.	ζ
0.492	0.00435
0.656	0.00393
0.980	0.00351

For pipes over one foot in diameter he recommends for use

$$\zeta = 0.003.$$

Replacing the hydraulic mean depth m by $\frac{1}{4}d$, its value for round pipes, and using $R = 53.22$ and $g = 32.16$, we have in place of equation (351)

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{430 Td} \right\}^{\frac{1}{2}} \quad \ldots \ldots \quad (353)$$

All of the dimensions are given in feet, but from the form of the equation it is evident that the pressures may be in any convenient units, for example, in pounds per square inch absolute.

For example, let us find the loss of pressure of 300 cubic feet per minute if delivered through a six-inch pipe a mile long, the initial pressure being 100 pounds by the gauge.

The velocity of the air will be

$$(300 \div 60) \div \frac{\pi d^2}{4} = 5 \div \frac{\pi(\frac{6}{12})^2}{4} = 25.5 \text{ feet.}$$

The terminal pressure will consequently be

$$p_2 = p_1 \left\{ 1 - \frac{\zeta u_1^2 L}{430 Td} \right\}^{\frac{1}{2}} = 114.7 \left\{ 1 - \frac{0.0044 \times \overline{25.5}^2 \times 5280}{430(460.7 + 70)\frac{1}{2}} \right\}^{\frac{1}{2}}$$
$$= 106.8 \text{ pounds,}$$

* *Neue Erfahrungen über die Kraftversorgung von Paris durch Druckluft*, 1891.

† *Development and Distribution of Power.*

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with 70° F. for the temperature of the atmosphere and with $\zeta = 0.0044$. Consequently the loss of pressure is about eight pounds.

Compressed-air Engines. — Engines for using compressed air differ from steam-engines only in details that depend on the nature of the working fluid. In some instances compressed air has been used in steam-engines without any change; for example, in Fig. 90 the dotted diagram was

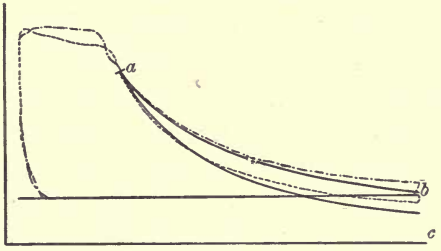

FIG. 90.

taken from the cylinder of an engine using compressed air, and the dot-and-dash diagram was taken from the same end of the cylinder when steam was used in it. The full line *ab* is a hyperbola and the line *ac* is the adiabatic line for a gas; both lines are drawn through the intersection of the expansion lines of the two diagrams.

Power of Compressed-air Engines. — The probable mean effective pressure attained in the cylinder of a compressed-air engine, or to be expected in a projected engine, may be found in the same manner as is used in designing a steam-engine. In Fig. 91 the expansion curve 1 2 and the compression curve 3 0 may be assumed to be adiabatic lines for a gas represented by the equation

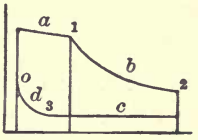

FIG. 91.

$$pv^{\kappa} = p_1 v_1^{\kappa},$$

and the area of the diagram may be found in the usual way, and therefrom the mean effective pressure can be determined.

Having the mean effective pressure, the power of a given engine or the size required for a given power may be determined directly. The method will be illustrated later by an example.

Air-consumption.—The air consumed by a given compressed-air engine may be calculated from the volume, pressure, and temperature at cut-off or release, and the volume, temperature, and pressure at compression, in the same way that the indicated consumption of a steam-engine is calculated; but in this case the indicated and actual consumption should be the same, since there is no change of state of the working fluid. Since the intrinsic energy of a gas is a function of the temperature only, the temperature will not be changed by loss of pressure in the valves and passages, and the air at cut-off will be cooler than in the supply-pipe, only on account of the chilling action of the walls of the cylinder during admission, which action cannot be energetic when the air is dry, and probably is not very important when the air is saturated.

Final Temperature.—If the expansion in a compressed-air engine is complete, i.e., if it is carried down to the pressure in the exhaust-pipe, then, assuming that there are no losses of pressure in valves and passages, the final temperature may be found by the equation

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{\frac{\kappa-1}{\kappa}}. \quad . \quad . \quad . \quad . \quad . \quad (354)$$

If the expansion is not complete, then the temperature at the end of expansion may be found by the equation

$$T_r = T_3\left(\frac{V_c}{V_r}\right)^{\kappa-1}, \quad . \quad . \quad . \quad . \quad . \quad (355)$$

in which V_c is the volume in the cylinder at cut-off and V_r at release, T_r is the absolute temperature at the end of expansion, and T_3 is the temperature at cut-off, assumed to be the

same as in the supply-pipe. T_r is not the temperature during back-pressure nor in the exhaust-pipe. When the exhaust-valve is opened at release the air will expand suddenly, and part of the air will be expelled at the expense of the energy in the air remaining—much as though that air expanded behind a piston, and the temperature in the cylinder during exhaust and at the beginning of compression may be calculated by equation (354). The temperature in the exhaust-pipe will not be so low, for the temperature of the escaping air will vary during the expulsion produced by sudden expansion, and will only at the end of that operation have the temperature T_4, while the energy expended on that air to give it velocity will be restored when the velocity is reduced to that in the exhaust pipe.

Volume of the Cylinder. —The determination of the volume of the cylinder of a compressed-air engine which uses a stated volume of air per minute is the converse of the determination of the air consumed by a given engine, and can be found by a similar process. We may calculate the volume of air, at the pressure in the supply-pipe, consumed per stroke by an engine having one unit of volume for its piston displacement, and therefrom find the number of units of volume of the piston displacement for the required engine.

Interchange of Heat.—The interchanges of heat between the walls of the cylinder of a compressed-air engine and the air working therein are of the same sort as those taking place between the steam and the walls of the cylinder of a steam-engine; that is to say, the walls absorb heat during admission and compression, if the latter is carried to a considerable degree, and yield heat during expansion and exhaust. Since the walls of the cylinder are never so warm as the entering air nor so cold as the air exhausted, the walls may absorb heat during the beginning of expansion and yield heat during the beginning of compression.

The amount of interchange of heat is much less in a compressed-air engine than in a steam-engine. With a moderate

expansion the interchanges of heat between dry air and the walls of the cylinder are insignificant. Moisture in the air increases the interchanges in a marked degree, but does not make them so large that they need be considered in ordinary calculations.

Moisture in the Cylinder.—The chief disadvantage in the use of moist compressed air—and it is fair to assume that compressed air is nearly if not quite saturated when it comes to the engine—is that the low temperature experienced when the range of pressures is considerable causes the moisture to freeze in the cylinder and clog the exhaust-valves. The difficulty may be overcome in part by making the valves and passages of large size. Freezing of the moisture may be prevented by injecting steam or hot water into the supply-pipe or the cylinder, or the air may be heated by passing it through externally heated pipes or by some similar device. In the application of compressed air to driving street-cars the air from the reservoir has been passed through hot water, and thereby made to take up enough hot moisture to prevent freezing. The study of gas-engines suggests a method of heating compressed air which it is believed has never been tried. The air supplied to a compressed-air engine, or a part of the air, could be caused to pass through a lamp of proper construction to give complete combustion, and the products of combustion passed to the engine with the air. Should such a device be used it would be advisable that the temperature of the air should be raised only to a moderate degree to avoid destruction of the lubricants in the cylinder, and the combustion at all hazards must be complete, or the cylinder would be fouled by unburned carbon.

Compound Air-engines.—When air is expanded to a considerable degree in a compressed-air engine a gain may be realized by dividing the expansion into two or more stages in as many cylinders, provided that the air can be economically reheated between the cylinders. The heat of the atmosphere or of water at the same temperature may sometimes be used

for this purpose. It is not known that machines of this construction have been used. If they were to be constructed the practical advantages of equal distribution of work and pressure would probably control the ratio of the volumes of the cylinders.

Calculation for a Compressed-air Engine.—Let it be required to find the dimensions for a compressed-air engine to develop 100 indicated horse-power at the pressure of 92 pounds by the gauge and at 70° F. Assume the clearance to be five per cent of the piston displacement, and assume the cut-off to be at half stroke, the release to be at the end of the stroke, and the compression at one-tenth of the stroke.

If the piston displacement is represented by D, then the volume in the cylinder at cut-off will be $0.30D$, that at release will be $1.05D$, and that at compression will be $0.15D$. The absolute pressures during supply and exhaust may be assumed to be 106.7 and 14.7 pounds per square inch. The work for one stroke of the piston will be

$$W = 144 \times 106.7 \times 0.25D + \frac{144 \times 106.7 \times 0.30D}{\kappa - 1}\left\{1 - \left(\frac{0.30}{1.05}\right)^{1.4-1}\right\}$$

$$- 144 \times 14.7 \times 0.9D - \frac{144 \times 14.7 \times 0.15D}{\kappa - 1}\left\{1 - \left(\frac{0.05}{0.15}\right)^{1.4-1}\right\}$$

$$= 144D(26.68 + 31.530 - 13.23 - 1.96) = 144 \times 43.02D.$$

The corresponding mean effective pressure is 43.02 pounds per square inch. If the engine is furnished with large ports and automatic valve-gear the actual mean effective pressure may be 0.9 of that just calculated, or 38.7 pounds per square inch.

For a piston displacement D the engine will develop at 150 revolutions per minute

$$\frac{144 \times 38.7D \times 2 \times 150}{33000} \text{ horse-power;}$$

and conversely to develop 100 horse-power the piston displacement must be

$$D = \frac{100 \times 33000}{144 \times 38.7 \times 2 \times 150} = 1.974 \text{ cubic feet,}$$

and with a stroke of 2 feet the effective area of the piston will be

$$1.974 \times 144 \div 2 = 142.1 \text{ square inches.}$$

If the piston-rod is 2 inches in diameter it will have an area of 3.14 square inches, so that the mean area of the piston will be 143.7 square inches, corresponding to a diameter of $13\frac{1}{2}$ inches.

We find, consequently, that an engine developing 100 horse-power under the given conditions will have a diameter of $13\frac{1}{2}$ inches and a stroke of 2 feet, provided that it runs at 150 revolutions per minute.

In order to determine the amount of air used by the engine we must consider that the air caught at compression is compressed to the full admission-pressure of 106.7 pounds absolute. Part of this compression is done by the piston and part by the entering air, but for our present purpose it is immaterial how it is done. The volume filled by air at atmospheric pressure when the exhaust-valve closes (including clearance) is 0.15 of the piston displacement. When the pressure is increased to 106.7 pounds the volume will be reduced to

$$0.15\left(\frac{14.7}{106.7}\right)^{\frac{1}{1.4}} = 0.017$$

of the piston displacement. The volume drawn in from the supply-pipe will consequently be

$$0.25 + 0.05 - 0.017 = 0.283$$

of the piston displacement. If the compression occurred sufficiently early to raise the pressure to that in the supply-pipe before the admission-valve opened, then only 0.25 of the

piston displacement would be used per stroke and a saving of about 13 per cent would be attained; in such case the mean effective pressure would be smaller and the size of the cylinder would be larger.

The air-consumption for the engine appears to be

$$2 \times 150 \times 0.283 \times \text{pist. displ.} = 2 \times 150 \times 0.283 \times 1.974 = 167.6$$

cubic feet per minute. The actual air-consumption will be somewhat less on account of loss of pressure in the valves and passages; it may be fair to assume 160 cubic feet per minute for the actual consumption.

In order to make one complete calculation for the use of compressed air for transmitting power the data for the compressed-air engine have been made to correspond with the results of calculations for an air-compressor on page 459 and for the loss of pressure in a pipe on page 466. Since there is a loss of pressure in flowing through the pipe at constant temperature, there is a corresponding increase of volume, so that the pipe delivers

$$300 \times 114.7 \div 106.7 = 322.6$$

cubic feet per minute. Our calculation for the air-consumption of an engine to deliver 100 horse-power gives about 160 cubic feet, from which it appears that the system of compressor, conducting-pipe, and compressed-air engine should deliver

$$100 \times 322.6 \div 160 = 200 + \text{horse-power.}$$

If the friction of the compressed-air engine is assumed to be ten per cent the power delivered by it to the main shaft (or to the machine driven directly from it) will be

$$200 \times .9 = 180 \text{ horse-power.}$$

The steam-power required to drive a simple compressor was found to be 520 horse-power; it consequently appears that

$$180 \div 520 = 0.34$$

of the indicated steam-power is actually obtained for doing work from the entire system of transmitting power. If, however, a compound compressor is used, then the indicated steam-power is 444, and of this

$$180 \div 444 = 0.40$$

will be obtained for doing work.

If, however, we consider that the power would in any case be developed in a steam-engine, and that the transmission system should properly include only the compressor-cylinder, the pipe, and the compressed-air engine, then our basis of comparison will be the indicated power of the compressor-cylinder. For the simple compressor we found the horse-power to be 442, which gives for the efficiency of transmission

$$180 \div 442 = 0.41,$$

while the compound compressor demanded only 377 horse-power, giving an efficiency of

$$180 \div 377 = 0.48.$$

It appeared that the failure to obtain complete compression involved a loss of about 13 per cent in the air-consumption. It may then be assumed that with complete compression our engine could deliver 200 horse-power to the main shaft In that case the efficiency of transmission when a compound compressor is used may be 0.53.

Efficiency of Compressed-air Transmission.—The preceding calculation exhibits the defect of compressed air as a means of transmitting power. It is possible that somewhat better results may be obtained by a better choice of pressures or proportions; Professor Unwin estimates that when used on a large scale from 0.44 to 0.51 of the indicated steam-power may be realized on the main shaft of the compressed-air engine. On the other hand, when compressed air is used in small motors, and especially in rock-drills and other mining-machinery, much less efficiency may be expected.

Experiments made by M. Graillot * of the Blanzy mines showed an efficiency of from 22 to 32 per cent. Experiments made by Mr. Daniel at Leeds gave an efficiency varying from 0.255 to 0.455, with pressures varying from 2.75 atmospheres to 1.33 atmospheres. An experiment made by Mr. Kraft † gave an efficiency of 0.137 for a small machine, using air at a pressure of five atmospheres without expansion.

Compressed air has been used for transmitting power either where power for compression is cheap and abundant, or where there are reasons why it is specially desirable, as in mining and tunnelling. It is now used to a considerable extent for driving hand-tools, such as drills, chipping-chisels, and calking-tools, in machine- and boiler-shops, and in shipyards. It is also used for operating cranes and other machines where power is used only at intervals, so that the condensation of steam (when used directly) is excessive, and where hydraulic power is liable to give trouble from freezing.

Compressed air has been used to a very considerable extent for transmitting power in Paris. The system appears to be expensive and to be used mainly on account of its convenience for delivering small powers or in places where the cold exhaust can be used for refrigeration. The trouble from freezing of moisture in the cylinder has been avoided by allowing the air to flow through a coil of pipe which is heated externally by a charcoal fire. Professor Unwin estimates that an efficiency of transmission of 0.75 may be attained under favorable conditions when the air is heated near the compressor, but he does not include the cost of fuel for reheating in this estimate.

Storage of Power by Compressed Air.—Reservoirs or cylinders charged with compressed air have been used to store power for driving street-cars. A system developed by Mekarski uses air at 350 to 450 pounds per square inch in reservoirs having a capacity of 75 cubic feet. The car also carries a tank

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* Pernolet, *L'Air Comprimé*, pp. 549, 550.

† *Revue universelle des Mines*, 2 série, tome vi.

of hot water at a temperature of about 350° F., through which the air passes on the way to the motor and by which it is heated and charged with steam. This use of hot water gives a secondary method of storing power and also avoids trouble from freezing in the motor-cylinders. Air at much higher pressures has been used for driving street-cars in New York City, but the particulars have not been given to the public.

The calculation for storage of power may be made in much the same way as that for the transmission of power; the chief difference is due to the fact that the air is reduced in pressure by passing it through a reducing-valve on the way from the reservoir to the motor. By the theory of perfect gases such a reduction of pressure should not cause any change of temperature, but the experiments of Joule and Thomson (page 72) show that there will be an appreciable, though not an important, loss of temperature when there is a large reduction of pressure. Thus at 70° F. or 21°.1 C. the loss of temperature for each 100 inches of mercury will be

$$0°.92 \times \left(\frac{273.7}{294.8}\right)^2 = 0°.79 \text{ C.} = 1\tfrac{3}{4}° \text{ F.}$$

Now 100 inches of mercury are equivalent to about 49 pounds to the square inch, so that 100 pounds difference of pressure will give about $3\frac{1}{2}°$ F. reduction of temperature and 1000 pounds difference of pressure will give about 35° F. reduction of temperature. The last figures are far beyond the limits of the experiments and the results are therefore crude. Again, the air in passing through the reducing-valve and the piping beyond will gain heat and consequently show a smaller reduction of temperature. The whole subject of loss of temperature due to throttling is more curious than useful and need not be considered in practical calculations.

For an example of the calculation for storage of power let us find the work required to store air at 450 pounds per square inch in a reservoir containing 75 cubic feet. Replacing the specific volume v_1 in equation (339) by the actual volume, we

have for the work of compression (not allowing for losses and imperfections)

$$W = 3 \times 14.7 \times 144 \times 75 \frac{1.4}{1.4 - 1} \left\{ \left(\frac{464.7}{14.7} \right)^{\frac{1.4-1}{3 \times 1.4}} - 1 \right\}$$
$$= 649300 \text{ foot-pounds.}$$

If the pressure is reduced to 50 pounds by the gauge before it is used the volume of air will be

$$75 \times 464.7 \div 64.7 = 539 \text{ cubic feet.}$$

The work for complete expansion of one pound to the pressure of the atmosphere will be

$$W = p_3 v_3 + \frac{p_3 v_3}{\kappa - 1} \left\{ 1 - \left(\frac{v_3}{v_4} \right)^{\kappa - 1} \right\} - p_4 v_4$$

$$= p_3 v_3 \frac{\kappa}{\kappa - 1} \left\{ 1 - \left(\frac{p_4}{p_3} \right)^{\frac{\kappa - 1}{\kappa}} \right\},$$

and the work for 539 cubic feet will be

$$144 \times 64.7 \times 539 \frac{1.4}{1.4 - 1} \left\{ 1 - \left(\frac{14.7}{64.7} \right)^{\frac{1.4-1}{1.4}} \right\} = 487300$$

foot-pounds, without allowing for losses or imperfections. The maximum efficiency of storing and restoring energy by the use of compressed air in this case is therefore

$$487300 \div 649300 = 0.75.$$

In practice the efficiency cannot be more than 0.50, if indeed it is so high.

It may not be out of place to call attention to a danger that may arise if air at high pressure is suddenly let into a

pipe which has oil mingled with the air in it or even adhering to the side of the pipe. The air in the pipe will be compressed and its temperature may become high enough to ignite the oil and cause an explosion. That this danger is not imaginary is shown by an explosion which occurred under such conditions in a pipe which was strong enough to withstand the air-pressure.

CHAPTER XVIII.

REFRIGERATING-MACHINES.

A REFRIGERATING-MACHINE is a device for producing low temperatures or for cooling some substance or space. It may be used for making ice or for maintaining a low temperature in a cellar or storehouse.

Refrigeration on a small scale may be obtained by the solution of certain salts; a familiar illustration is the solution of common salt with ice, another is the solution of sal ammoniac in water. Certain refrigerating-machines depend on the rapid absorption of some volatile liquid, for example, of ammonia by water; if the machine is to work continuously there must be some arrangement for redistilling the liquid from the absorbent. The most recent and powerful refrigerating-machines are reversed heat-engines. They withdraw the working substance (air or ammonia) from the cold-room or cooling-coil, compress it, and deliver it to a cooler or condenser. Thus they take heat from a cold substance, do work and add heat, and finally reject the sum of the heat drawn in and the heat equivalent of the work done. These reversed heat-engines, however, are very far from being reversible engines, not only on account of imperfect valve action and losses of pressure, but because the walls of the compressor-cylinder have an appreciable effect on the working substance.

Two forms of refrigerating-machines are in common use, air refrigerating-machines and ammonia refrigerating-machines. Sometimes sulphur dioxide or some other volatile fluid is used instead of ammonia.

Air Refrigerating-machine.—The general arrangement of an air refrigerating-machine is shown by Fig. 92. It consists of a compression-cylinder *A*, an expansion-cylinder *B* of smaller size, and a cooler *C*. It is commonly used to keep the atmosphere in a cold-storage room at a low temperature, and has certain advantages for this purpose, especially on ship-board. The air from the storage-room comes to the compressor at or about freezing-point, is compressed to two or three atmospheres and delivered to the cooler, which has the same form as a surface-condenser, with cooling water entering at *e* and leaving at *f*. The diaphragm *mn* is intended to improve the circulation of the cooling water. From the cooler the air, usually somewhat warmer than the atmosphere, goes to the expansion-cylinder *B*, in which it is expanded nearly

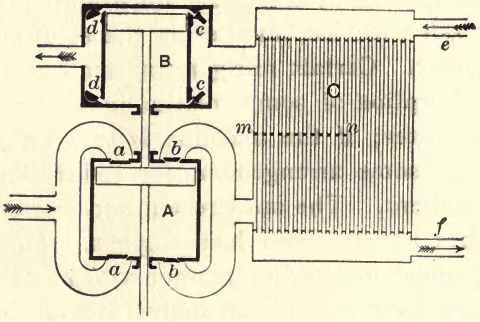

FIG. 92.

to the pressure of the air and cooled to a low temperature, and then delivered to the storage-room. The inlet-valves *a*, *a* and the delivery-valves *b*, *b* of the compressor are moved by the air itself; the admission-valves *c*, *c* and the exhaust-valves *d*, *d* of the expansion-cylinder are like those of a steam-engine and must be moved by the machine. The difference between the work done on the air in the compressor and that done by the air in the expansion-cylinder, together with the friction work of the whole machine, must be supplied by a steam-engine or other motor.

It is customary to provide the compression-cylinder with a water-jacket to prevent overheating, and frequently a spray of water is thrown into the cylinder to reduce the heating and the work of compression. Sometimes the cooler *C*, Fig. 92, is replaced by an apparatus resembling a steam-engine jet-condenser, in which the air is cooled by a spray of water. In any case it is essential that the moisture in the air, as well as the water injected, should be efficiently removed before the air is delivered to the expansion-cylinder; otherwise snow will form in that cylinder and interfere with the action of the machine. Various mechanical devices have been used to collect and remove water from the air, but air may be saturated with moisture after it has passed such a device. The Bell-Coleman Company use a jet-cooler with provision for collecting and withdrawing water, and then pass the air through pipes in the cold-room on the way to the expansion-cylinder. The cold-room is maintained at a temperature a little above freezing-point, so that the moisture in the air is condensed upon the sides of the pipes and drains back into the cooler. The same machine as made by Menck and Hambrock is provided with a device for removing moisture from the air that is shown by Fig. 93. Air from the cooler comes in by the pipe *a*, is distributed by the annular perforated pipe *b*, and passes out to the expansion-cylinder by the pipe *c*. The chamber *E* is surrounded by a jacket through which passes the cold air on the way from the expansion-cylinder to the cold-room. Since the air in the jacket is many degrees below freezing-point, the walls of the chamber *E* are quickly covered with frost, which accumulates till a considerable thickness is attained; afterwards the moisture condenses and runs down to the bottom of the chamber, from whence it is withdrawn. A coil of steam-pipe *dd* is provided for thawing ice and snow that may accumulate at the bottom of the chamber. Since the same air is used continuously, being taken from the cold-room, chilled, and returned, the effect of these devices is to remove the moisture from the air in the cold-room and to

maintain a cold, dry atmosphere in it, which is well adapted to preserving all kinds of perishable provisions.

When an air refrigerating-machine is used as described the pressure in the cold-room is necessarily that of the atmosphere, and the size of the machine is large as compared with its performance. The performance may be increased by running the machine on a closed cycle with higher pressures; for example, the cold air may be delivered to a coil of pipe in a

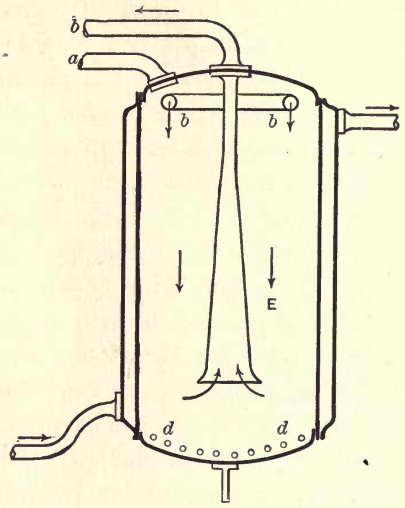

FIG. 93.

non-freezing salt solution, from which the air abstracts heat through the walls of the pipe and then passes to the compressor to be used over again. The machine may then be used to produce ice, or the brine may be used for cooling spaces or liquids. A machine has been used for producing ice on a small scale, without cooling water, on the reverse of this principle: that is, atmospheric air is first expanded and chilled and delivered to a coil of pipe in a salt solution, then the air is drawn from this coil, after absorbing heat from the brine, compressed to atmospheric pressure, and expelled.

Proportion of Air of Refrigerating-machines.—The performance of a refrigerating-machine may be stated in terms of the number of thermal units withdrawn in a unit of time, or in terms of the weight of ice produced. The latent heat of fusion of ice may be taken to be 80 calories or 144 B. T. U.

Let the pressure at which the air enters the compression-cylinder be p_1, that at which it leaves be p_2; let the pressure at cut-off in the expanding-cylinder be p_3 and that of the back-pressure in the same be p_4; let the temperatures corresponding to these pressures be t_1, t_2, t_3, and t_4, or, reckoned from the absolute zero, T_1, T_2, T_3, and T_4. With proper valve-gear and large, short pipes communicating with the cold-chamber p_4 may be assumed to be equal to p_1 and equal to the pressure in that chamber. Also t_1 may be assumed to be the temperature maintained in the cold-chamber, and t_3 may be taken to be the temperature of the air leaving the cooler. With a good cut-off mechanism and large passages p_3 may be assumed to be nearly the same as that of the air supplied to the expanding-cylinder. Owing to the resistance to the passage of the air through the cooler and the connecting pipes and passages, p_3 is considerably less than p_2.

It is essential for best action of the machine that the expansion and compression of the expanding-cylinder shall be complete. The compression may be made complete by setting the exhaust-valve so that the compression shall raise the pressure in the clearance-space to the admission-pressure p_3 at the instant when the admission-valve opens. The expansion can be made complete only by giving correct proportions to the expanding- and compression-cylinders.

The expansion in the expanding-cylinder may be assumed to be adiabatic, so that

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\kappa - 1}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (356)$$

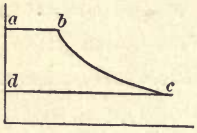

FIG. 94.

Were the compression also adiabatic the temperature t_2 could be determined in a similar manner; but the air is usually cooled during compression, and contains more or less vapor, so that the temperature at the end of compression cannot be determined from the pressure alone, even though the equation of the expansion curve be known.

Let the air passing through the refrigerating-machine per minute be M; then the heat withdrawn from the cold-room is

$$Q_1 = Mc_p(t_1 - t_4). \quad . \quad . \quad . \quad . \quad . \quad (357)$$

The work of compressing M pounds of air from the pressure p_1 to the pressure p_2 in a compressor without clearance is (Fig. 94)

$$W_c = M\left\{p_2v_2 + \int_{v_2}^{v_1} pdv - p_1v_1\right\};$$

$$\therefore \quad W_c = M\left\{p_2v_2 + \frac{p_1v_1}{n-1}\left[\left(\frac{v_1}{v_2}\right)^{n-1} - 1\right] - p_1v_1\right\},$$

$$\therefore \quad W_c = M\left\{p_1v_1\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \frac{p_1v_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] - p_1v_1\right\},$$

$$\therefore \quad W_c = Mp_1v_1\frac{n}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}, \quad . \quad . \quad . \quad . \quad . \quad (358)$$

provided that the compression curve can be represented by an exponential equation. If the compression can be assumed to be adiabatic

$$W_c = Mp_1v_1\frac{\kappa}{\kappa-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right\} = \frac{Mc_p}{A}(t_2 - t_1); \quad (359)$$

for in such case we have the equations

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}, \qquad AR = c_p - c_v = c_p\frac{\kappa-1}{\kappa}.$$

If the expansion is complete in the expanding-cylinder, as should always be the case, then the equation for the work done by the air will have the same form as equation (358) or (359), replacing t_1 and p by t_4 and p_4, and t_2 and p_2 by t_3 and p_3; so that

$$W_e = Mp_4v_4\frac{n}{n-1}\left\{\left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} - 1\right\}, \quad . \quad . \quad . \quad . \quad (360)$$

and for adiabatic expansion

$$W_e = \frac{Mc_p}{A}(t_3 - t_4). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (361)$$

The difference between the works of compression and expansion is the net work required for producing refrigeration; consequently

$$W = W_c - W_e = \frac{Mc_p}{A}\{t_2 - t_1 - t_3 + t_4\}; \quad . \quad (362)$$

or, replacing M by its value from equation (357),

$$W = \frac{Q_1}{A}\frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (363)$$

The net horse-power required to abstract Q_1 thermal units per minute is consequently

$$P_n = \frac{778Q_1}{33000}\frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4}, \quad . \quad . \quad . \quad . \quad . \quad (364)$$

where t_1 is the temperature of the air drawn into the compressor and t_2 is the temperature of the air forced by the compressor into the cooler, and t_3 is the temperature of the air supplied to the expanding-cylinder and t_4 is the temperature of the cold air leaving the expanding-cylinder. The gross horse-power developed in the steam-engine which drives the refrigerating-machine is likely to be half again as much as the net horse-power or even larger. The relation of the gross

and the net horse-powers for any air refrigerating-machine may readily be obtained by indicating the steam- and air-cylinders, and may serve as a basis for calculating other machines.

The heat carried away by the cooling water is

$$Q_2 = Q_1 + AW. \quad . \quad . \quad . \quad . \quad . \quad . \quad (365)$$

If compression and expansion are adiabatic, then

$$Q_2 = Mc_p(t_1 - t_4 + t_2 + t_4 - t_1 - t_3) = Mc_p(t_2 - t_3); \quad (366)$$

or, replacing M by its value from equation (357),

$$Q_2 = Q_1 \frac{t_2 - t_3}{t_1 - t_4}. \quad . \quad . \quad . \quad . \quad . \quad (367)$$

If the initial and final temperatures of the cooling water are t_i and t_k, and if q_i snd q_k are the corresponding heats of the liquid, then the weight of cooling water per minute is

$$G = \frac{Q_2}{q_k - q_i} = Q_1 \frac{t_2 - t_3}{(t_1 - t_4)(q_k - q_i)}. \quad . \quad . \quad (368)$$

The compressor-cylinder must draw in M pounds of air per minute at the pressure p_1 and the temperature t_1, that is, with the specific volume v_1; consequently its apparent piston displacement without clearance will be, at N revolutions per minute,

$$V_c = \frac{Mv_1}{2N} = \frac{MRT_1}{2Np_1}, \quad . \quad . \quad . \quad . \quad . \quad (369)$$

for the characteristic equation gives

$$p_1v_1 = RT_1.$$

Replacing M by its value from equation (357), we have

$$D_c = \frac{Q_1RT_1}{2Nc_pp_1(t_1 - t_4)}. \quad . \quad . \quad . \quad . \quad (370)$$

Since all the air delivered by the compressor must pass through the expanding-cylinder, its apparent piston displacement will be

$$D_e = D_c \frac{p_1 T_4}{p_4 T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (371)$$

If , the pressure of the air entering the compression-cylinder, is equal to p_4, that of the air leaving the expanding-cylinder (as may be nearly true with large and direct pipes for carrying the air to and from the cold-room) equation (371) will reduce to

$$D_e = D_c \frac{T_4}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (372)$$

Both the compressor- and the expanding-cylinder will have a clearance, that of the expanding-cylinder being the larger. As is shown on page 447, the piston displacement for an air-compressor with a clearance may be obtained by dividing the apparent piston displacement by the factor

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} + \frac{1}{m}.$$

If the expansion and compression of the expanding-cylinder are complete the same factor may be applied to it. For a refrigerating-machine n may be replaced by κ for both cylinders. To allow for losses of pressure and for imperfect valve action the piston displacements for both compressor- and expanding-cylinders must be increased by an amount which must be determined by practice; five or ten per cent increase in volume will probably suffice. In practice the expansion in the expanding-cylinder is seldom complete. A little deficiency at this part of the diagram will not have a large effect on the capacity of the machine, and will prevent the formation of a loop in the indicator-diagram; but a large drop at the release of the expanding-cylinder will diminish both the capacity and the efficiency of the machine.

The temperature t_4 and the capacity of the machine may be controlled by varying the cut-off of the expanding-cylinder. If the cut-off is shortened the pressure p_2 will be increased, and consequently T_4 will be diminished. This will make D_e, the piston displacement of the expanding-cylinder, smaller. A machine should be designed with the proper proportions for its full capacity, and then, when running at reduced capacity, the expansion in the expanding-cylinder will not be quite complete.

Calculation for an Air-compression Machine.—Required the dimensions and power for an air refrigerating-machine to produce an effect equal to the melting of 200 pounds of ice per hour. Let the pressure in the cold-chamber be 14.7 pounds per square inch and the temperature 32° F. Let the pressure of the air delivered by the compressor-cylinder be 39.4 pounds by the gauge or 55.1 pounds absolute, and let there be ten pounds loss of pressure due to the resistance of the cooler and pipes and passages between the compressor- and the expanding-cylinder. Let the initial and final temperatures of the cooling water be 60° F. and 80° F., and let the temperature of the air coming from the cooler be 90° F. Let the machine make 60 revolutions per minute.

With adiabatic expansion and compression the temperatures of the air coming from the compressor- and discharged from the expanding-cylinder will be

$$T_3 = 492.7 \left(\frac{54.1}{14.7}\right)^{\frac{0.4}{1.4}} = 714.9\,; \qquad \therefore \quad t_3 = 254°.2 \text{ F.}$$

$$T_4 = 460.7 + 90 \left(\frac{14.7}{44.1}\right)^{\frac{0.4}{1.4}} = 402.3\,; \qquad \therefore \quad t_4 = -\,58°.4 \text{ F.}$$

The melting of 200 pounds of ice is equivalent to

$$200 \times 144 \div 60 = 480 \text{ B. T. U.}$$

per minute; consequently the net horse-power of the machine is by equation (364)

$$P_n = \frac{778 Q_1}{33000} \frac{t_2 + t_4 - t_1 - t_3}{t_1 - t_4}$$

$$= \frac{778 \times 480}{33000} \times \frac{254.2 - 58.4 - 32 - 90}{32 + 58.4}$$

$$= \frac{778 \times 480 \times 73.8}{33000 \times 90.4} = 9.24 \text{ H. P.},$$

and the indicated power of the steam-engine may be assumed to be 14 horse-power.

By equation (370) the apparent piston displacement of the compressor without clearance will be

$$D_c = \frac{Q_1 R T_1}{2 N c_p p_1 (t_1 - t_4)}$$

$$= \frac{480 \times 53.22 \times 492.7}{2 \times 60 \times 0.2375 \times 144 \times 14.7 (32 + 58.4)} = 2.33 \text{ cu. ft.}$$

By equation (372) the apparent piston displacement of the expanding-cylinder without clearance will be

$$D_e = D_c \frac{T_4}{T_1} = 2.33 \times \frac{402.3}{492.7} = 1.90 \text{ cubic feet.}$$

If the clearance of the compressor-cylinder is 0.02 of its piston displacement, then the factor for clearance by equation (316) is

$$1 - \frac{1}{m}\left(\frac{p_2}{p_1}\right)^{\frac{1}{m}} + \frac{1}{m} = 1 - \frac{2}{100}\left(\frac{54.1}{14.7}\right)^{\frac{1}{1.4}} + \frac{2}{100} = 0.979,$$

so that the piston displacement becomes

$$2.33 \div 0.979 = 2.38 \text{ cubic feet.}$$

If, further, the clearance of the expander-cylinder is 0.05 of its piston displacement the factor for clearance becomes

$$1 - \frac{5}{100}\left(\frac{44.1}{14.7}\right)^{\frac{1}{1.4}} + \frac{5}{100} = 0.963,$$

which makes the piston displacement

$$1.90 \div 0.963 = 1.97 \text{ cubic feet.}$$

If now we allow ten per cent for imperfections we will get for the dimensions: stroke 2 feet, diameter of the compressor-cylinder 15½ inches, and diameter of the expanding-cylinder 14 inches.

Compression Refrigerating-machine.—The arrangement of a refrigerating-machine using a volatile liquid and its vapor is shown by Fig. 95. The essential parts are the compressor *A*, the condenser *B*, the valve *D*, and the vaporizer *C*. The compressor draws in vapor at a low pressure and temperature, compresses it, and delivers it to the condenser, which consists of coils of pipe surrounded by cooling water that enters at *e*

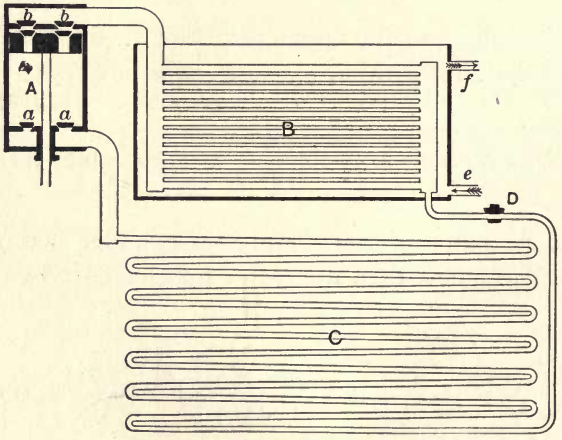

FIG. 95.

and leaves at *f*. The vapor is condensed, and the resulting liquid gathers in a reservoir in the bottom, from whence it is

led by a small pipe having a regulating-valve D to the vaporizer or refrigerator. The refrigerator is also made up of coils of pipe, in which the volatile liquid vaporizes. The coils may be used directly for cooling spaces, or they may be immersed in a tank of brine, which may be used for cooling spaces or for making ice. Fig. 95 shows the compressor with one single-acting vertical cylinder which has head-valves, foot-valves, and valves in the piston. Single-acting compressors commonly have two cylinders; horizontal compressors usually have one double-acting cylinder. Some vertical compressors are double-acting.

The cycle which has been stated for the compression refrigerating-machine is incomplete, because the working fluid is allowed to flow through the expansion-cock into the expanding-coils without doing work. To make the cycle complete there should be a small expanding-cylinder in which the liquid could do work on the way from the condenser to the vaporizing-coils; but the work gained in such a cylinder would be insignificant, and it would lead to complications and difficulties.

Proportions of Compression Refrigerating-machines.— The liquid condensed in the coils of the condenser flows to the expansion-cock with the temperature t_1 and has in it the heat q_1. In passing through the expansion-cock there is a partial vaporization, but no heat is gained or lost. The vapor flowing from the expansion-coils at the temperature t_2 and the pressure p_2 is usually dry and saturated, or perhaps slightly superheated, as it approaches the compressor. Each pound consequently carries from the expanding-coils the total heat λ_2. Consequently the heat withdrawn from the expanding-coil by a machine using M pounds of fluid per minute is

$$Q_1 = M(\lambda_2 - q_1). \quad . \quad . \quad . \quad . \quad . \quad (374)$$

The compressor-cylinder is always cooled by a water-jacket, but it is not probable that such a jacket has much

effect on the working substance, which enters the cylinder dry and is superheated by compression. We may consequently calculate the temperature of the vapor delivered by the compressor by aid of equation (181), page 135, giving

$$T_s = T_2\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (374)$$

As has already been pointed out, the vapor approaching the compressor may be treated as though it were dry and saturated, each pound having the total heat λ_2. The vapor discharged by the compressor at the temperature t_s and the pressure p_1 will have the heat

$$c_p(t_s - t_1) + \lambda_1.$$

The heat added to each pound of fluid by the compressor is consequently

$$c_p(t_s - t_1) + \lambda_1 - \lambda_2,$$

and an approximate calculation of the horse-power of the compressor may be made by the equation

$$P_c = \frac{778 M\{c_p(t_s - t_1) + \lambda_1 - \lambda_2\}}{33000}; \quad . \quad . \quad (375)$$

or, substituting for M from equation (374),

$$P_c = \frac{778 Q_1\{c_p(t_s - t_1) + \lambda_1 - \lambda_2\}}{33000(\lambda_2 - q_1)}. \quad . \quad . \quad (376)$$

The power thus calculated must be multiplied by a factor to be found by experiment in order to find the actual power of the compressor. Allowance must be made for friction to find the indicated power of the steam-engine which drives the motor; for this purpose it will be sufficient to add ten or fifteen per cent of the power of the compressor.

The heat in the fluid discharged by compressor is equal to the sum of the heat brought from the vaporizing-coils and the

heat equivalent of the work of the compressor. The heat that must be carried away by the cooling water per minute is consequently

$$Q_2 = M(\lambda_2 - q_1) + M\{c_p(t_s - t_1) + \lambda_1 - \lambda_2;$$

$$\therefore\ Q_2 = M\{c_p(t_s - t_1) + r_1\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (376)$$

where r_1 is the heat of vaporization at the pressure p_1.

If the cooling water has the initial temperature t_i and the final temperature t_k, and if q_i and q_k are the corresponding heats of the liquid for water, then the weight of cooling water used per minute will be

$$G = \frac{Mc_p(t_s - t_1) + r_1}{q_k - q_i}. \quad . \quad . \quad . \quad . \quad (377)$$

If the vapor at the beginning of compression can be assumed to be dry and saturated, then the volume of the piston displacement of a compressor without clearance, and making N strokes per minute, is

$$D = \frac{Mv_2}{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (378)$$

To allow for clearance, the volume thus found may be divided by the factor

$$1 - \frac{1}{m}\left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} + \frac{1}{m},$$

as is explained on page 447. The volume thus found is further to be multiplied by a factor to allow for inaccuracies and imperfections.

The vapors used in compression-machines are liable to be mingled with air or moisture, and in such case the performance of the machine is impaired. To allow for such action the size and power of the machine must be increased in practice above

those given by calculation. Proper precautions ought to be taken to prevent such action from becoming of importance.

Calculation for a Compression Refrigerating-machine.— Let it be required to find the dimensions and power for an ammonia refrigerating-machine to produce 2000 pounds of ice per hour from water at 80° F. Let the temperature of the brine in the freezing-tank be 15° F., and the temperature in the condenser be 85° F. Assume that the machine will have one double-acting compressor, and that it will make 80 revolutions per minute.

The heat of the liquid at 80° F. is 48.09 B. T. U., and the heat of liquefaction of ice is 144, so that the heat which must be withdrawn to cool and freeze one pound of water will be

$$48.09 + 144. = 192.09 \text{ B. T. U.}$$

If we allow 50 per cent loss for radiation, conduction, and melting the ice from the freezing-cans, the heat which the machine must withdraw for each pound of ice will be about 300 B. T. U.; consequently the capacity of the machine will be

$$Q_1 = 2000 \times 300 \div 60 = 10000 \text{ B. T. U. per minute.}$$

The pressures corresponding to 15° and 85° F. are 42.43 and 165.47 pounds absolute per square inch, so that by equation (374)

$$T_s = T_2\left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} = (15 + 460.7)\left(\frac{165.47}{42.43}\right)^{\frac{1}{4}} = 668.5.$$

$$\therefore\ t_s = 668.5 - 460.7 = 207°.8 \text{ F.}$$

The-horse-power of the compressor is

$$P_c = \frac{778 Q_1 \{C_p(t_s - t_1) + \lambda_1 - \lambda_2\}}{33000(\lambda_2 - q_1)}$$

$$= \frac{778 \times 10000\{0.50836(207.8 - 85) + 556 - 535\}}{33000(535 - 58)} = 41.2.$$

If we allow 10 per cent for imperfections the compressor will require 45 horse-power. If further 15 per cent is allowed for friction the steam-engine must develop 53 horse-power.

From equation (374) the weight of ammonia used per minute is

$$M = Q_1 \div (\lambda_2 - q_1) = 10000 \div (535 - 58) = 21 \text{ pounds};$$

and by equation (378) the piston displacement for the compressor will be

$$D = \frac{Mv_2}{N} = \frac{21 \times 6.93}{2 \times 80} = 0.91 \text{ cubic feet.}$$

If ten per cent is allowed for clearance and imperfect valve action the piston displacement will be one cubic foot, and the diameter may be made 10½ inches and the stroke 20 inches.

Fluids Available.—The fluids that have been used in compression refrigerating-machines are ether, sulphurous acid, ammonia, and a mixture of sulphurous acid and carbonic acid, known as Pictet's fluid. The pressures of the vapors of these fluids at several temperatures, and also the pressure of the vapors of methylic ether and carbonic acid, are given in the following table:

PRESSURES OF VAPORS, MM. OF MERCURY.

Temperatures, degrees Centigrade.	Ether.	Sulphur Dioxide.	Methyl-ether.	Ammonia.	Carbon Dioxide.	Pictet's Fluid.
— 30	.....	287.5	576.5	866.1	.....	585
— 20	68.9	479.5	882.0	1392.1	15142	745
— 10	114.7	762.5	1306.6	2144.6	20340	1018
0	184.4	1165.1	1879.0	3183.3	26907	1391
10	286.8	1719.6	2629.0	4574.0	34999	1938
20	432.8	2462.1	3586.0	6387.8	44717	2584
30	634.8	3431.8	4778.0	8701.0	56119	3382
40	907.0	4670.2	.....	11595.3	69184	4347

Ether was used in the early compression-machines, but at the temperatures maintained in the refrigerator the pressure is small and the specific volume large, so that the machines, like air refrigerating-machines, were either feeble or bulky. More-

over, air was liable to leak into the machine and unduly heat the compressor cylinder. Sulphur dioxide has been used successfully, but it has the disadvantage that sulphuric acid may be formed by the leakage of moisture into the machine, in which case rapid corrosion occurs. Ammonia has been extensively used in the more recent machines with good results. When distilled from an aqueous solution it is liable to contain considerable moisture. As is shown by the table, Pictet's fluid has a pressure at low temperature intermediate between the pressures of sulphur dioxide and ammonia, and the pressure increases slowly with the temperature. It has been used by the inventor only, and does not appear in practice to have any advantage over ammonia.

Absorption Refrigerating Apparatus.—Fig. 96 gives an ideal diagram of a continuous absorption refrigerating apparatus. It consists of the following essential parts: (1) the generator B, containing a concentrated solution of ammonia in water, from which the ammonia is driven by heat; (2) the condenser C, consisting of a coil of pipe in a tank, through which cold water is circulated; (3) the valve V, for regulating the pressures in C and in I; (4) the refrigerator I, consisting of a coil of pipe in a tank containing a non-freezing salt solution; (5) the absorber A, containing a dilute solution of ammonia, in which the vapor of ammonia is absorbed; and (6) the pump P for transferring the solution from the bottom of A to the top of B; there is also a pipe connecting the bottom of B with the top of A. It is apparent that the condenser and refrigerator or vaporizer correspond to the parts B and C of Fig. 95, and that the absorber and generator take the place of the compressor. The pipes connecting A and B are arranged to take the most concentrated solution from A to B, and to return the solution from which the ammonia has been driven, from B to A. In practice the generator B is placed over a furnace, or is heated by a coil of steam-pipe, to drive off the ammonia. Also, arrangements are made for transferring heat from the hot liquid flowing from B to A to

the cold liquid flowing from *A* to *B*. As the ammonia is distilled from water in *B* the vapor driven off contains some moisture, which causes an unavoidable loss of efficiency.

The earliest absorption apparatus, made by Carré, consisted of a cylindrical receptacle containing a solution of ammonia, and acting alternately as generator and absorber, in open communication through a pipe with a vessel of double

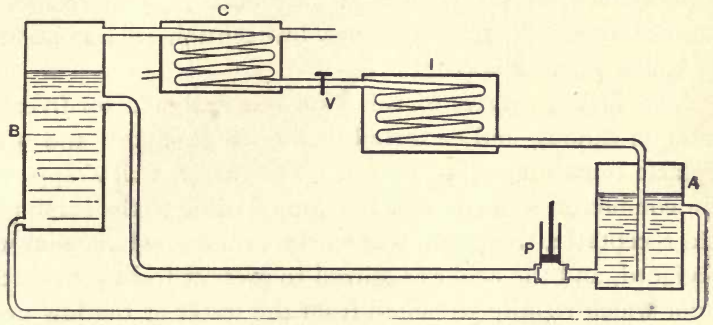

FIG. 96.

conical form, acting alternately as condenser and refrigerator. In use, the generator was placed on a furnace and the condenser in a tank of cold water, and the ammonia driven off from the solution condensed between the inner and outer conical surfaces of the condenser. When a sufficient amount of liquid ammonia had collected, the vessel containing the solution was transferred from the furnace to the cold-water tank, and became thereby changed into the absorber. The condenser at the same time became the vaporizer or refrigerator, and after receiving a mould containing water to be frozen, was securely wrapped with non-conducting material. Apparatus of this kind is only fitted for work on a small scale, and is inefficient.

An adaptation of Carré's apparatus has been used in refrigerator-cars for carrying perishable freight. In the car are placed two receptacles—one containing liquid ammonia, which maintains a low temperature by vaporization; and the other

containing water, to absorb the ammonia as it formed. At the end of the route, or when necessary, the receptacles are recharged—one with liquid ammonia and the other with fresh water. The ammonia in the rejected solution is regained by distillation.

Vacuum Refrigerating Apparatus.—A form of absorption apparatus uses water for the volatile liquid and concentrated sulphuric acid for the absorbent. From the fact that vapor of water at freezing-point has a very low tension such apparatus are called vacuum apparatus.

The first apparatus of this kind was designed for freezing water in carafes, and consisted of a good air-pump and a receptacle containing oil of vitriol. The carafe, well wrapped in non-conductor, was attached to a pipe leading to the sulphuric-acid receptacle, the pump was worked till a good vacuum was produced, and the acid was stirred to present fresh acid to the vapor which rapidly streamed from the water at the low pressure produced. The vaporization of about one-sixth of the weight of the water was found to be sufficient to freeze the remainder.

An ideal sketch of a continuous vacuum apparatus is shown by Fig. 97. At B is an air-pump capable of producing a vacuum of one or two mm. of mercury in the chamber AC. At H there is a tank of concentrated sulphuric acid, from which a spray is delivered at J. The acid absorbs the vapor found in the chamber at the low pressure existing there, gathers in the tank J, and flows out through the pipe K, which is of sufficient length to deliver the acid against atmospheric pressure in the tank L. The dilute acid is reconcentrated and returned to the tank H. At G is a pipe supplying fresh water which passes through the water-injector s and throws a jet of salt solution into the chamber at A. The finely divided jet loses fresh water by vaporization, is chilled, and gathers in the bottom of the chamber. The salt solution flows through the pipe F in the cold-chamber EE, taking up heat on the way, and is again thrown into the chamber with a fresh supply of water

from the pipe *G*. At *N* and *N* are screens to prevent splashing of water into the upper part of the chamber.

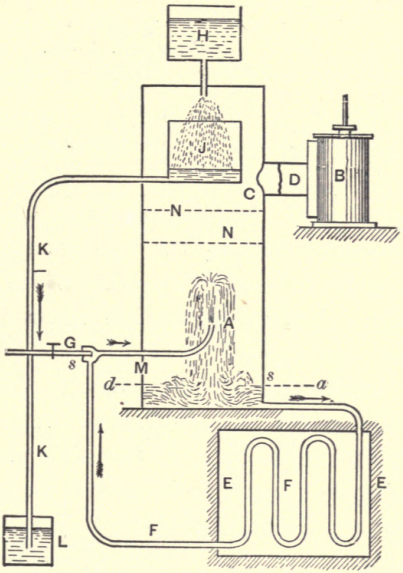

FIG. 97.

Tests of an Air Refrigerating-machine.—An air refrigerating-machine, constructed under the Bell-Coleman patent, was tested by Professor Schröter * at an abattoir in Hamburg, where it was used to maintain a low temperature in a storage-room. The machine is horizontal, and has the pistons for the expansion- and compression-cylinders on one piston-rod, the expansion-cylinder being nearer the crank. Power is furnished by a steam-engine acting on a crank at the other end of the main shaft and at right angles to the crank driving the air-pistons. Both the steam-cylinder and the expansion-cylinder have distribution slide-valves, with independent cut-off valves. The main dimensions are given in the following table:

* *Untersuchungen an Kältemachinen*, 1887.

DIMENSIONS BELL-COLEMAN MACHINE.

	Steam Cylinder.		Compression Cylinder.		Expansion Cylinder.	
	Head end.	Crank end.	Head end.	Crank end.	Head end.	Crank end.
Diameter of piston, cm.	53	53	71	71	53	53
" " piston-rod, cm.	8.1	6.9	9.0	6.8	9.0	9.0
Stroke, m	0.605	0.605	0.605	0.605	0.605	0.605
Clearance, per cent of piston displacement.	5.9	5.8	1.4	1.4	8.9	8.9

Water is sprayed into the compression-cylinder, and the air is further cooled by passing through an apparatus resembling a steam-engine jet-condenser, after which it is dried by passing it through a system of pipes in the cold-room before it passes to the expansion-cylinder.

In the tests indicators were attached to each end of the several cylinders, and the temperature of the air was taken at entrance to and exit from each of the air-cylinders. Specimens of the indicator-diagrams from the air-cylinders show for the compressor a slight reduction of pressure during admission and some irregularity during expulsion, and for the expansion-cylinder a little wire-drawing at cut-off, and a good expansion and compression, though neither is complete. No attempt was made to measure the amount and temperatures of the cooling water.

The data and results of the tests and the calculations are given in Table LI.

Tests of Compression-machines.—In Table LII are given the data and results of tests on three refrigerating-machines on the Linde system using ammonia, and of a machine on Pictet's system using Pictet's fluid, all by Professor Schröter. The tests on machines used for making ice were necessarily of considerable length, but the tests on machines used for cooling liquids were of shorter duration.

The cooling water when measured was gauged on a weir or through an orifice. In the tests 3 to 6 on a machine used for cooling fresh water the heat withdrawn was determined by

TABLE LI.

TESTS ON BELL-COLEMAN MACHINE.

Number of Test.	I.	II.	III.
Duration in hours	6	1.63	2.92
Revolutions per minute	65.05	61.2	63.5
Temperatures of air, degrees Centigrade:			
At entrance to compression cylinder	19.3	17.5	19.1
At exit from " "	27.3	26.8	27.2
At entrance to expansion "	19.0	16.6	19.1
At exit from " "	− 47.0	− 47.0	− 47 0
Mean effective pressure, kgs. per sq. cm.:			
Steam-cylinder: head end	2.263	2.336	2.343
crank end	2.239	2.294	2.301
Compression cylinder: head end	1.900	1.861	1.879
crank end	1.869	1.825	1.906
Expansion cylinder: head end	1.592	1.589	1.626
crank end	1.615	1.594	1.624
Indicated horse-power:			
Steam-cylinder	85.12	82.35	85.71
Compression cylinder	128.85	118.55	126.01
Expansion cylinder	60.10	56.12	59.46
Mean pressure during expulsion from compression cylinder, kgs.	3.35	3.25	3.40
Mean pressure during admission to expansion cylinder, kgs.	2.82	2.83	2.84
Difference	0.53	0.42	0.56
Calculation from compression diagram:			
Absolute pressure at end of stroke, kgs	1.04	1.04	1.04
Absolute pressure at opening of admission-valve, kg.:			
Head end	0.783	0.788	0.764
Crank end	0.765	0.749	0.765
Volume at admission, per cent of piston displacement:			
Head end	6.15	5.95	6.03
Crank end	8.50	8.41	7.91
Weight of air discharged per stroke, kg.:			
Head end	0.2744	0.2764	0.2750
Crank end	0.2716	0.2742	0.2730
Weight of air discharged per revolution, kg	0.546	0.551	0.548
Calculation from expansion diagram:			
Absolute pressure at release, kgs.:			
Head end	1.32	1.31	1.33
Crank end	1.45	1.44	1.46
Absolute pressure at compression, kgs.:			
Head end	1.14	1.14	1.17
Crank end	1.20	1.19	1.22
Volume at release, per cent of p. d.:			
Head end	104.65	104.7	104.8
Crank end	106.1	106.3	106.4
Volume at compression, per cent of p. d.:			
Head end	16.5	16.0	16.6
Crank end	19.8	19.6	20.6
Air used per stroke, kg.:			
Head end	0.234	0.233	0.238
Crank end	0.254	0.254	0.255
Air used per revolution	0.488	0.487	0.493
Difference of weights, calculated by compression and expansion diagrams, kg	0.058	0.064	0.055
In per cent of the former	10.6	11.6	10.0
Mean weight of air per revolution, kg	0.514	0.519	0.520
Elevation of temperature at constant pressure, degrees Centigrade	66.3	64.5	66.1
Heat withdrawn per H. P. per hour, calories	371	354	363

taking the temperatures of the water cooled, and by gauging the flow through an orifice, for which the coefficient of flow was determined by direct experiment. The heat withdrawn in the tests 7 and 8 was estimated by comparison with the tests 3 to 6. The net production of ice in the tests 1 and 2 was

TABLE LII.

TESTS ON COMPRESSION REFRIGERATING-MACHINES.

By Professor SCHRÖTER, *Untersuchungen an Kältemaschinen.*

Number.	System of the machine.	Application.	Dimensions of the steam cylinder.			Dimensions of the compression cylinder.			Duration of test.	Revolutions per minute, steam engine.	Mean effective pressure. Steam cylinder, kgs. per sq. cm.		Indicated horse-power of steam cylinder.	Revolutions per minute, compressor.	Mean effective pressure compressor, kgs. per sq. cm.	
			Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.	Diameter of piston, mm.	Diameter of piston-rod, mm.	Stroke, mm.			Head end.	Crank end.			Head end.	Crank end.
1	Linde.	Ice-making.	371.25	55.5	800	325	63	540	36 min.	68.70	2.13	1.97	53.6	64.8	....	....
2	"	"	"	"	"	"	"	"	34 "	65.40	2.94	2.38	66.1	59.8	4.01	3.84
3	"	Cooling fresh water.	400	....	602	250	55	420	106 "	....	....	....	....	54.7	5.29	5.45
4	"	"	"	....	"	"	"	"	50 "	....	....	....	....	55.1	5.42	5.66
5	"	"	"	....	"	"	"	"	46 "	....	....	....	....	59.1	5.44	5.62
6	"	"	"	....	"	"	"	"	35 "	....	....	....	....	49.6	5.46	5.58
7	"	Cooling brine.	330	52	740	"	"	"	3 hours.	....	....	....	26.1	65.15	3.08	3.12
8	"	"	"	"	"	"	"	"	3 "	....	....	....	34.5	65.8	4.24	4.28
9	Pictet.	Ice-making.	450	68	900	430	65	900	8.5 "	64.2	2.19	2.38	91.2	64.2	1.33	1.49
10	"	"	"	"	"	"	"	"	11.08 "	64.7	2.28	2.42	94.5	64.7	1.55	1.77
11	"	"	"	"	"	"	"	"	9.83 "	64.5	2.41	2.53	99.2	64.5	1.70	1.87
12	"	"	"	"	"	"	"	"	4.00 "	64.0	....	....	....	64.0	1.91	2.17

TABLE LII.—*Continued.*

Number.	Indicated horse-power of compressor.	Absolute pressures of vapor, kilos. per sq. centimeter.				Cooling water.				Ice produced per hour.					Temperature of water or brine cooled.		Heat withdrawn calories.	
		In compressor during expulsion.	In condenser.	In compressor during admission.	In vaporizer.	Cubic meters per hour.	Initial temperature.	Final temperature.	Heat carried away per hour.	Temperature of water supplied, degrees C.	Gross product per hour, kilos.	Net product per hour, kilos.	Per compressor horse-power, per hour, gross, kilos.	Per compressor horse-power, per hour, net, kilos.	At entrance.	At exit.	Heat withdrawn per hour.	Per compressor horse-power per hour.
1	....	....	6.99	....	2.76	....	....	....	....	9.0	1803	1663	....	...	−4.4	−4.4	162631	4444
2	45.9	9.58	9.31	2.50	2.64	....	....	....	....	8.3	1597	1456	34.8	31.7	−5.9	−5.9	144209	3120
3	26.27	....	13.66	....	4.85 4.55	....	11.19	22.56	....	....	....	....	....	....	11.19	2.95	85339	3249
4	27.30	....	14.06	....	4.90 4.53	9.179	11.2	23.58	113819	....	...	....	....	....	11.2	2.38	91620	3367
5	29.23	....	14.11	....	4.91 4.55	....	11.2	23.04	....	....	...	....	....	....	11.2	2.24	89777	3072
6	24.49	....	13.78	....	4.27 4.83	6.097	11.1	26.10	91455	....	....	....	....	....	11.1	4.71	79909	3263
7	18.1	8.13	7.87	2.36	2.63	18.30	8.77	12.41	66630	.. .	....	....	....	....	−9.50	−9.97	66680	3684
8	25.8	10.68	10.41	2.97	3.24	6.958	8.82	20.45	80921	....	....	....	....	....	−3.1	−4.1	80624	3086
9	52.01	3.77	3.22	0.45	0.82	30.7	10.15	14.0	118195	11.3	876	789	16.8	15.2	−18.2	−18.2	87074	1674
10	61.70	4.11	3.50	0.63	1.03	30.7	10.1	15.95	179595	11.3	1544	1396	25.0	22.6	−10.0	−10.0	147143	2385
11	66.42	4.23	3.62	0.73	1.15	30.7	10.15	17.20	216435	11.3	1868	1722	28.2	25.9	−9.7	−9.7	175225	2638
12	75.02	5.81	5.11	0.67	1.06	8.36	10.3	31.1	173888	11.3	1545	1391	20.6	18.5	−6.05	−6.05	146929	1958

determined directly; and in the test 2 the loss from melting during the removal from the moulds was found by direct experiment to be 8.45 per cent. By comparison with this the loss by melting in the first test was estimated to be 7.7 per cent. The gross production of ice in the refrigerator was calculated from the net production by aid of these figures. In the tests 9 to 12 on the Pictet machine the gross production was determined from the weight of water supplied, and the net production from the weight of ice withdrawn.

A separate experiment on the machine used for cooling brine gave the following results for the distribution of power:

Total horse-power	57.1
Power expended on compressor	19.5
" " " centrifugal pump	9.8
" " " water-pump	3.6

The centrifugal pump was used for circulating the brine through a system of pipes used for cooling a cellar of a brewery. The water-pump supplied cooling water to the condenser and for other purposes.

A similar test on the Pictet machine gave:

Power of engine alone	7.9 H. P.
" " " and intermediate gear	16.6 "
" " " gear, and pump	20.0 "

From the above data the following table was arranged for the several tests on this machine:

INDICATED AND EFFECTIVE WORK.

Number of Test.	9	10	11	12
Indicated work without compressor	19.9	20.0	20.0	19.9
" " engine alone	7.9	8.0	7.9	7.9
Effective work of steam-engine	77.1	80.1	84.6	94.4
Indicated work of steam-engine	91.2	94.5	99.2	109.8
Mechanical efficiency of steam-engine	0.84	0.85	0.85	0.85
Power absorbed by intermediate gearing	11.2	11.2	11.2	11.2
Power absorbed by compressor	65.9(?)	68.9	73.4	83.2
Indicated power of compressor	52.0	61.7	66.4	75.0
Mechanical efficiency of compressor	0.79(?)	0.89	0.90	0.90

In 1888 comparative tests were made by Professor Schröter on a Linde and on a Pictet refrigerating-machine, in a special building provided by the Linde Company which had every convenience and facility for exact work. The following table gives the principal dimensions of the machines:

PRINCIPAL DIMENSIONS OF LINDE AND PICTET REFRIGERATING-MACHINES.

	Linde.	Pictet.
Diameter of steam-cylinder, cm	30.55	31.63
do. compressor-cylinder, cm	25.03	28.6
do. steam piston-rod, cm	4.85	5
do. compressor-rod, cm	5.5	5
Stroke of steam-piston, cm	70	62
do. compressor, cm	42	62
Diameter of pipe in vaporizers, external, mm	40.5	44
do. internal, mm	32	36
Length of pipe in first vaporizer, m	556.5	538.2
do. second vaporizer, m	558.5	538.2
Diameter of pipe in condenser, external, mm	38.5	44
do. internal, mm	30	36
Length of pipe in condenser, m	556.2	483.1

The Linde machine used ammonia and was allowed to draw a mixture of liquid and vapor into the compressor, so that no water-jacket was required. The Pictet machine used Pictet's fluid and had the compressor cooled by a water-jacket.

The data and results of the tests are given in Tables LIII and LIV. Five tests on each machine were made with only one of the two vaporizers in use, and three were made with both in use. The temperature of the salt solution or brine, from which heat was withdrawn by the vaporizers, was allowed to vary about three degrees centigrade from entrance to exit. The entrance temperatures were intended to be 6° C., —2° C., — 10° C., and — 18° C. The cooling water was supplied to the condenser at about 9°.5 C. for all tests, and for all but one it left the condenser with a temperature of nearly 20° C.; the fifth test on each machine was made with the exit temperature of the cooling water at about 35° C.

TABLE LIII.

TESTS ON LINDE AMMONIA REFRIGERATING-MACHINE.

By Professor M. SCHRÖTER, *Vergleichende Versuche an Kältemaschinen*, 1890.

	One vaporizer.					Two vaporizers.		
	I	II	III	IV	V	VI	VII	VIII
Steam-engine:								
Revolutions per minute	44.9	45.1	45.1	44.8	45.0	45.2	45.1	44.7
Horse-power	18.14	18.26	17.03	15.70	24.41	18.13	18.27	15.86
Compressor:								
Horse-power	15.53	15.20	14.31	12 63	21.86	16.20	16.06	13.00
Heat-equivalent, calories per hour	9890	9690	9110	8040	13920	10320	10230	8280
Mechanical efficiency	0.856	0.833	0.840	0.805	0.895	0.894	0.879	0.820
Pressure in condenser, kilograms per square centimetre	9.52	9.24	9.000	8.89	14.03	9.55	9.24	8.83
do. vaporizer, kilograms per square centimetre	3.89	2.95	2.13	1.56	2.95	4.23	3.17	1.68
Vaporizer:								
Mean temperature of brine, entrance	6.00	−2.02	− 9.99	−17.92	−2.03	+6.49	−2.04	−18.00
do. exit	2.89	−5.02	−12.91	−20.82	−5.01	+2.91	−5.01	−20.93
Brine per hour, litres	29569	22893	16168	11030	19598	29247	25567	11884
Specific heat per litre	0.850	0.846	0.843	0.840	0.845	0.850	0.848	0.848
Calories yielded per hour	78170	58100	39800	26870	49350	89000	64390	29530
Initial temperature of brine, entrance	5.98	−2.05	− 9.95	−17.97	−2.03	+6.51	−2.10	−18.06
do. exit	2.89	−5.02	−12.94	−20.83	−5.00	+2.91	−5.02	−20.96
Final temperature of brine, entrance	5.97	2.04	9.97	17.96	2.03	6.53	2.11	18.07
do. exit	2.94	5.04	12.89	20.83	5.01	2.91	5.01	20.94
Correction for change of brine temperature	−30	+10	−20	−10	+10	−20	0	−10
Calories per hour, corrected	78140	58110	39780	26860	49360	88980	64390	29520
Condenser:								
Mean temperature of cooling-water, entrance	9.56	9.54	9.61	9.61	9.68	9.23	9.00	9.08
do. exit	19.76	19.63	19.84	19.72	35.33	19.68	19.62	19.81
Cooling-water per hour, litres	8790	6837	4867	3526	2442	9512	6975	3556
Calories abstracted per hour	89660	68990	49790	35650	62640	99400	74070	38160
Initial temperature of water, entrance	9.56	9.55	9.61	9.64	9.68	9.23	8.95	9.11
do. exit	19.74	19.42	19.82	19.79	35.45	19.57	19.62	19.72
Final temperature of water, entrance	9.57	9.54	9.60	9.56	9.65	9 23	9.03	9.07
do. exit	19.74	19.45	19.89	19.88	35.44	19.54	19.62	19.69
Correction for cooling-water	+10	+10	+30	+10	−20	−20	+40	−40
Calories per hour, corrected	89670	69000	49820	35660	62620	99380	74110	38120
Error in heat account, per cent	−1.8	−1.8	−1.9	−2.1	+1	−0.1	+0.7	−0.8
Refrigerative effect, calories per horse-power per hour	4308	3182	2336	1711	2022	4908	3524	1861

TABLE LIV.

TESTS ON PICTET REFRIGERATING-MACHINE.

By Professor M. SCHRÖTER, *Vergleichende Versuche an Kältemaschinen.*

	One vaporizer.					Two vaporizers.		
	I	II	III	IV	V	VI	VII	VIII
Steam-engine:								
Revolutions per minute	57.0	56.8	57.1	57.6	59.3	57.3	57.5	57.8
Indicated horse-power	21.81	20.88	18.75	15.93	27.56	21.87	20.74	16.15
Compressor:								
Horse-power	16.82	16.10	14.26	11.83	22.91	18.81	18.08	12.74
Heat equivalent, calories per hour	10710	10250	9080	7530	14590	11980	11510	8110
Mechanical efficiency	0.771	0.771	0.761	0.743	0.831	0.860	0.872	0.789
Pressure in condenser, kilograms per square centimetre	3.99	3.91	3.84	4.25	6.39	4.29	4.17	4.13
do. vaporizer, kilograms per square centimetre	1.47	1.05	0.68	0.17	1.05	1.57	1.10	0.47
Vaporizer:								
Mean temperature of brine, entrance	6.10	−1.96	− 9.92	−17.93	−2.04	6.38	−2.00	−18.01
do. exit	3.08	−4 98	−12.91	−20.96	−5 01	3.07	−5.00	−20.90
Brine per hour, litres	29837	20848	13740	6723	18669	29802	22711	8405
Specific heat per litre	0.850	0.847	0.845	0.841	0.846	0.850	0.848	0.847
Calories yielded per hour	76590	53330	34720	17130	46910	83850	57780	20570
Initial temperature of brine, entrance	6.09	−2.02	− 9.91	−18.00	−1.99	6.31	−1.89	−18.15
do. exit	3.03	4.99	−12.91	−21.00	−5.02	3.07	−5.04	−20.92
Final temperature of brine, entrance	6.11	−2.04	− 9.94	−18.00	−2.05	6.33	−1.90	−18.14
do. exit	3.05	−4.98	−12.88	−21.00	−4.96	3.03	−5.00	−20.88
Correction for change of brine temperature	−100	+30	0	0	0	+60	−80	−140
Calories per hour, corrected	76490	53360	34720	17130	46910	83910	57700	20430
Condenser:								
Mean temperature of cooling-water, entrance	9.65	9.60	9.61	9.68	9.68	9.62	9.57	9.78
do. from condenser	19.72	19.70	19.59	19.51	35.18	19.65	19.60	19.92
do. from jacket	15.5	15.6	16.8	16.7	18.6	16.1	15.9	17.1
Cooling water per hour, litres, condenser	8356	6020	4058	2228	2068	9198	6498	2625
do. jacket	427	429	431	426	427	423	426	428
Calories abstracted per hour	86620	63370	43600	24880	56530	95010	67850	29740
Initial temperature of condensing-water, entrance	9.57	9.64	9.58	9.68	9.73	9.64	9.59	9.83
do. exit	19.71	19.72	19.37	19.52	35.08	19.71	19.92	19.92
Final temperature of condensing-water, entrance	9.67	9.57	9.61	9.72	9.72	9.61	9.59	9.78
do. exit	19.71	19.64	19.35	19.59	35.01	19.71	19.87	19.82
Correction for cooling water	+100	−160	+10	+110	−80	−30	−50	−160
Calories per hour, corrected	86720	63210	43610	24990	56450	94980	67800	29580
Error in heat account, per cent	+0.6	+0.6	+0.4	−1.3	+8.9	+1.0	+2.1	−3.5
Refrigerative effect, calories per horse-power per hour	3507	2556	1852	1075	1702	3837	2782	1265

The pressure in the compressor depended, of course, on the temperatures of the brine and the cooling-water. For all the tests except the fifth on each machine, the maximum pressure of the working substance was nearly constant: about 9 kilograms per square centimetre for ammonia and about 4 kilograms for Pectet's fluid. The fifth test had considerably higher pressure, corresponding to the higher temperature in the condenser. The minimum pressure of the working substance of course diminished as the brine temperature fell.

The heat yielded per hour to the ammonia in the vaporizer was calculated by multiplying together the amount of brine used in an hour, the specific heat of the brine, and its increase of temperature. But the initial and final temperatures were not quite constant, and so a correction was applied as indicated in the tables. The heat abstracted from the ammonia in the condenser was calculated from the water used per hour and its increase of temperature. The calculation for Pectet's machine involves also the jacket-water and its increase of temperature. A correction is applied for the variations of initial and final temperatures of the cooling-water. If the heat equivalent of the work of the compressor is added to the heat yielded by the vaporizer the sum should be equal to the heat abstracted by the cooling-water. The per cent of difference between these two calculations of the heat abstracted by the cooling-water is a measure of the accuracy of the tests.

The refrigerative effect is obtained by dividing the heat yielded by the vaporizer by the horse-power of the steam-cylinder. The first four tests with constant temperature in the condenser show a regular decrease in the refrigerative effect for each machine as the temperature of the brine and the minimum pressure of the working substance is reduced. The three tests with the entire vaporizing surface in use show a like result. The fifth test, with a higher temperature in the condenser, shows a less refrigerative effect than the second test, which has nearly the same brine temperatures. These results

are in concordance with the idea that a refrigerating-machine is a reversed heat-engine; for a heat-engine will have a higher efficiency and will use less heat per horse-power when the range of temperatures is increased, and *per contra*, a refrigerating-machine will be able to transfer less heat per horse-power as the range of temperatures is increased.

Table LV gives the data and results of tests made by Professor Denton on an ammonia refrigerating-machine. The only items requiring explanation are the refrigerative effect

TABLE LV.

TESTS ON AMMONIA REFRIGERATING-MACHINE.

By Professor J. E. DENTON, *Trans. Am. Soc. Mech. Engr.*, vol. xii, p. 326.

	I	II	III	IV
Pressure above atmosphere, pounds per square inch:				
Ammonia from compressor	151	152	147	161
Ammonia back-pressure	28	8.2	13	27.5
Barometer, inches of mercury	30.07	29.59	29.87	30.01
Temperature, degrees Fahrenheit:				
Brine, inlet	36.76	6.27	14.29	
do. outlet	28.86	2.03	2.29	28.45
Condensing-water, inlet	44.65	56.65	46.9	54.00
do. outlet	83.66	85.4	85.46	82.86
Jacket-water, inlet	44.65	56.7	46.9	54.3
Ammonia-vapor leaving brine-tank	34.2	14.7	3.0	29.2
do. entering compressor	39	25	10.13	34
do. leaving compressor	213	263	239	221
do. do. calculated	229	304	260	237
do. entering condenser	200	218	209	168
Brine, pounds per minute	2281	2173	942.8	2374
Specific gravity	1.163	1.174	1.174	1.174
Specific heat	0.82	0.78	0.78	0.783
Ammonia, lbs. per min. by metre		14.68	16.67	28.32
do. from compressor displacement			22.10	34.51
Heat account, B. T. U. per minute:				
Given to ammonia by brine	14776	71876	8824	14647
do. compressor	27860	2320	2518	3020
do. atmosphere	140	147	167	141
Total received by ammonia	17702	9653	11409	17708
Taken from ammonia by condenser	17242	9056	9910	17359
do. jackets	608	712	656	406
do. atmosphere	182	338	250	252
Total taken from ammonia	18032	10106	10816	18017
Error, per cent	2	5	3.5	2
Power, etc.:				
Revolutions per minute	58.09	57.7	57.88	58.89
Horse-power steam-cylinder	85.0	71.7	73.6	88.6
do. compressor	65.7	54.7	59.4	71.2
Mechanical efficiency	0.81	0.83	0.86	0.83
Refrigerative effect:				
Tons of ice (melted) in 24 hours	74.8	36.43	44.64	74.56
B. T. U. abstracted from brine per horse-power, minutes	174	197	197	196
Pounds of ice (melted) per pound of coal	24.1	14.1	17.27	23.37

and the calculated temperature of the vapor leaving the condenser; the latter was calculated by the equation

$$T_s = T_2 \left(\frac{p_1}{p_2}\right)^{0.24}$$

and shows both the cooling effect of the jacket and the error in assuming an adiabatic compression. The refrigerative effect was obtained by dividing the B.T.U. given to the ammonia in a minute by the horse-power of the steam-cylinder. The tons per horse-power in 24 hours was obtained by multiplying the refrigerative effect in thermal units per minute by the number of minutes in a day and then dividing the product by 2000 (the pounds in a short ton) and by 144 (the heat of melting a pound of ice). The pounds of ice per pound of coal was based on an assumed consumption of three pounds of coal per horse-power per hour, and was calculated by multiplying the B.T.U. per horse-power per minute by 60 and dividing by 3×144.

The main dimensions of the machine were:

Diameter of ammonia cylinder (single-acting)	12 inches.
Stroke of ammonia cylinder (single-acting)	30 "
Diameter of steam-cylinder	18 "
Stroke of steam-cylinder	36 "
Diameter of pipe for vaporizer and condenser	1 "
Length of pipe in vaporizer	8000 feet.
do. condenser	5000 "

Test of an Absorption-machine.—The principal data and the results of a test made by Professor J. E. Denton* on an absorption ammonia refrigerating-machine are given in Table LVI. The machine is applied to chill a room of about 400,000 cubic feet capacity at a pork-packing establishment at New Haven, Conn. In connection with this test the specific heat of the brine, which served as a carrier of heat from the cold room to the ammonia, was determined by direct

* *Trans. Am. Soc. Mech. Eng.*, vol. x, May, 1889.

TABLE LVI.

TEST OF AN ABSORPTION-MACHINE.

SEVEN DAYS' CONTINUOUS TEST, SEPT. 11–18, 1888.

Average pressures above atmosphere in lbs. per sq. in.	Generator	150.77
	Steam	47.70
	Cooler	23.69
	Absorber	23.4
Average temperatures in Fahrenheit degrees.	Atmosphere in vicinity of machine	80
	Generator	272°
	Brine: Inlet	21.205
	Brine: Outlet	16.16
	Condenser: Inlet	54½
	Condenser: Outlet	80
	Absorber: Inlet	80
	Absorber: Outlet	111
	Heater: Upper outlet to generator	212
	Heater: Lower " " absorber	178
	Heater: Inlet from absorber	132
	Inlet from generator	272°
	Water returned to main boilers from steam coil	260
Average range of temperatures Fahr. degrees.	Condenser	25½
	Absorber	31
	Brine	5.13
Brine circulated per hour.	Cubic feet	1,633.7
	Pounds	119,260
Specific heat of brine		0.800
Cooling capacity of machine in tons of ice per day of 24 hours		40.67
Steam consumption per hour, to volatilize ammonia, and to operate ammonia pump ... lbs.		1,986
British thermal units.	Eliminated: Per pound of brine	4.104
	Eliminated: Total per hour	481,260
	Of refrigerating effect per pound of steam consumption	243
	Rejected: At condenser, per hour	918,000
	Rejected: At absorber "	1,116,000
	Per pound of steam: On entering generator coil	1,203
	Per pound of steam: On leaving generator coil	271
	Consumed by generator per lb. of steam condensed	932
Condensing water per hour, in lbs.		36,000
Equivalent ice production per pound of coal, if one pound of coal evaporates ten pounds of steam at boiler		17.1
Calories, refrigerating effect per kilogramme of steam consumed.		135
Approximate coil surface in sq. ft.	Condensing coil	870
	Absorber "	350
	Steam "	200

experiment. The brine chilled and the cooling water used were measured with meters, which were afterwards tested under the conditions of the experiment.

It is interesting to compare the refrigerative effects expressed in pounds of ice per pound of coal. On this basis the compression-machine tested by Professor Denton has an advantage of

$$\frac{24.1 - 17.1}{24.1} \times 100 = 19 \text{ per cent.}$$

But this comparison is really unfair to the compression-machine, for its steam-engine is assumed to require a consumption of three pounds of coal per horse-power per hour, while the calculation for the absorption-machine is based on the assumption that a pound of coal can evaporate ten pounds of water; but an automatic condensing-engine of the given power should be able to run on 20 or 22 pounds of steam per horse-power per hour.

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